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Models of network excitation control

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Abstract

This paper studies models of centralized, decentralized and distributed control of excitation in a network of interacting purposeful agents. As examples, we analyze models of threshold behavior and mob control.

Keywords: social network, network excitation, threshold behavior, centralized control, decentralized control, distributed control, mob control

1. Introduction

Consider a set of interconnected agents having mutual influence on their decision-making. Variations in the states of some agents at an initial instant accordingly modify the state of other agents. The nature and character of such dynamics depend on the practical interpretation of a corresponding network. Among possible interpretations, we mention propagation of excitation in biological networks (e.g., neural networks [21]), failure models (in the general case, structural dynamics models) in information and control systems and complex engineering systems, models of innovation diffusions, information security models, penetration/infection models, consensus models and others, see an overview in [3].

The control problem of purposeful "excitation" of a network possesses the following statement. Find a set of agents to apply an initial control action such that a network reaches a required state. This abstract setting covers informational control problems in social networks [3], [6], control problems for collective threshold behavior [1], [11], [12], information security problems [3], etc. For definiteness, further exposition runs in terms of social networks.

Let $N = \{1, 2, ..., n\}$ stand for a finite set of *agents*; they form a *social network* described by a directed graph $\Gamma = (N, E)$, where $E \subseteq N \times N$ denotes the set of arcs. Each agent is in one of two states, "0" or "1" (passivity or activity, being unexcited or excited, respectively). Designate by $y_i \in \{0; 1\}$ the state of agent i ($i \in N$) and by $y = (y_1, y_2, \dots, y_n)$ the vector of agents' states. For convenience, transition from passivity to activity is called the "excitation" of an agent.

Assume that, initially, all agents appear passive and the dynamics of the network is described by a mapping $\Phi: 2^N \to 2^N$. Here $\Phi(S) \subseteq N$ indicates the set of agents having state "1" at the end of the transient process caused by network "excitation"; the latter represents the variation (switching from passivity to activity) in the states of agents from the set (*coalition*) $S \subseteq N$, which takes place at the initial instant. We emphasize that control actions are applied singly.

Concerning the mapping $\Phi(\cdot)$, suppose that it enjoys the following properties:

A.1 (reflexivity). $\forall\, S \subseteq N$: $S \subseteq \Phi(S)$;

A.2 (monotonicity). $\forall\, S, U \subseteq N$ such that $S \subseteq U$: $\Phi(S) \subseteq \Phi(U)$.

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A.3. (convexity). $\forall S, U \subseteq N$ such that $S \cap U = \varnothing$: $\Phi(S) \cup \Phi(U) \subseteq \Phi(S \cup U)$.

For a given mapping $\Phi(\cdot)$, it is possible to define a function $\hat{G} = \{0; 1\}^n \to \{0; 1\}^n$, which associates the vector y of initial states of agents with the vector of their final states:

$$\hat{G}_i(y) = \begin{cases} 1, \text{ if } i \in \Phi(\{j \in N \mid y_j = 1\}) \\ 0, \text{ otherwise.} \end{cases}.$$

Similarly, we easily define the states of agents "excited indirectly":

$$G_i(y) = \begin{cases} 1, \text{ if } \hat{G}_i(y) = 1 \text{ and } y_i = 0 \\ 0, \text{ otherwise} \end{cases}, \ i \in N.$$

The paper is organized as follows. Section 2 poses the centralized control problem of network excitation and focuses on a special case with threshold behavior of agents. Section 3 gives a formal statement of the decentralized control problem when agents make independent decisions on their "excitation." Moreover, the issue of implementability of efficient or given states of a network is investigated from the game-theoretic view. The problem of mob control serves as a possible application. The conclusion outlines promising research directions for models of network excitation control.

2. Centralized control problem

Consider given functions C: $2^N \to \Re^1$ and H: $2^N \to \Re^1$. The former characterizes the costs $C(S)$ of initial variation in the states of agents from a coalition $S \in 2^N$, while the latter describes the income $H(W)$ from the resulting "excitation" of a coalition $W \in 2^N$. The subjects incurring these costs depend on a specific statement of the problem.

The goal function of a control subject (a *Principal*) is the difference between the income and the costs. For the Principal, the *centralized control problem* lies in choosing a set of initially excited agents to maximize the goal function $v(S)$:

$$v(S) = H(\Phi(S)) - C(S) \to \max_{S \subseteq N}. \quad (1)$$

In this setting, the Principal incurs the costs and receives the income.

In the general case (without additional assumptions on the properties of the functions $C(\cdot)$ and $H(\cdot)$, and on the mapping $\Phi(\cdot)$), obtaining a solution $S^* \subseteq N$ of the discrete problem (1) requires exhausting all 2^n possible coalitions. The design of efficient solution methods for this problem makes an independent field of investigations (we refer to [17], [23], [29], etc. for several successful statements of optimization problems for system's staff, which employ rather simple algorithms). The state S^* of the social network, maximizing the goal function (1), will be called *efficient*.

A special case engages the cost function and income function being additive with respect to agents:

$$u(S) = \sum_{i \in \Phi(S)} H_i - \sum_{j \in S} c_j, \quad (2)$$

where $(c_i, H_i)_{i \in N}$ are known nonnegative constants. For the time being, we deal with the additive case (2) for simplicity.

In the centralized control problem, agents (network nodes) are passive in some sense. Notably, the Principal "excites" agents from a set S, and then this excitation propagates according to the operator $\Phi(\cdot)$.

Alternative formulations of the control problem are possible, e.g., income maximization under limited costs (the so-called knapsack problem if the cost function and income function enjoy the additive property with respect to agents) or costs minimization for a given income.

Discrete problems of the form (1) demonstrate high computational complexity for large networks (i.e., networks with very many agents). Therefore, in such cases networks are treated as random graphs with specified probabilistic characteristics [1], [1], [7], [8] and the control problem is stated in terms of expected values (e.g., optimization of the expected number of excited agents), see [1].

Example: threshold behavior. Agents in a certain network have mutual *influence* on each other. Arc (i, j) from node i to node j corresponds to the influence of agent i on agent j (we believe that loops are absent). Denote by $N^{in}(i) = \{j \in N \mid \exists$

$(j; i) \in E\}$ the set of "*neighbors*," i.e., agents having an influence on agent i ("initiators"). By analogy, let $N^{out}(i) = \{j \in N \mid \exists (i; j) \in E\}$ designate the set of agents ("followers") being influenced by agent i, $n^{out}(i) = |N^{out}(i)\}$, $n^{in}(i) = |N^{in}(i)|$.

The process of collective decision-making by agents can be described through different models (see the surveys in [1], [3], [15], consensus models [6], de Groot's model [19]). Consider a special case of threshold behavior adopted by agents [1], [9]:

$$y_i^t = \begin{cases} 1, \frac{1}{n-1} \sum_{j \in N^{in}(i)} y_j^{t-1} \geq \theta_i \\ y_i^{t-1}, \text{otherwise} \end{cases}. \tag{3}$$

Here y_i^t is the state of agent i at instant t, and $\theta_i \in [0; 1]$ represents the threshold of this agent, $t = 1, 2, \ldots$ The initial conditions are given: $y_i^0 = y_i$, $i \in N$. The model (3) presupposes that, being once excited (by a control action or under the impact of excited neighbors), an agent never becomes "passive." Obviously, any graph Γ with the dynamics (3) enjoys the following properties. The number of active agents forms a nondecreasing function of time, the transient process terminates at most after n steps, the correspondence between the initial and end states meets A.1-A.3.

It seems interesting to study the special case of *threshold behavior with unity thresholds* ($\theta_i = 1$, $i \in N$). In other words, agent's excitation results from excitation of, at least, one of his initiators. The corresponding mapping $\Phi(\cdot)$ is "*linear*": $\forall$ $S, U \subseteq N$ such that $S \cap U = \varnothing$ we have $\Phi(S) \cup \Phi(U) = \Phi(S \cup U)$, i.e., $\Phi_0(S) = \bigcup_{i \in S} \Phi_0(\{i\})$. The function (2) becomes superadditive, *viz*., $\forall$ $S, U \subseteq N$ such that $S \cap U = \varnothing$ we obtain $u(S \cup U) \leq u(S) + u(U)$.

Let the graph Γ be acyclic and the specific costs of initial excitation of any agent make up c. For each agent $i \in N$, find the set M_i of all his "*indirect followers*," i.e., agents (including agent i) connected to this agent via paths in the graph. Evaluate $h_i = \sum_{j \in M_i} H_j$, which is the payoff from excitation of agent i. Owing to the acyclic property of the graph and homogeneous costs, a reasonable strategy lies in excitation of agents having no initiators (denote the set of such agents by $M \subseteq N$). Then the problem (1) acquires the form

$$\sum_{j \in \bigcup_{i \in S} M_i} H_j - c\,|S| \to \max_{S \subseteq N}.$$

Despite a series of simplifying assumptions (threshold behavior, unit thresholds, homogeneous costs and acyclic graph), the resulting problem of centralized control still admits no analytical solution and requires exhausting all subsets of the set M. It is possible to apply different heuristics, e.g., to believe that the "optimal" excitation covers agents whose payoff exceeds the costs: $S^* = \{i \in M \mid h_i \geq c\}$. Another approach consists in sorting of agents from the set M in the descending order of the quantities h_i and adding them in the desired set starting from agent 1 (until the corresponding increase in the "payoff" becomes smaller than the specific costs).

Therefore, centralized control problems for network excitation admit simple solutions merely in some cases. Now, we concentrate on possible statements and solution methods of *decentralized control problems* when agents make independent decisions on their "excitation."

3. Decentralized control problem

Consider a given *mechanism* (decision-making procedure [23]) $\sigma = \{\sigma_i(G(y)) \geq 0\}_{i \in N}$ of payoff allocation among agents. According to the mechanism σ, additional payoffs can be received only by agents excited at the initial instant. Allocation concerns the "payoff" from indirect excitation of other agents.

Suppose that agents represent active subjects in the following sense. Under the known mechanism σ, at the initial instant they simultaneously and independently choose their states (activity or passivity). Further dynamics of agents' states is still described by the operator $\Phi(\cdot)$.

The goal function of agent i (denoted by $f_i(y)$) makes the difference between his payoff and costs:

$$f_i(y) = \sigma_i(G(y)) + (H_i - c_i)\, y_i,\ i \in N. \tag{4}$$

If an agent makes the decision on his excitation, he incurs excitation costs and obtains the payoff H_i plus an additional incentive from the Principal (according to the payoff allocation mechanism). *Self-excitation* is the profitability of agent's excitation regardless of the payoff allocation mechanism. It follows from (4) that, if $H_i > c_i$, then agent i self-excites. In the sequel, we believe that $c_i > H_i$, $i \in N$, to avoid the "self-excitation" of active agents (except the cases when other conditions are clearly stated).

There exist different payoff allocation mechanisms for agents. Typical examples are *balanced* mechanisms (the total agents' payoff equals the Principal's payoff from "indirect excitation"), namely, the *uniform allocation mechanism*:

$$\sigma_i(G(y)) = \frac{\sum_{j \in N} H_j G_j(y)}{\sum_{j \in N} y_j} y_i, \; i \in N, \tag{5}$$

the mechanism of costs-proportional allocation:

$$\sigma_i(G(y)) = \frac{\sum_{j \in N} H_j G_j(y)}{\sum_{k \in N} c_k y_k} c_i y_i, \; i \in N \tag{6}$$

and *the mechanism of limit contribution-proportional allocation*:

$$\sigma_i(G(y)) = \frac{[\sum_{j \in N} H_j G_j(y) - \sum_{j \in N} H_j G_j(y_{-i}, 0)]}{\sum_{k \in N} y_k [\sum_{j \in N} H_j G_j(y) - \sum_{j \in N} H_j G_j(y_{-k}, 0)]} \sum_{j \in N} H_j G_j(y) y_i, \; i \in N. \tag{7}$$

According to the expressions (4)-(7), the Principal reallocates the payoff from indirect excitation of other agents by initially excited agents among the latter. Moreover, indirect excitation causes no costs.

By definition, a vector y^* is a *Nash equilibrium* [25] in the game of agents, if

$$\sigma_i(G(y^*)) + (H_i - c_i)\, y^*_i \geq \sigma_i(G(y^*_{-i}, 1 - y^*_i)) + (H_i - c_i)\,(1 - y^*_i), \; i \in N, \tag{8}$$

where $y_{-i} = (y_1, \ldots, y_{i-1}, y_{i+1}, \ldots, y_n)$ forms the opponents' action profile of agent i.

In a Nash equilibrium, active agents benefit nothing by switching their state to passivity (provided that other agents keep their states unchanged), whereas passive agents benefit nothing by switching their states to activity. In other words, active agents (i.e., agents $i \in N$ such that $y_i^* = 1$) satisfy the condition

$$\sigma_i(G(y^*)) + H_i \geq c_i. \tag{9}$$

On the other hand, passive agents (i.e., agents $i \in N$ such that $y_i^* = 0$) meet the inequality

$$\sigma_i(G(y^*_{-i}, 1)) + H_i \leq c_i. \tag{10}$$

Write down and analyze the conditions (9)-(10) for the mechanism (5). In the case of the uniform allocation mechanism, active agents in the equilibrium satisfy the condition

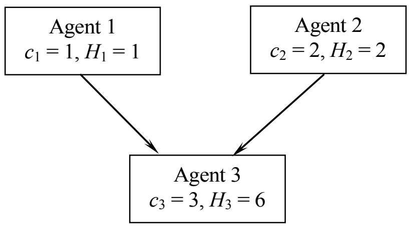
$$\frac{\sum_{j \in N} H_j G_j(y^*)}{\sum_{j \in N} y_j^*} + H_i \geq c_i, \tag{11}$$

whereas passive agents in the equilibrium meet the inequality

$$c_i \geq \frac{\sum_{j \in N} H_j G_j(y^*_{-i}, 1)}{\sum_{j \in N} y^*_j + 1} + H_i. \tag{12}$$

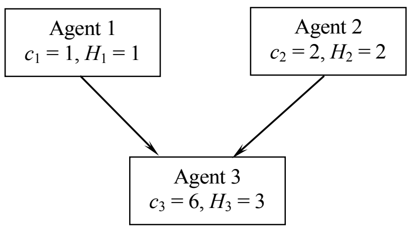
What is the connection between the solution set of the centralized control problem and the set of Nash equilibria? When is it possible to *implement* the efficient state of the network (see criteria (1) and (2)) by centralized control? Which hypotheses are true: (a) an efficient state forms a Nash equilibrium, (b) at least, one equilibrium state appears efficient? Consider two simple examples showing that the sets of efficient states and Nash equilibria have nontrivial connection and both hypotheses fail.

Example 1. There are three agents with unit thresholds (see (3)). Their costs and payoffs are presented below.



Note that the condition $c_i > H_i$, $i \in N$, takes no place. The list of efficient state vectors comprises (0; 1; 0), (1; 0; 0) and (1; 1; 0). Regardless of the payoff allocation mechanism, Nash equilibria are the rest nontrivial state vectors. Therefore, all four Nash equilibria appear inefficient and all efficient states are nonequilibrium states. •

Example 2. Within the framework of Example 1, we modify the costs and payoff of agent 3 as follows:



Again, efficient state vectors are (0; 1; 0), (1; 0; 0) and (1; 1; 0). The unique Nash equilibrium–the vector (1; 1; 0)–is efficient.

And so, the issue of choosing payoff allocation mechanisms for decentralized implementation of efficient network states remains unsettled in the general case.

The mechanisms (5)-(7) are balanced, i.e., the Principal allocates the whole (!) payoff from “indirect network excitation” among initially active agents. Such mechanisms can be called *motivational control mechanisms* (see [29])–the Principal stimulates agents to choose certain states as equilibria in their game. An alternative approach lies in stronger *institutional control* [29], which aims at establishing certain constraints and norms of agents’ activity by the Principal. We examine a corresponding model in the network excitation model.

Institutional control. Let the goal functions of agents have the form

$$f_i(y) = s_i(y) + (H_i - c_i)\, y_i, \tag{13}$$

where $s_i(y) \geq 0$ means an incentive paid by the Principal to agent i ($i \in N$). Generally, this incentive depends on the state vector (action profile) of all agents. Following [23], [29], the structure $s(y) = \{s_i(y)\}_{i \in N}$ represents the vector-form *incentive function of agents* from the Principal. The dependence of agents’ incentives on the action profile of all agents is a special case of the above payoff allocation mechanism.

Fix some set V of agents. Consider the following "institutional control" problem. Find the minimal vector-form incentive function (in the sense of the total costs of the Principal), which "excites" the given set V of agents as a Nash equilibrium $y^*(s(\cdot))$ in their game. Formally, this problem admits the following statement:

$$\begin{cases} \sum_{i \in N} s_i(y^*) \to \min_{s(\cdot)} \\ y_i^*(s(\cdot)) = 1, i \in V, \\ y_j^*(s(\cdot)) = 0, j \notin V. \end{cases} \tag{14}$$

At the first glance, the problem (14) seems complicated; however, it admits rather easy solution by the *decomposition theorem of agents' game* [29]. Consider the following vector-form incentive function:

$$s_i^*(y) = \begin{cases} c_i - H_i + \varepsilon_i, \text{ if } i \in V \text{ and } y_i = 1 \\ 0, \text{ otherwise} \end{cases}, \tag{15}$$

where ε_i ($i \in V$) are arbitrarily small strictly positive constants. If we nullify these constants, agents become indifferent between activity and passivity, and all conclusions below remain in force under the hypothesis of benevolence [29]. Clearly, if the Principal applies the mechanism (15), the choice of unit actions by agents from the set V (and only by them!) forms a unique *dominant strategies' equilibrium* in the game of agents with the goal functions (13). Moreover, the mechanism (15) represents the ε_V-optimal solution of the problem (14), where $\varepsilon_V = \sum_{i \in V} \varepsilon_i$.

We make an important terminological remark. Formally, the problem (14) is a motivational control problem; nevertheless, its solution (15) can be interpreted as institutional control. The Principal establishes rather strict norms of activity for agents: any deviation from assigned behavior causes penalties (their incentives vanish).

The Principal's goal function $F(\cdot)$ makes the difference between the payoff from exciting the set $\Phi(V)$ of agents and the total costs (15) to implement the excitation of agents from the set V (see formula (2)). In other words,

$$F(V) = \sum_{i \in \Phi(V)} H_i - \sum_{j \in V} s_j(y^*) = \sum_{i \in \Phi(V)} H_i + \sum_{j \in V} H_j - \sum_{i \in V} c_i - \varepsilon_V. \tag{16}$$

Comparison of the expressions (16) and (2) brings to the following conclusion. Under sufficiently small values of ε_V (see the discussion above), one obtains the condition $F(V) \geq u(V)$. It would seem that the decentralized control problem is completely solved! Really, we have constructed a partial solution of this problem, since in the mechanisms (5)-(7) the Principal does not explicitly specify the actions expected from agents. Thus, agents can play their game "independently" (the basic idea of control decentralization (see the Conclusion) consists in designing a certain procedure of autonomous interaction of agents, which leads to the choice of most efficient action vector in the sense of some centralized criterion). The "mechanism" (15) explicitly states the actions expected by the Principal from different agents. Furthermore, the Principal still has to solve the centralized control problem (1). Notably, knowing his optimal payoff (16) (in the sense of the minimal costs to motivate excited agents), the Principal has to define a coalition for initial excitation:

$$F(V) \to \max_{V \subseteq N} . \tag{17}$$

Therefore, a positive side of decentralized mechanisms is that agents may possess incomplete information (they do not have to compute the Nash equilibrium (8) or solve the discrete optimization problems (1) or (17)). And a drawback is the complexity (or even infeasibility) of constructing an efficient decentralized mechanism.

Among advantages of the centralized mechanism, we mention that all "cognitive" (informational and computational) costs belong to the Principal; however, these costs may appear appreciably high.

As an example, let us consider the mob control problem.

An example: mob "excitation." The paper [12] considered a model of mob control, where agents make decisions (choose between their activity or passivity) depending on the number of active agents. The Principal's efficiency criterion is the number (or share) of active agents. In terms of the model analyzed in the present paper, the problem of mob control admits the following statement: choose a set of initially excited agents to maximize (to minimize, etc.) the number of indirectly excited agents such that the costs of control meet a given budget constraint C_0:

$$\begin{cases} |\Phi(S)| \to \max_{S \subseteq N}, \\ C(S) \le C_0. \end{cases} \tag{18}$$

Let the behavior of agents be described by the expression (3); moreover, assume that the communication graph of agents appears complete. Renumber agents in the ascending order of their thresholds: $\theta_1 \le \ldots \le \theta_n$. Denote by

$$P(x) = \frac{1}{n}\left|\{i \in N : \theta_i < x\}\right|$$

the distribution function of agents' thresholds and by $\{x_t\}_{t \ge 0}$ the sequence of shares of active agents (in discrete time, where t indicates current instant (step)).

Suppose that we know the share x_0 of agents acting at step 0. The share of agents whose thresholds do not exceed x_0 makes up $P(x_0)$. And so, at step 1 we have $x_1 = \max\{x_0; P(x_0)\}$. At the next step, the share x_2 of active agents is defined by $x_2 = \max\{x_1; P(x_1)\}$ (the thresholds of agents in this share are not greater than x_1). Arguing by analogy, one easily obtains the following recurrent formula for the behavioral dynamics of the set of agents [9], [18]:

$$x_{k+1} = \max\{x_k; P(x_k)\}. \tag{19}$$

The equilibria in the system (19) are determined by the initial point x_0 and the points of intersection of the curve $P(\cdot)$ and the bisecting line of quadrant I: $P(x) = x$. Possible stable equilibria are the points, where the curve $P(\cdot)$ crosses the bisecting line, approaching it "from above." Fig. 1 shows an example of the distribution function of agents' thresholds in the continuous-time case (see the lower fragment of the figure). The points x_2^* and x_4^* are stable.

Designate by $\Psi(P(\cdot)) \subseteq [0;1]$ the set of roots of the equation $P(x) = x$ (it appears nonempty, since one of the roots equals 1). Next, define $x^*(x_0) = \min\{y \in \Psi(P(\cdot)) : y > x_0\}$. For instance, the point x_0 in Fig. 1 satisfies the condition $x^*(x_0) = x_2^*$. It follows from (19) and the equilibrium stability conditions that

$$\Phi(x_0) = \begin{cases} x_0, \text{ if } P(x_0) \le x_0, \\ x^*(x_0), \text{ if } P(x_0) > x_0. \end{cases} \tag{20}$$

Let a nondecreasing function $c(x_0)$ specify the initial excitation costs of a given share $x_0 \in [0; 1]$ of agents (within the framework of our model of collective behavior, excitation first applies to agents with smaller thresholds). By virtue of (20), the problem (18) acquires the form

$$\begin{cases} \Phi(x_0) \to \max_{x_0 \in [0;1]}, \\ c(x_0) \le C_0. \end{cases} \tag{21}$$

We can solve the inverse problem, i.e., find the minimal costs $C_{\min}(x)$ of an initial excitation such that the final share of excited agents in mob is not smaller than a given level $x \in [0; 1]$. See Fig. 1 for an example of its solution.

Stochastic models of mob control are examined in [1].

By assumption, the Principal seeks to maximize the number of excited agents. If the goal lies in mob activity minimization, the corresponding problems are posed and solved by analogy, since analysis of stables states and their dependence on the model parameters (see Fig. 1) allows characterizing the relationship between the resulting and initial states.

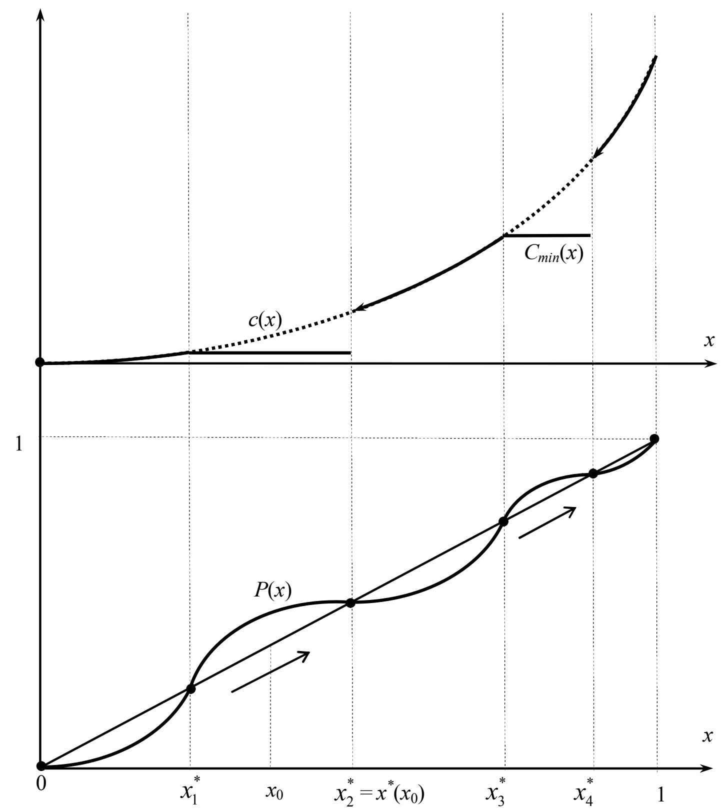


Fig. 1. The distribution function of agents' thresholds and cost functions

4. Conclusion

Therefore, we have considered the general statement of the network "excitation" problem, established the nontrivial character of the decentralized implementation of an efficient equilibrium and suggested an efficient mechanism of institutional control.

It is possible to identify several directions of promising research. First, the matter concerns analyzing the influence of certain properties of the communication graph of agents and their decision-making principles on the properties of optimal solutions in control problems. In the present paper, both these factors (the structure of agents' communication and the models of their cooperative behavior) are "embedded" in the operator $\Phi(\cdot)$. Explicit formulations and exploration of special cases (acyclic graphs, concrete threshold models and other models of agents' decision-making) may yield nontrivial results with practical interpretations. Furthermore, the setting with several control subjects naturally brings to models of informational contagion [1], [20], [28].

Second, as far as the standard triad of control types [29] comprises institutional control, motivational control and informational control (the first and second types have been mentioned above), certain interest belongs to analysis of informational control models [27] in network excitation problems.

Third, the presence of a network of agents making strategic decisions suggests involving some results from the theory of cooperative games on graphs (a communication graph restricts the capabilities of coalition formation [5], [13], [26], [30] and interaction among agents [14]). However, in network excitation problems, communication graphs are connected with the agents' ability to influence other agents (i.e., communication graphs define the goal functions of agents, *ergo* the characteristic function of corresponding cooperative games), rather than with the feasibility of forming certain coalitions.

Fourth, it is necessary to study in detail the payoff allocation mechanisms of the form (5)-(7), including their analysis within the framework of social choice theory (e.g., see [24]). Here we mean characterization of their classes possessing given properties (for instance, an important property of efficient states' implementation).

And finally, many prospects are associated with further decentralization of the network "excitation" problem. The approaches of algorithmic game theory [4], [31] and distributed optimization [9], [10] prompt the following idea. We should endeavor to find simple procedures of local behavior of agents, leading to the optimal state within the initial problem of the form (1) and (17), which has very high algorithmic or computational complexity.

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