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PLANNING AND CONTROLLING MULTILEVEL MAN-MACHINE ORGANIZATION SYSTEMS UNDER RANDOM DISTURBANCES

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PREFACE

In the loving memory of Prof. Abraham Mehrez
without whom this book would never appear

In recent years problems associated with developing various organization systems (OS) have been discussed extensively in scientific literature. Those problems become more and more urgent since nowadays modern OS are characterized by [168]:

- increasing both the systems' complexity and the number of their hierarchical levels;
- various random disturbances which affect the systems' realization (especially in project management);
- evaluating the progress of an OS usually only at preset inspection (control) points since it is impossible or too costly to measure the system's output continuously.

By examining the existing literature (see, e.g., [5, 168]) one can draw a conclusion that there exist at least three main shortcomings in the area of analyzing and synthesizing modern OS:

I. The existing quality techniques are not applicable to OS since they deal only with finished products and services. The developed utility theory [125-127] is restricted to solving market competitive problems alone. Thus, all existing models focus on analyzing the competitive quality of OS's outcome products rather than dealing with the quality of the systems' functioning, i.e., with OS in their entirety. This may result in heavy financial losses, e.g., when excellent project objectives are achieved by a badly organized project's realization.

   Thus, a conclusion can be drawn that the existing utility theory cannot be used as the system's quality techniques. In order to fill in the gap, we have undertaken research in the area of *estimating the quality of the system itself*, e.g., the system's public utility. We will consider a complicated organization system which functions under random disturbances. Such a system usually comprises a variety of qualitative and quantitative attributes, characteristics and parameters, which enable the system's functioning. The problem arises to determine a generalized (usually quantitative) value which covers all essential system's parameters and can be regarded to as a system's qualitative estimate. We will henceforth call such a generalized value *the system's utility*.

II. Another conclusion which can be drawn, is as follows:

   It goes without saying that a large (multilevel) OS, like any other system, has to be planned and controlled. This, in turn, requires developing a corresponding multilevel unified *mathematical model* comprising local interconnected models at each level.

The latter have to determine both control actions (e.g., resource reallocation) and various decision making. It can be well-recognized that no real OS comprise such models as yet. Moreover, important OS such as construction, maintenance, service, agriculture, socioeconomics, safety engineering, do not implement even local optimization models at any level. More formalized OS like man-machine production and project management comprise various separate local models. However, the latter do not form a multilevel unified on-line control model with cyclic information. They are practically unable either monitoring the system's object under random disturbances, or estimating the quality of the system's functioning.

In order to fill in this gap as well, we will create and outline in the monograph a new multi-parametric on-line control optimization model in order to maximize the system's utility as a generalized quality measure of the system's functioning. Since such a model is, in essence, a trade-off compromise between the system's parameters, we will henceforth call that model *harmonization model* (HM).

III. The last major shortcoming of the existing theory of OS is that practically no attempts have been made to create proper analysis and synthesis models for hierarchical complicated OS, e.g., holding corporations including marketability problems, or highly complicated multilevel production and project management systems (especially by means of on-line control). We will do our best to widespread the utility theory on those systems' functioning, including the outcome product's life cycle. Newly developed models will be outlined in our monograph, including models for strategic holding corporations and three-level on-line control models of project- and production management.

It can be well-recognized that the majority of man-machine OS under random disturbances are innovative in nature. Indeed, they deal either with:

- developing new hi-tech products, unique installations with no prototypes in the past, modern safety engineering devices, multilevel stochastic project management systems, etc., or with
- improving existing OS which function under random disturbances.

Taking into account that since both cases usually result in obtaining novel results, a conclusion can be drawn that OS under consideration have a strong innovation tendency.

The structure of the monograph is as follows:

In Chapter 1 the description of man-machine OS is presented. A special emphasis is drawn on the four essential characteristics of the organization - the system's content (personnel, equipment and resources), its structure (man-machine OS comprise usually 2-4 hierarchical levels), its communications (i.e., the information flaws) and the decision-making procedures. Standard control actions typical for the majority of modern man-machine OS, are presented as well.

It goes without saying that a single monograph is virtually unable to reflect the vast spectrum of man-machine OS being operated under random disturbances. Thus, we will focus our mind on the following man-machine OS: project management, construction, safety engineering, man-machine production systems with variable speeds, maintenance systems (relative to safety engineering), compound and multi-attribute OS, strategic OS (multilevel holding corporations and marketing OS). A special place occupy the so-called hierarchical active systems actually referring to man-machine production OS with a high influence of the human factor [32-38].

In Chapter 2 a justification for using beta-distribution for calculating man-machine operations' duration is outlined. The presented material examines both cases of a single processor and several identical processors. Emphasis is made on particular "family members" of the beta-distribution variety, e.g., the Freshe distribution law [69, 92], which proves to be very efficient for estimating time durations not for a single activity but for a whole fragment. It is shown that the Freshe law is stable to operations of both convolution and maximization.

In Chapter 3 control concepts in multilevel man-machine OS are outlined. Several essential control concepts, namely plan assignment, coordination, planning trajectories, various resource parameters, etc., are all introduced as basic techniques in planning and regulating production OS under random disturbances.

In Chapter 4 we present models for determining inspection points in various OS. Both cases of one-level and multilevel OS are considered.

Chapter 5 presents newly obtained results in developing optimization models for multiparametrical OS under random disturbances. The results may be applied to a wide variety of man-machine OS including those pointed out above. The newly developed theory enables both estimating and optimizing utility values of any man-machine OS. The optimizing process (as mentioned above, referred to as harmonization) is carried out on two levels and comprises a modified simulation model at the lower level accompanied by a search coordinate descent algorithm at the upper one.

Chapter 6 outlines risk management problems associated with man-machine OS. New combined models including risk management and harmonization models, are presented.

Chapter 7 houses a cluster of newly developed cost-optimization models related to a specific but very important case of man-machine OS with several production speeds (rates of advancement to accomplishing the system's objective) which depend on the degree of intensity of the system's functioning. It covers many representative man-machine OS, e.g., construction OS where a construction team may be employed different hours a day.

In the next Chapter 8 the theoretical results outlined in Chapter 5, are applied to estimating the quality of PERT-COST projects' network (both single and multiple projects) under random disturbances. For multiple projects networks, both cases of projects with equal and different significances are examined. The chapter contains numerical examples obtained by means of harmonization modeling (HM).

In Chapter 9 the HM approach has been applied to safety engineering fault tree cost-reliability optimization. Corresponding numerical results are outlined as well.

Chapters 10 and 11 relate to maintenance cost-reliability optimization, mostly with safety engineering implications. Two different models are examined - the HM versus the “look-ahead” predicting model.

Chapter 12 covers both HM and heuristic modeling for the case of construction OS. Several different cost-optimization models for a three-level and a two-level OS are considered and developed.

In Chapter 13 a hierarchical active man-machine OS [32-38] based on essential human influence is described. The basic idea is as follows: since the system’s elements on neighboring levels (being in fact subordinated) are not contradictory in essence, the corresponding assignment plans, together with the revenue obtained by each other, must be optimal (or, at least, quasi-optimal!) for both of them. Several examples illustrating this general idea for PERT-COST project management systems, are being outlined in Chapters 13 and 17.

Chapter 14 describes a compound multi-attribute case of OS referring mostly to strategic management in order to maximize the outcome product’s utility on the basis of designing subproducts.

Chapter 15 outlines a cost optimization model for a truly strategic OS, namely, a HM for a complex holding corporation, comprising several subsidiary corporations.

Chapter 16 presents optimization models in strategic marketability and, in this course, outlines an algorithm determining the minimal R&D project’s budget to enable accomplishing the project on time subject to the reliability constraint, thus resulting in the maximal marketability value.

Chapter 17 outlines a hierarchical on-line control model for stochastic project management [70, 104]. A multilevel control model for several stochastic network projects is suggested; at any control point the model determines:

- optimal budget values assigned from the company to each project,
- optimal budget reallocation among the project's activities,
- optimal control points to inspect each project,
- optimal resource delivery schedule for project activities,

and comprises two conflicting objectives:

- minimize the total number of control points for all projects, and
- maximize the probability of meeting the deadline of the slowest project.

In the last section of the chapter we have combined the results outlined above with human factor influence techniques referred to in Chapter 13. A conclusion can be drawn that the combination of the theory of active systems [32-38] and the results outlined in Chapters 5 and 8, may raise essentially the quality of the hierarchical project management system as well as the human behavior of the system's personnel.

In Chapter 18 a hierarchical on-line control model for production management is presented. The approach to solving interaction problems between different levels in hierarchical control systems is based on the conception of emergency, introduced by Golenko-Ginzburg and Sinuany-Stern [79]. By using the idea that hierarchical levels can interact only in special situations, so-called emergency points, one can decompose a general and complex multi-level problem of optimal production control into a sequence of one-level problems. This approach is applied to a control model for three-level man-machine production system [78, 91, 102, 124]. The system comprises the factory level, several sections and multiple production units. A newly developed approximate method for solving reallocation problems is suggested. The method is a combination of the coordinate descent method and a high-speed iterative algorithm with an implemented switching procedure based on two objectives: the unit and the product criteria.

Thus, it can be well-recognized that the monograph's content is subdivided into four different parts:

I. Chapters 1-7 present various concepts for time-, resource- and control parameters in man-machine OS, including the general approach to solving optimization problems. Practically speaking, we present hereby our ideology.

II. Chapters 8-12 describe the results obtained by applying the developed theory to several most important man-machine OS: project management, construction, safety engineering, various production OS, etc.

III. Chapters 13-16 cover problems of strategic management with a higher level of hierarchy and, as a result, more complicated cost-optimization problems.

IV. In the last two chapters (17-18) the newly developed multilevel hierarchical on-line control models for project management and production management are outlined.

In conclusion, let us cite Prof. W.G. Scott (see [5], p. 119) who four decades ago wrote the following words:

"Modern organization theory needs tools of analysis and a conceptual framework uniquely of its own, but it must also allow for the incorporation of relevant contributions of many fields. It may be that the framework will come from general system theory. New areas of research such as decision theory, information theory, and cybernetics also offer reasonable expectations of analytical and conceptual tools. Modern organization theory represents a frontier of research which has great significance for management."

We hope that our monograph will help meeting some of those prophetic targets which have not been reached as yet.

PART I

BASIC CONCEPTS OF MAN-MACHINE ORGANIZATION SYSTEMS UNDER RANDOM DISTURBANCES

Chapter 1. Organization Systems' Description

§1.1 Standard two-level organization system

Consider a standard two-level organization system (OS) comprising a control device at the upper level and n elements E_k, $1 \le k \le n$, at the lower level. In order to outline the problem the following additional terms have to be introduced.

1.1.1 Notation

Let us denote:

- R_S - the system's resources at the beginning of OS functioning (pregiven);
- R_{st} - remaining system resources at moment t (observed via inspection);
- R_{kt} - resources assigned to element E_k at moment $t \ge 0$ (to be determined);
- D_k - the due date for E_k to meet its target (pregiven);
- V_k - target amount (the planned program) for E_k to be reached at D_k (pregiven);
- V_{kt} - the actual output of E_k observed at moment t (a random value obtained via inspection);
- p_k - the chance constraint, i.e., the least permissible probability of meeting the element's target on time;
- $v_k(R_{kt})$ - the random speed of element E_k depending parametrically on R_{kt};
- $R_{k\min}$ - the minimal resource capacities;
- $R_{k\max}$ - the maximal resource capacities;

1.1.2 The problem

The optimization problem to be solved at each emergency point t^*, when it is anticipated that a certain element E_k cannot meet its local target on time subject to its chance constraint, is as follows:

For all non-accomplished elements determine new resource capacities $\{R_{kt^*}\}$ to minimize the objective

$$\min \sum_k R_{kt^*} \tag{1.1.1}$$

subject to

$$P\left\{V_{kt^*} + v_k\left(R_{kt^*}\right)\left(D_k - t^*\right) \ge V_k\right\} \ge p_k, \tag{1.1.2}$$

$$R_{k\min} \le R_{kt^*} \le R_{k\max},\ 1 \le k \le n, \tag{1.1.3}$$

$$\sum_k R_{kt^*} = R_{st}. \tag{1.1.4}$$

Note that here we assume the simplest case of generalized resources (e.g., budget assignment) versus the case of detailed resources. Moreover, we will not consider the case of several element's speeds for one and the same resource capacity R_k. Those different speeds may be achieved by intensification of the element's functioning, although the time unit cost would under such circumstances certainly increase.

The problem (1.1.1-1.1.4) may have many modifications, but the principal conclusions outlined below remain the same:

A. The information flows at moment $t = 0$ go first downstairs, while at all emergency moments they develop upstairs, from element's level to the system manager, and afterwards (after the new resource reallocation) proceed again downstairs. Such a switching procedure remains in the course of the system's functioning.

B. Decision-making, i.e., control actions, results in reallocating remaining resources among non-accomplished elements. They are implemented at the upper level only.

In more complicated cases, i.e., in case of a three-level OS (e.g., several network projects of PERT-COST type [101, 104]), decision-making is carried out on two levels: at the upper one (resource reallocation among the projects) and at the project level (resource reallocation among project activities). Control actions may also result in changing the element's speed (in case of several speeds for one and the same resource capacity) and in re-scheduling the starting times for system's elements (if required).

All these problems are being outlined in the below sections of the monograph.

§1.2 General multi-parametrical harmonization problem

We suggest calling the system's utility a weighted linear function of the system's parameters with constant coefficients. The parameters are divided into:

- independent parameters, where for each parameter its value may be preset and may vary independently on other parameters' values, and
- dependent parameters whose values may not depend uniquely on the values of independent parameters. However, when optimized (for the same values of independent parameters), they are solely dependent on those values.

Both independent and dependent parameters together with the coefficients of the utility function are externally pregiven.

If an organization system functions under random disturbances and comprises n_1 independent basic parameters $R_i^{(ind)},\ 1 \le i \le n_1$, and n_2 dependent basic parameters $R_j^{(dep)},\ 1 \le j \le n_2$, the harmonization problem boils down to maximize the system's utility

$$U_S = \left(\sum_{i=1}^{n_1} \alpha_i R_i^{(ind)} + \sum_{j=1}^{n_2} \alpha_j R_j^{(dep)} \right) \tag{1.2.1}$$

subject to certain restrictions. We suggest determining the optimal vector

$$\vec{R}_* = \left(R_{1*}^{(ind)}, R_{2*}^{(ind)}, \dots , R_{n_1 *}^{(ind)}, R_{1*}^{(dep)}, R_{2*}^{(dep)}, \dots , R_{n_2 *}^{(dep)} \right) \tag{1.2.2}$$

which delivers maximization to the system's utility U_S, by means of the following sequential stages:

Stage I - implement a look-over algorithm to examine all feasible combinations of independent basic values $\left\{ R_i^{(ind)} \right\}$;

Stage II - determine optimal values $\left\{ R_{j\ opt}^{(dep)} \right\}$ for all dependent parameters by means of values $\left\{ R_i^{(ind)} \right\}$ obtained at the previous stage; for each j-th dependent parameter an individual optimization model (called henceforth the partial harmonization model PHM_j), is used. The latter enables the optimality of each value $R_{j\ opt}^{(dep)}$ which solely depends on the combination $\left\{ R_i^{(ind)} \right\}$;

Stage III - calculate the system's utility U_S via (1.2.1) for the combination

$$\left\{ R_i^{(ind)} \right\},\ \left\{ R_{j\ opt}^{(dep)} \right\} \tag{1.2.3}$$

obtained at *Stages I* and *II*;

Stage IV - calculate the optimal system's utility by determining the optimal combination (1.2.2) for all independent and dependent parameters which delivers the maximum to U_s.

If, due to the high number of possible combinations $\left\{ R_i^{(ind)} \right\}$, implementing *Stage I* requires a lot of computational time, we suggest to use a simplified heuristic search procedure, e.g., a cyclic coordinate search algorithm.

Thus, we suggest an approximate harmonization's problem solution as follows. At the first stage a relatively simple search algorithm in the area of independent parameters, e.g., the cyclic coordinate descent method, is implemented. At the second stage, in order to evaluate the optimal value of each dependent parameter, an optimization problem PHM_j,

$1 \leq j \leq n_2$, has to be solved. Thus, the idea is to obtain independent parameters' values at the first stage and to use them as input values of all partial harmonization models at the second stage.

PHM is usually a stochastic optimization model which is solved on the basis of simulation modeling. However, in certain cases, e.g., reliability and safety engineering problems, various *PHM* require more complicated formulations. In such cases we suggest to use additional heuristic models in order to implement realistic quantitative links between the system's attributes. For various dependent parameters the *PHM* may be formulated and solved by means of expert information.

§1.3 Applications to safety engineering and project management

Projects with pregiven structure (in the form of PERT-COST network models) comprise 3÷4 basic parameters as follows:

- the budget assigned to the project (independent parameter);
- the project's due date (independent parameter);
- the project's reliability, i.e., the probability of meeting the project's due date on time (dependent parameter);
- a safety engineering parameter, i.e., the probability of a hazardous failure in the course of carrying out the project (dependent parameter).

Two different cases are considered:

- case of one project;
- case of several projects with different or equal importance and significance.

The developed research considers the following models:

1.3.1 Harmonization models in reliability and safety engineering

A hierarchical technical system functioning under random disturbances and being subject to critical failures at the bottom level which may result in an accident or a hazardous condition including environmental safety violations at the upper level, is considered. If a certain failure at the bottom level occurs, it may affect some system elements at higher hierarchical levels, and, thus, cause a hazardous failure at the top level. All logical relations between the system's elements are formalized by a fault tree simulation model which is externally pregiven. Certain primary elements at the bottom level, together with their corresponding primary failures, can be refined by undertaking technical improvement. The list of the latter is pregiven as well. It is possible by means of fault tree simulation to evaluate the increment of the system's reliability by implementing any set of technical improvements. The harmonization models center on determining an optimal sub-set of technical improvements in order:

- either to maximize the system's reliability subject to a restricted budget assigned for the improvements' implementation, or
- to minimize the system's budget subject to a reliability value restricted from below.

Both *PHM* are optimized by a combination of heuristic decision-making rules and a fault tree simulation.

Two different cases are considered:

- a simplified cost-sensitivity trade-off model to solve cost-reliability problems for a complicated hazardous technical system with two basic parameters: cost and reliability, and
- more complicated trade-off harmonization problems where the system's utility, cost expenditures, reliability values and other basic parameters are linked together by means of sensitivity relations.

1.3.2 Harmonization models for a single network project

A PERT-COST type project with random activity durations is considered. The project comprises several essential parameters which practically define the quality of the project as a whole:

- the budget assigned to the project (C);
- the project's due date (D);
- the project's reliability, i.e., the probability of meeting the project's due date on time (R).

To establish the utility of the project, the concept of the project's utility is introduced. This is carried out within the framework of the general theory of optimal harmonization models for multi-parametric organization systems. In order to maximize the project's utility, a three-parametric harmonization model is developed. The model results in a certain trade-off between essential project's parameters and is, thus, a compromise optimization model. The model's algorithm is a unification of a cyclic coordinate search algorithm in the two-dimensional area (cost- and time values) and a partial harmonization model to maximize the project's reliability subject to the preset budget and due date values. The *PHM* comprises a heuristic procedure to reassign the budget among project's activities, and a simulation model of the project's realization.

1.3.3 Harmonization model in project management with safety engineering concepts

The harmonization model is extended by supplementing its basic parameters by a new essential parameter defining the utility of the project as a whole, namely, the probability P of a hazardous failure in the course of carrying out the project. On the basis of expert information we came to the conclusion that a hazardous failure capable of jeopardizing environmental or personnel's safety, depends mostly on the following project's control actions:

- decreasing the project's due date, and
- increasing the intensity of the project's realization without undertaking proper safety engineering measures.

A formalized four-parametric harmonization model accompanied by a heuristic solution has been developed. The model is based on a cost – time – reliability – safety trade-off.

Parameters C and D are input values of the model. Value R is optimized by means of a heuristic procedure. Value P is calculated on the basis of dependency $P(C,D)$ obtained by means of statistical analysis and expert information.

Optimizing the harmonization model is carried out by solving the main problem (to determine an optimal budget value C and an optimal due date D) and two subsidiary problems as following:

- solving the optimal *PHM* problem, i.e., maximizing $R(C,D)$, and
- calculating $P(C,D)$ on the basis of expert information.

Note that *Problem III* is the first harmonization model of mixed type, where optimization techniques, simulation model and expert information meet together.

1.3.4 Harmonization models for several network projects

A highly complicated project management system including several simultaneously realized PERT-COST type network projects, is considered. The projects are of different importance and significance; for each k-th project its corresponding priority index η_k is pregiven. The total budget at the project management disposal to carry out all the projects is limited. Given for each project its priority value, the problem is to determine optimal budget assignments and optimal due dates of accomplishing each project, to maximize the weighted sum of the projects' utilities, i.e., the objective

$$J_1 = Max \sum_{k=1}^{n} (\eta_k \cdot U_k), \tag{1.3.1}$$

where n is the number of projects, and U_k is the k-th project's utility. The problem centers on maximizing the system's utility by implementing for each project the harmonization model. The system's harmonization model comprises two levels. At the upper level a high-speed look-over search algorithm is implemented, together with a partial harmonization model to determine the projects' reliability values. At the lower level a linear programming model is imbedded under certain assumptions.

Another harmonization model covers projects of equal importance. This results in changing the objective,

$$J_2 = \underset{\{C_k, D_k\}}{Max} \; \underset{k}{Min} \; U_k, \qquad (1.3.2)$$

in order to maximize the utility of the project with the least utility value. Similarly to model (1.3.1), harmonization model (1.3.2) comprises two levels with the upper level identical to that in model (1.3.1). As to the lower level, modified linear programming methods are implemented.

1.3.5 Interactions and linkage with other OS

Harmonization techniques depend solely on the organization system's model (SM) by means of which trade-off optimization and utility estimation is carried out. Changing the system's model results in entire changes of HM including all PHM.

For the case of Project Management the system's model is given in the form of a PERT-COST network with random activity durations. The latter depend parametrically on budget values assigned to those activities. All PERT-COST networks are presented in a formalized shape and do not depend on the nature of the project. Unlike risk management models, those SM do not deal with such risk factors as technology, design changes, market regulations and policies, etc., although they usually comprise probabilistic parameters which may affect those risk factors. All harmonization models which are based on such formalized network projects, deal with only one risk factor, namely, the risk not to meet the project's due date on time because of random durations of the project's activities. Besides optimizing the project's utility, HM may serve as *a risk assessment technique*. Being, in essence, operation research (OR) models (like fault tree models, various models of mathematical programming, etc.), *HM, thus, are not similar to general risk management methods which are based on a variety of engineering, economic and political aspects*. However, *HM may be compared with similar risk assessment techniques*, i.e., similar OR models, which are used in risk analysis. E.g., *in cases when certain comparative alternatives and scenarios in project risk analysis can be presented in the form of PERT-COST sub-networks, harmonization models may be applied to analyze those sub-networks, including optimization and calculation of their utility values*. Using HM as a risk assessment technique can be justified since harmonization models, being applied to PERT-COST projects, are essentially more effective than the traditional deterministic time – cost trade-offs which are used as yet in project risk analysis.

For hierarchical production plants with the possibility of hazardous failures at the top level the system's model is given in the form of a fault tree simulation model, together with a list of possible technical improvements for primary elements at the bottom level. The developed harmonization models refer to risk assessment models at the stage of optimization. Those models cannot be compared with similar research on safety engineering since no developments on multi-parametric trade-off optimization have been outlined in prior references as yet.

§1.4 Human factors in active systems

1.4.1 The significance of human factors

In the last three decades by means of the scientific school of V.N. Burkov [32-38] a newly developed theory of the so-called active organization systems has been suggested. The theory, in essence, is based on human decision-making by implementing a competitive game between several human collectives. The theory of active systems comprises organizational and economic mechanisms of managing projects of various types. The theory is based on human's behavior and takes into account the reliability of information, obtained from executors in the course of projects' realization, and economic motivations of executors. Thus, the "human factor", substantially affecting the process of project realization and management, is directly considered.

The problems met in the course of development of the theory of organization systems include the difficulties of applying formal methods, dimensions of the tasks involved, the immense number of interconnections and factors that do not support direct monitoring, the hierarchic structure of the management-control system, etc. Among the fundamental characteristics of organization systems is the goal-directed nature of the operation of their constituent subsystems.

The goal-directed operation of organization systems is dictated by the involvement of human behavior in them. Moreover, the presence of man imparts a certain “activity” to the controlled process. The significance of this attribute lies in the ability of man to foresee the control functions from the side of the control element as well as the actions of other components of the system and, with this knowledge, to select (within the scope of available alternatives) his “actions and behavioral strategy” with a view towards attaining goals of one kind or another. Management practice gives us many examples of how the “activity” effect is manifested in organization systems. For example, in centralized-planning industrial systems, the activity of the separate subsystems (associations, enterprises or corporations, institutes, companies, etc.) without coordination of targets results in such negative effects as excessive requisitions for resources and necessary finances, overpricing of products, breakdown of plans with respect to individual “unfavorable” types of products (including new technologies) while meeting the general aggregate indices of planning specifications; underestimation of production capabilities of enterprises in the synthesis of plans; overestimation of design fulfillment periods and, conversely, competition for the improvement of quality and efficiency and reduction of costs, etc. Looking at the enterprise and its subdivisions and judging from the materials of widespread publications, here again we can cite a long list of manifestations of the “activity” of subsystems.

One general conclusion is obvious: The allowance for target-directed factors in the operation of the controlled processes in organization systems and the corresponding manifestations of their “activity” is a necessary condition for the formulation of a realistic organization theory. Of course, the consideration of these factors increases the difficulty of formulating an appropriate formal theory. However, to ignore them, in our opinion, renders highly problematical the possibility of even synthesizing adequately realistic mathematical models equipped to investigate and solve, for example, such important economic policy problems as the selection of a system of rating indices for the results of the functioning of economic elements, the "horizontal" and "vertical" coordination of subsystem plans, the reliability of information, etc. These problems are certainly timely even today (we can mention, e.g., the "shaft" problem, the problem of "profitable" versus "unprofitable" operations, the sometimes unjustified overstocking of reserves, the search for the most effective forms of competition, etc.).

Until now, management and control theory has not dealt with "active" target-directed processes. The theory of organization management with allowance for the "activity factor" (theory of active systems) has been in the process of development, and its status is summarized in several publications by Burkov [32-38]. The main focus of the present section is on new trends in research on the theory of active systems and practical studies in progress at the present time. A number of concepts and results of the work covered in this section have proved exceedingly useful in the development of the theory of active systems.

1.4.2 Methods of description of organizations

At present, the greatest progress has been achieved in the investigation of two-level "fan-type" organization systems, which we therefore consider a logical starting point for our discussion.

The description of an organization system in the theory of active systems is based on its structural concept and the models and mechanism of its operation [34-37].

The structure of the two-level organization system comprises: the center (top-level management element), its subordinate active elements, and the "external medium" element. The model is understood to be a description of the organization system and its constituent elements in terms of vectors of states and constraints on those elements. It is customarily assumed that the center is a purely administrative organ. Then the system model represents a description of the states of lower-level elements as well as local and global constraints on them. The description of the center, on the other hand, is given in terms of a description of the actions that it can exercise with respect to organization of the system operating process and in the actual operating process of the system.

The operating mechanism of a two-level system is the set of rules (procedures, functions) regulating the actions of all elements of the system in the course of its operation. The formal description of the operating mechanism of an active system [32-33] is specified by a description of:

- the mode of generation of data about the model (to be used in the event of imperfect information availability to the center) and states of the elements;

- the law of generation of control parameters in the system (control function);
- the objective function of the system in the large (assuming that it coincides with the objective function of the center) and the objective functions of its constituent elements;
- additional constraints introduced into the system, alone with the sequence of actions of the elements adopted in the system in connection with information communication and selection of the states of the system.

The efficiency of operation of each active element in a given period is estimated in terms of the value attained by its objective function, and that of the system in the large in terms of the value attained by the objective function of the system. Of course, formal description of the objective functions of the elements and the system does not by any means pose a simple problem. In developing descriptions of this kind, it is necessary to consider factors of economic, social, and ethical consideration. It can be well-recognized that at present, the greatest success has been attained in formalization of the economic components of the objective function of active elements. This fact already enables us to implement the developed approach to economic systems and, in particular, to analysis of the management mechanism (with regard to the foregoing remarks).

It is understood that, generally speaking, an arbitrary state of the system, accessible under local constraints, is not necessarily accessible under the global constraints as well. An operating mechanism is called *feasible* if it ensures satisfaction of the following conditions:

a) the state acquired by the total system as a result of any locally admissible selection of states by the elements is admissible, i.e., satisfies the global constraints;

b) the state acquired by the total system as a result of any "rational" locally admissible selection of states by the elements is admissible, i.e., satisfies the global constraints.

Due to the specification of hypotheses on the behavior of the elements, case (a) corresponds to the "strong" and case (b) to the "weak" feasibility condition of the operating mechanism. If condition (a) holds, then condition (b) is automatically satisfied, but the contrary may generally prove to be incorrect. For condition (a) to be satisfied, it is sufficient to construct operating mechanisms (or, more precisely, constraints and laws of generation of the control parameters) such that the condition of independence of the system elements is satisfied.

As conceived, the developed methods of description make it possible to reflect many important features of real active OS in economics and industry. For example, the structure of an active OS reflects the inherent "subordination hierarchy". The system model provides a means for specifying the description of the system in terms of natural and financial indices as well as the relationships between elements. The description of the operating mechanism makes it possible to reflect:

- the procedure used in the active OS for generation of management information;
- methods of generation of the control parameters - plans, prices, and norms;
- methods of financial control and economic stimulation;
- organizational and financial constraints on the activity of organizations and their subdivisions;
- real-time control methods;
- methods of organization of competition;
- and in general the entire set of organizational, legal, economic, and financial rules governing the operation of the active OS in economics and industry.

It can also be assumed that the future outlook will include the possibility of formalizing and investigating a number of characteristics of the structures, models, and operating mechanisms with regard to the quantitative and qualitative characteristics implemented in investigation and description of real organizations. There are already existing indications of a certain positive experience of this nature (formalization of the degree of management centralization).

1.4.3 Analysis and synthesis of operating mechanisms

One of the central problems considered in the first studies on the theory of active systems was the investigation and assessment of the effectiveness of a number of operating mechanisms in two-level active OS. The problem was treated under the following assumptions:

a) the center is fully informed, i.e., the center is fully aware of the set of possible states (or objective functions) of the elements correct to a finite-dimensional vector of parameters;

b) the center strives to be fully informed and, for this purpose, organizes a certain procedure for the generation of estimates of parameters unknown to it;

c) the operating mechanism of the system ensures independent selection of states by the elements;

d) the operation (functioning) of the system can have either a recurrent or a non-recurrent character.

Also, hypotheses have been advanced regarding the activity of target-directed elements. It has been assumed that any element is familiar with the operating mechanism of the system, can select any state from the set of its admissible values, can communicate to the center unreliable estimates of parameters unknown to it, and take into account not only the targets of the present, but also those of future operating periods.

The statements of the analysis and synthesis problems have been formalized with the application of a number of modern game-theoretic concepts. The operation of the active system is treated as a game in which the players are the center and the active elements. The center is given the first move, which boils down to selecting the operating mechanism of the active system. After the center's move (i.e., with a given operating mechanism), the game is played between active elements. The strategies of the active elements in the game are to report information to the center and to select their states. The problem of analysis of the operating mechanism is to determine the values of the objective function and other indices characterizing the operation of the system and available in the decisions of the game between active elements, as well as to analyze the properties of the actual game decision. Here the decisions of the game between active elements are interpreted as situations that can be realized in the system when the elements act rationally in accordance with their criteria and the possibilities (strategies) and information available to them. If the operation of the system is recurrent in nature, then global stability requirements are additionally imposed on the decisions of the game [32, 35]. The synthesis problem entails determining for the active system an operating mechanism that satisfies certain predetermined properties (which necessarily include the feasibility condition) and has maximum efficiency (in the sense of the value attained by the objective function of the system in the decisions of the game between active elements). The solutions of the synthesis problem are sought be the method of selection and detailed investigation of the properties of the "good" (from the economic and practical points of view) operating mechanisms. It is interesting to note that satisfactory (or good) results of operation can be achieved in cases where "sufficiently complete" planning of the state vector of the system and an effective system of penalties and incentives are instituted or "matching" of the interests of the center and the active elements is realized in one sense or another.

1.4.4 New research trends

Work is currently in progress on a number of new directions in the theory of active systems. Understandably, these efforts will aid in expanding the sphere of potential practical applications of the theory. We discuss these new directions in the present section.

Extension of hypotheses on the informedness of the center. It is apparent that the "informedness (or awareness) index" of the center regarding the models of the elements in real active OS can vary its meaning within extremely broad limits, from the case of full "informedness" of the center to the possibility of total "uninformedness". This has motivated a certain expansion of the set of hypotheses regarding the informedness of the center as treated in the theory of active systems (through inclusion in that set of the case of full informedness of the center regarding models of the elements and the case in which the center has no knowledge even of parametric representation of their models) and the development of corresponding statements of the management problem for those cases, bearing in mind their subsequent investigation.

Degree of centralization of the operating mechanisms in active systems. It is proposed that the problem of centralization of planning for a specified set of indices characterizing the state of the system and its elements be solved by comparing the corresponding criteria of efficiency of operation of the system. In the theory of active systems, the concept of the degree of centralization of an operating mechanism was first proposed in [32-33], where it was defined in terms of the set of planning indices, the "strength" of the responsibility of the elements for deviations of their states from the plan, and the constraints imposed by the center on the selection of states of the elements. This approach made it possible to introduce a partial order relation into the set of operating mechanisms, to formalize the problem of selecting the optimal degree of management centralization, to propose one version of classification of the operating mechanisms described in literature (uncontrollable and controllable markets, partially and completely centralized planning) and to perform a comparative analysis of those mechanisms. Following are the principal results obtained in this direction:

- a theorem on the growth of operating efficiency of the system with increasing degree of centralization of the operating mechanism (without regard for management expenditures, which increase with the degree of centralization);
- estimates of the cost of "decentralization" under conditions of full and partial informedness.

These tools make it possible, through the synthesis of an efficiency function and loss function depending on the degree of centralization, to solve the problem widely discussed in literature: *to determine the optimal degree of management centralization in the active OS*.

Active systems with dependent elements. An active system is called a system with dependent elements if its operating mechanism does not provide independent selection of states by the active elements. The strong feasibility condition can be satisfied in systems with dependent elements by invoking such mechanisms as specification of the sequence of "moves" of the elements and the introduction of a special rule for constraints on the elements' selection of their states (auctions, priority queues, random queues, quota constraints, etc.) or the introduction of a system for real-time control of the state-selection process. Only a few simple examples of active systems with dependent elements have been investigated so far [32].

Of special interest in the practical regard is the reduction of the problem of analysis of systems with active elements to the case of independent elements by means of behavioral hypotheses. For example, an element may assume that orders from external suppliers will be filled in the required quantity on schedule (see also the principle of coordination by prognosis of interactions). For "proper" operating mechanisms (i.e., which ensure the reliability of information and performance of plans), such a hypothesis makes it possible to apply the procedure of investigation of systems with independent elements. The first steps in the investigation of systems with dependent elements, clearly, are also best taken in the investigation of the case of full informedness of the center regarding the element models. Thus, the strategy of each active element in this case merely boils down to a state selection (on the basis of the criterion of maximization of the objective function), which is facilitated by game-theoretic analysis.

Satisfaction of the weak feasibility condition requires the solution of the following problems:

a) to prove the existence of control parameters (also called equilibrium parameters) that will, in conjunction with the principles adopted by the elements for selection of rational strategies, ensure the selection of feasible states by the elements in the given system;

b) to develop algorithms for computation of the equilibrium control functions or to develop iterative procedures for creating equilibrium control functions that "work" with the participation of the elements. A number of procedures of this kind have been investigated in studies on iterative planning and management (or, as they are also called, on decomposition methods).

Active systems with "dependent" operating periods and adaptive control schemes. Active systems with "dependent" operating periods include systems whose operation has a recurrent character, where the results of operation (reported information and states selected by elements) of the current operating period affect the "payoff" of the elements not only in the current, but also in subsequent operating periods (for a fixed system model). This type of situation arises when the center uses adaptive procedures for the generation of control parameters, adaptive procedures for the reconstruction of unknown (to the center) parameters of the models of active elements, or some combination of these procedures with other procedures for the generation of data and control parameters [32]. With this kind of interdependence between individual operating periods of the system, it may prove useful for the active element to "sacrifice a little" in the current operating period in order to "gain" more in subsequent periods. One of the key issues in this context is how the active element takes future into consideration. Several modes of formalization of "assimilating the future" in the criteria of the active element (sum of the "payoffs" for several periods, discontinuation of the sum of the "payoffs" after several periods, etc.) are being outlined in [32].

At present, investigation of several aspects of the operation of active systems with dependent periods has already begun. For example, the concept of the decisions of the game between active elements in a system with recurrent and dependent periods has been developed, and decisions of the element game subject to certain behavioral hypotheses have been determined [32].

It is essential to note that the additional difficulties (both for the center in the investigation and organization of efficient system operation and for the active elements in deciding a rational behavior) arising in connection with an interdependence between individual operating periods are largely associated with the difficulties of “assimilation of the future” in the criteria of the active elements. Accordingly, it is also instructive to examine operating mechanisms whereby the decision-making principle of the active element in the case of “coupled” operating periods does not depend (or depends only slightly) on the “mode of assimilation of the future” in its criteria. In particular, such operating mechanisms include the already-mentioned “progressive” operating mechanisms [32-33]. It can be well-recognized that with regard to the adaptive approach to data generation (or a countermeasure approach with heavy penalties) progressive mechanisms ensure reliable information on the element models.

Iterative control schemes in active systems. Iterative schemes are those for which the control parameters are generated in the planning stage by the organization of a multiple-step (iterative) procedure with the recruitment in each step of additional information requested by lower-level elements. A necessary factor of iterative schemes in this interpretation is that the stage of implementation of a state of the system not set in before the generation of control parameters in the system has been terminated.

Investigations of operating mechanisms with iterative control-parameter generation schemes within the framework of the theory of active systems are conducted with the intention of developing methods for their game-theoretic analysis and synthesis. And although iterative schemes have been studied for some time, a number of unsolved fundamental problems remain.

A great many publications have been devoted to the iterative method of generating control parameters (belonging to the family of decomposition methods). The substantiation of convergence and efficiency of such methods rests most significantly on certain hypotheses concerning the behavior of a subsystem in reporting of information. The legitimacy of this approach does not elicit any doubt with regard to the use of iterative methods for the solution of mathematical programming problems. However, if one is concerned with the treatment of iterative methods as planning methods for two-level active OS, the question of the efficiency of a particular method requires further critical examination. For example, at least two questions requiring further critical examination are indicated in [32-33]:

1) Will the decreed objective functions prescribed by the elements in a certain iterative scheme be consistent with the state of affairs in real OS?
2) Are the hypotheses adopted in certain iterative schemes regarding the behaviour of the elements in reporting of information rationally according to the criteria of the elements?

Thus, it is reasonable to assume that each element seeks to generate a “favorable” value of the control parameters established by the center specifically in the last iteration. Naturally, such a hypothesis on the behavior of the elements does not necessarily imply that the element will report in every iteration the plan optimizing their criteria in each step of the iterative procedure; i.e., whatever hypothesis about the “locally optimal” behavior of the elements in each iteration serves as the basis for substantiation of convergence and optimality of iterative schemes for the generation of control parameters may be unsatisfied.

Dynamic models in active systems. As this subtitle implies, we are concerned here with active systems in which the system model varies from one operating period to another. Such situations may evolve from a number of possible reasons listed below:

- global constraints (for example, an exogenic resource, a finite production level) “link” several operating periods at once;
- the sets of possible states of the elements vary (due to exogenic factors, scientific progress) from one operating period to another;
- the set of possible states of the system in the current period depends on the set of its possible states in the preceding operating period.

Only the first steps have been made towards investigation of the operation of active systems with dynamic models. For example, it has been shown (initially in the PLAN model [32] and later in a more general case[38]) that a number of results previously obtained for a static model can be carried over to the case of a dynamic model by enlarging the state space of the system. In the same vein, a number of problems are analyzed in conjunction with the aggregate description of sets of possible states of the system in the operating mechanisms of dynamic models. The already-mentioned problem of taking into consideration the “long-range forecasts” of the elements also arises in the case of dynamic models of active systems. This problem appears, for example, due to inconsistency between long-range forecasts of the center and the active elements. The center’s choice of a “planning horizon” (fixed or “sliding”) poses a substantial problem of its own.

Use of aggregate plans and aggregate descriptions of models of active elements in multilevel active systems. The methodology of extending the approach of the theory of active systems to these cases is discussed in part in [32-38]. The transition to operating mechanisms that apply a particular aggregation procedure makes it necessary to solve the problems of determining the “errors of aggregation”, which are rather complex from the mathematical standpoint. Some progress has been made in this direction for the case of full informedness of the center regarding the models of the active elements. The use of information-aggregation procedures in multilevel active systems is investigated in [32-33]. Cases of “ideal aggregation” are discussed in examples in [32-37]. It is important to note the new approach, first explored in a simple model in [32], to the determination of the optimal “aggregation condition”. It can be well-recognized that the larger the parameters used to describe the models of the elements, the more precise will be the description, but at the same time the more difficult the control process (in the sense that it is more difficult to ensure reliability of estimation of the parameters, execution of plans, and, accordingly, high operating efficiency of the system). In light of these two conflicting conditions, there is an optimal aggregation level ensuring maximum operating efficiency of the system subject to the condition of information reliability (within the aggregation error limits).

In the following Chapters 13 and 17 we will outline some more detailed examples of improving the efficiency of a multilevel OS by taking into account human factors influence.

Chapter 2. Justification of Using Probability Laws for Operations of Organization Systems under Random Disturbances

§2.1 Introduction

In various organization systems, e.g., in PERT analysis [42, 52, 58, 67, 69, 111, 128, 132, 134, 146, 166, 182-184, etc.] the activity-time distribution is assumed to be a beta-distribution, and the mean value and variance of the activity time are estimated on the basis of the "optimistic", "most likely" and "pessimistic" completion times, which are subjectively determined by an analyst. The creators of PERT (e.g., [42, 44, 59-60, 146, etc.]) worked out the basic concepts of PERT analysis, and suggested the estimates of the mean and variance values

$$\mu = \frac{1}{6}(a + 4m + b), \qquad (2.1.1)$$

$$\sigma^2 = \frac{1}{36}(b - a)^2, \qquad (2.1.2)$$

subject to the assumption that the probability density function (p.d.f.) of the activity time is

$$f(y) = \frac{\Gamma(\alpha+\beta)}{\Gamma(\alpha)\,\Gamma(\beta)} \frac{(y-a)^{\alpha-1}(b-y)^{\beta-1}}{(b-a)^{\alpha+\beta-1}}, \quad a < y < b, \quad \alpha,\beta > 0. \qquad (2.1.3)$$

Here a is the optimistic time, b - the pessimistic time, and m stands for the most likely (modal) time.

Since in PERT applications parameters a and b of p.d.f. (2.1.3) are either known or subjectively determined, we can always transform the density function to a standard form,

$$f(x) = \frac{\Gamma(\alpha+\beta)}{\Gamma(\alpha)\,\Gamma(\beta)}\, x^{\alpha-1}(1-x)^{\beta-1}, \quad 0 < x < 1, \quad \alpha,\beta > 0, \qquad (2.1.4)$$

where $x = \dfrac{y-a}{b-a}$ has the following parameters:

$$\mu_x = \frac{\mu_y - a}{b-a}, \quad \sigma_x = \frac{\sigma_y}{b-a}, \quad m_x = \frac{m_y - a}{b-a}. \qquad (2.1.5)$$

Let $\alpha - 1 = p$, $\beta - 1 = q$. Then p.d.f. (2.1.4) becomes

$$f(x) = \frac{\Gamma(p+q+2)}{\Gamma(p+1)\,\Gamma(q+1)}\, x^p(1-x)^q, \quad 0 < x < 1, \quad p,\ q > -1, \qquad (2.1.6)$$

with the mean, variance and mode as follows:

$$\mu_x = \frac{p+1}{p+q+2}, \tag{2.1.7}$$

$$\sigma_x^2 = \frac{(p+1)(q+1)}{p+q+2}, \tag{2.1.8}$$

$$m_x = \frac{p}{p+q}. \tag{2.1.9}$$

From (2.1.6) and (2.1.9) it can be obtained

$$f(x) = \frac{\Gamma(p+q+2)}{\Gamma(p+1)\,\Gamma(q+1)}\; x^p (1-x)^{p(1/m_x-1)}. \tag{2.1.10}$$

Thus, value m_x, being obtained from the analyst's subjective knowledge, indicates the density function. On the basis of statistical analysis and some other intuitive arguments, the creators of PERT assumed that $p+q \cong 4$. It is from that assertion that estimates (2.1.1) and (2.1.2) were finally obtained, according to (2.1.6-2.1.9).

Although the basic concepts of PERT analysis have been worked out many years ago [42, 146], they are open till now to considerable criticism. Numerous attempts have been made to improve the main PERT assumptions for calculating the mean μ_x and variance σ_x^2 of the activity-time on the basis of the analyst's subjective estimates. In recent years, a very sharp discussion [65, 74] has taken place in order to raise the level of theoretical justifications for estimates (2.1.1) and (2.1.2).

Grubbs [111] pointed out the lack of theoretical justification and the unavoidable defects of the PERT statements, since estimates (2.1.1) and (2.1.2) are, indeed, "rough" and cannot be obtained from (2.1.3) on the basis of values a, m and b determined by the analyst. Moder [142-143] noted that there is a tendency to choose the most likely activity – time m much closer to the optimistic value a than to the pessimistic one, b, since the latter is usually difficult to determine and thus is taken conservatively large. Moreover, it is shown [67] that value m, being subjectively determined, has approximately one and the same relative location point in $[a,b]$ for different activities. This provides an opportunity to simplify the PERT analysis at the expense of some additional assumptions. McCrimmon and Ryavec [136], Lukaszewicz [134] and Welsh [182] examined various errors introduced by the PERT assumptions, and came to the conclusion that these errors may be as great as 33%. Murray [146] and Donaldson [52] suggested some modifications of the PERT analysis, but the main contradictions nevertheless remained. Farnum and Stanton [58] presented an interesting improvement of estimates (2.1.1) and (2.1.2) for cases when the modal value m is close to the upper or lower limits of the distribution. This modification, however, makes the distribution law rather uncertain, and causes substantial difficulties to simulate the activity network.

Upon analyzing the most reasonable assumptions in PERT analysis, specific groups of assumptions can be considered, namely:

Option I [73-74]

(1) Assume a beta-distribution with pregiven values a, m and b.
(2) Restrict the set of possible beta-distributions to those for which

$$\sigma^2 = \frac{1}{36}(b-a)^2 .$$

(3) Approximate the mean by formula (2.1.1).

Option II [65]

(1) Same as for Option I.
(2) Restrict the set of possible beta-distributions to those for which $\alpha+\beta=4$.
(3) Approximate the variance by formula (2.1.2).

In our opinion, both Options I and II result in considerably rough approximations at stage (3) because of very hard restrictions at stage (2). These restrictions leave too little degrees of freedom for the next stage in order to obtain an accurate approximation for the entire distribution range.

In order to refine the estimates, two further options can be introduced as follows:

(1) Relaxing the restrictions at stage (2);
or
(2) Obtaining a more precise approximation at stage (3) by partitioning the distribution range, i.e., by introducing a piecewise approximation.

Option III [58] is based upon the second alternative and results in the following:

(1) Same as for Option I.
(2) Same as for Option I.
(3) Same as for Option I.
(4) Single out subinterval $\left[a^*, b^*\right] \in [0,1]$, where the estimate for a standardized beta density with $x = \dfrac{y-a}{b-a}$ provides a close approximation ($a^* = 0.13$, $b^* = 0.87$).
(5) In both intervals $\left[0, a^*\right]$ and $\left[b^*, 1\right]$ re-estimate values μ and σ^2 as constrained by the value of the mode.

Thus, Option III is an extension of Option I. Its main shortcoming is the difficulty of implementation in practical PERT applications, since raising the accuracy of the approximation makes the latter more complicated. In particular, to estimate or simulate the activity – time, one has to use three alternative estimates or three alternative beta-distributions, respectively.

Option IV [166-167] is facilitated by means of the first alternative, as follows:

(1) Same as for Options I, II and III.
(2) Restrict the set of possible beta-distributions to those for which value $k = \alpha + \beta$ is a constant (but not predetermined, $k = 4$, as in Option II).
(3) Restrict the set of possible beta-distributions to those for which the alternative variance value is equal to $\frac{(b-a)^2}{36}$.
(4) Determine value k and calculate estimates μ and σ^2 on the basis of a, m and b.

A comparative analysis [73] leads to the conclusion that for non-extreme values m Options I-IV provide for the same accuracy. For extreme values (since values b are often estimated conservatively large, extreme values m are usually located in the lower tail of the distribution), Option III delivers a better accuracy, than the other ones. As to Option IV, it results in estimates which, being as simple as the PERT ones, are more accurate.

A conclusion can be drawn [67] upon analyzing over a lengthy period different network projects that the "most likely" activity – time estimate is practically useless. Other statistical experiments [67] lead to the conclusion that additional assumptions $p = 1$ and $q = 2$ are reasonable, since they simplify the PERT analysis without compromising the accuracy estimates for the project as a whole. Thus, the p.d.f. in the PERT statements can be modified to a simpler one

$$f(x) = \frac{12}{(b-a)^4} (x-a)(b-x)^2, \tag{2.1.11}$$

with the mean, variance and mode as follows:

$$\mu_x = 0.2(3a+2b), \tag{2.1.12}$$

$$\sigma_x^2 = 0.04(b-a)^2, \tag{2.1.13}$$

$$m_x = \frac{2a+b}{3}. \tag{2.1.14}$$

This simplified modification has been used successfully in [67-69].

Besides the beta-distribution p.d.f. (2.1.4), other density functions have been examined as well [31]. Williams [183-184] accepts, besides the asymmetric beta p.d.f., symmetric p.d.f., e.g. normal and triangle distributions. However, an overwhelming majority of publications in the area of PERT analysis consider that an activity-time p.d.f. has to be asymmetric with finite upper and lower limits of the distribution. In addition, the following properties are usually accepted a priori in all project management systems which actually deal with network planning and control:

- the activity-time p.d.f. is a continuous curve;

- the activity-time p.d.f. has only one mode;
- both points of intersection of the activity-time p.d.f. with the abscissa axis are non-negative ones.

In this chapter, we will justify the beta-distribution p.d.f for an activity – time duration applicable to the cases when either one, or several identical processors, i.e., several generalized resource units, are operating the activity [9, 67].

§2.2 Case of one processor to operate a man-machine activity

We will consider a man-machine operation which is carried out by one processor, i.e., by one resource unit. The processor may be a machine, a proving ground, a department in a design office, etc.

Assume that the operation starts to be processed at a pregiven moment T_0. The completion moment F of the operation is a random value with distribution range $\left[T_1, T_2\right]$. Moment T_1 is the operation's completion moment on condition that the operation will be processed without breaks and without delays, i.e., value T_1 is a pregiven deterministic value. Assume, further, that the interval $\left[T_0, T_1\right]$ is subdivided into n equal elementary periods with length $\left(T_1 - T_0\right)/n$. If within the first elementary period $\left[T_0, T_0 + \left(T_1 - T_0\right)/n\right]$ a break occurs, it causes a delay of length $\Delta = \left(T_2 - T_1\right)/n$. The operation stops to be processed within the period of delay in order to undertake necessary refinements, and later on proceeds functioning with the finishing time of the first elementary period

$$T_0 + \left(T_1 - T_0\right)/n + \left(T_2 - T_1\right)/n = T_0 + \left(T_2 - T_0\right)/n.$$

It is assumed that there cannot be more than one break in each elementary period. The probability of a break at the very beginning of the operation is set to be p. However, in the course of carrying out the operation, the latter possesses certain features of self-adaptivity, as follows:

- the occurrence of a break within a certain elementary period results in increasing the probability of a new break at the next period by value η, and
- on the contrary, the absence of a break within a certain period decreases the probability of a new break within the next period, practically by the same value.

2.2.1 The concept of self-adaptivity

The probabilistic self-adaptivity can be formalized as follows:

Denote A_i^k the event of occurrence of a break within the $(i+1)$-th elementary period, on condition, that within the i preceding elementary periods k breaks occurred, $1 \le k \le i \le n$. It is assumed that relation

$$P\left(A_i^k\right) = \frac{p + k \cdot \eta}{1 + i \cdot \eta} \tag{2.2.1}$$

holds. Note that (2.2.1) is, indeed, a realistic assumption.

Relation (2.2.1) enables obtaining an important assertion. Let $P\left(A_i^0\right)$ be the probability of the occurrence of a break within the $(i+1)$-th period on condition, that there have been no breaks at all as yet. Since

$$P\left(A_i^0\right) = \frac{p}{1 + i \cdot p}, \tag{2.2.2}$$

it can be well-recognized that relation

$$\frac{P\left(A_i^{k+1}\right) - P\left(A_i^k\right)}{P\left(A_i^0\right)} = \frac{\eta}{p} \tag{2.2.3}$$

holds. Thus, an assertion can be formulated as follows:

Assertion. Self-adaptivity (2.2.1) results in a probability law for delays with a constant ratio (2.2.3) for a single delay.

2.2.2 Calculating the activity – time distribution

Let us calculate the probability $P_{m,n}$ of obtaining m delays within n elementary periods, i.e., the probability of completing the operation at the moment

$$F = T_1 + m \cdot \Delta = T_1 + \frac{m}{n}\left(T_2 - T_1\right).$$

The number of sequences of n elements with m delays within the period $\left[T_0, F\right]$ is equal C_n^m, while the probability of each such sequence equals

$$\frac{\left[\prod_{i=0}^{m-1} \left(p + i\eta\right)\right]\left[\prod_{i=0}^{n-m-1}\left(1 - \eta + i\eta\right)\right]}{\prod_{i=0}^{n-1}\left(1 + i\eta\right)}. \tag{2.2.4}$$

Relation (2.2.4) stems from the fact that if breaks occurred within h periods and did not occur within k periods, the probability of the occurrence of the delay at the next period is equal

$$\frac{p + h\eta}{1 + \left(k+h\right)\eta}, \tag{2.2.5}$$

while the probability of the delay's non-appearance at the next period satisfies

$$\frac{1-\eta+k\eta}{1+(k+h)\eta} \,. \tag{2.2.6}$$

Using (2.2.4-2.2.6), we finally obtain

$$P_{m,n} = C_n^m \frac{\left[\prod_{i=0}^{m-1}(p+i\eta)\right]\left[\prod_{i=0}^{n-m-1}(1-\eta+k\eta)\right]}{\prod_{i=0}^{n-1}(1+i\eta)} \,. \tag{2.2.7}$$

Note that $\eta=0$, i.e., the absence of self-adaptivity, results in a regular binomial distribution.

Let us now obtain the limit value $P_{m,n}$ on condition that $n\to\infty$. From relation (2.2.7) we obtain

$$\frac{P_{m+1,n}}{P_{m,n}} = \frac{n-m}{m+1}\,\frac{p+m\eta}{1-p+(n-m-1)\eta} \,. \tag{2.2.8}$$

Denoting $\frac{p}{\eta}=\alpha$, $\frac{p}{\eta}\left(\frac{1}{p}-1\right)=\beta$, we obtain

$$\frac{P_{m+1,n}-P_{m,n}}{P_{m,n}} = \frac{(\alpha-1)n+(2-\alpha-\beta)m-\beta+1}{(m+1)(\beta+n-m-1)} = \frac{(\alpha-1)+(2-\alpha-\beta)\frac{m}{n}+\frac{1-\beta}{n}}{n\frac{m+1}{n}\left(1-\frac{m+1}{n}+\frac{\beta}{n}\right)} \,.$$

Denoting $m/n=x$, $(m+1)/n=x+\Delta x$, $P_{m,n}=y$, $P_{m+1,n}=y+\Delta y$, via convergence $n\to\infty$ or $\Delta x\to 0$ and, later on, by means of integration, we finally obtain

$$y = C\,x^{\alpha-1}(1-x)^{\beta-1} \,. \tag{2.2.9}$$

It can be well-recognized that the p.d.f. of random value $\xi=\lim_{n\to\infty}\frac{m}{n}$ satisfies

$$p_\xi(x) = \frac{1}{B(\alpha,\beta)}\,x^{\alpha-1}(1-x)^{\beta-1} \,, \tag{2.2.10}$$

where $B(\alpha,\beta)$ represents the Euler's function. Thus, relation (2.2.10) practically coincides with (2.1.10).

Thus, ξ is a random value with the beta-distribution activity – time p.d.f. By transforming $x = (y-a)/(b-a)$, we obtain the well-known p.d.f. (2.1.3).

Thus, under certain realistic assumptions, the following conclusions can be drawn from the Section:

1. Under certain realistic assumptions we have proven theoretically that the activity-time distribution satisfies the beta-distribution with p.d.f. (2.1.3) being used in PERT analysis.
2. Changing more or less the implemented assumptions, we may alter to a certain extent the structure of the p.d.f. At the same time, its essential features (e.g., asymmetry, unimodality, etc.) remain unchanged.
3. The outlined above results can be applied to semi-automated activities, where the presence of man-machine influence under random disturbances is, indeed, very essential. Those activities are likely to be considered in organization systems (e.g. in project management), but not in fully automated plants.

§2.3 Case of several processors

To present the results, we will require additional definitions.

Call an operation area W an accessible area open to several identical processors in charge of operating simultaneously a certain activity. Call a specific operation area Z a part of an operation area open to one processor only. Thus, relation

$$\frac{W}{X} = Z \tag{2.3.1}$$

holds, where X stands for the number of processors being implemented in W. Call, further, an optimal specific operation area Z^{opt} in case it enables the processor's work with its maximal labor productivity. Note that the term "maximal labor productivity" denotes the maximal part (usually in percentages) of the volume V of the work to accomplish the activity by means of one processor per time unit. It can be well-recognized that setting value Z^{opt} (for a pregiven operation area W) results in determining the optimal number of processors X^{opt} satisfying

$$X^{opt} = \frac{W}{Z^{opt}}. \tag{2.3.2}$$

If for a certain activity with preset operation area W value Z becomes less than Z^{opt}, the result is both decreasing the processor's labor productivity and increasing the number of processors. This means, in turn, that for a preset W increasing the number of processors results at first in increasing the processor's labor productivity up to a certain value X^{opt}. The subsequent increase of value X results in decreasing the processor's labor productivity. A long variety of statistical experiments including time-studies [4, 9, 67] leads to the conclusion that the production speed to process an activity by using several processors is at first a linear function of the increasing number of processors, i.e., relation

$$v = \frac{dV}{dt} = \eta^{opt} \cdot X, \quad 0 < X \le X^{opt}, \tag{2.3.3}$$

holds, η^{opt} being the processor's maximal labor productivity. However, an additional increase of the number of processors, beginning from X^{opt}, results in two contradictory tendencies:

- the labor productivity of a routine processor starts decreasing, i.e.,
  $$X > X^{opt} \Rightarrow \eta < \eta^{opt}; \tag{2.3.4}$$
  within a relatively short interval $\left[X^{opt}, X^{max} \right]$ the production speed v *proceeds increasing, but not linearly*, since increasing value X slightly overbalances at first the decrease of the labor productivity value η (see Fig. 2.1). It goes without saying that the length and the structure of interval $\left[X^{opt}, X^{max} \right]$ depends on the activity under consideration;
- by increasing value $X > X^{max}$ production speed v decreases due to a significant decrease in labor productivity η.

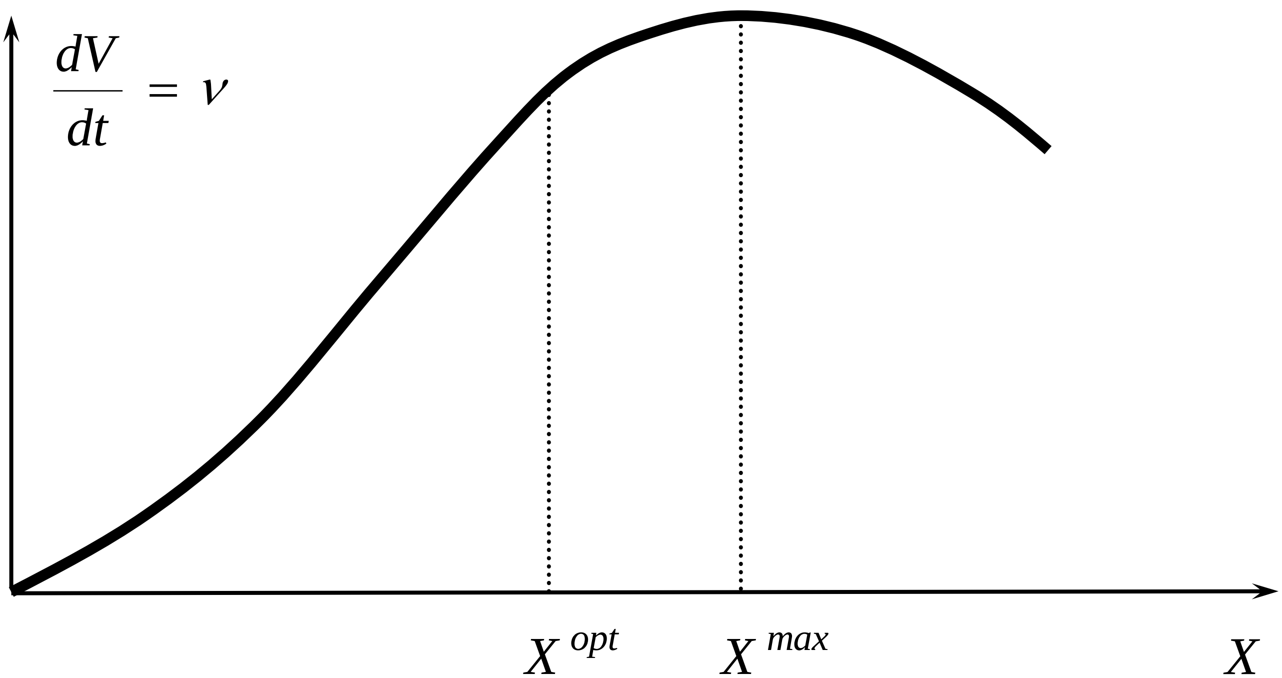

Figure 2.1. <u>The dependence of production speed on the number of processors</u>

Note that we can describe the dependence $\nu = f(X)$, $X^{opt} \le X < \infty$, by choosing a function

$$\nu = aX^b e^{cX}, \tag{2.3.5}$$

where parameters a, b and c are unknown and have to be determined.

Thus, the general problem is to "sew together" two curves

$$\begin{cases} \nu = \eta^{opt} \cdot X, & 0 < X \le X^{opt} \\ \nu = aX^b e^{cX}, & X^{opt} \le X < \infty \end{cases} \tag{2.3.6}$$

in order to determine the production speed according to its properties outlined above.

2.3.1 *<u>The model</u>*

We suggest estimating speed (2.3.6) as follows:

<u>Stage 1</u>. Determine the bundle of straight lines passing the point $A(X^{opt}, \nu^{opt})$:

$$y - \nu^{opt} = \eta^{opt}(X - X^{opt}). \tag{2.3.7}$$

The first derivative $\frac{dy}{dx}$ from (2.3.7) is equal η^{opt}, while the first derivative $\frac{dy}{dx}$ from $y = aX^b e^{cX}$, is equal

$$y' = aX^{b-1}e^{cX}(b + cX). \tag{2.3.8}$$

In order to obtain a smooth function from the "sewed together" functions (2.3.4) and (2.3.5), their first derivatives at the point $X = X^{opt}$ have to be equal, i.e., relation

$$\eta^{opt} = a(X^{opt})^{b-1} e^{cX^{opt}} (b + cX^{opt}) \tag{2.3.9}$$

holds. Taking into account (2.3.9), as well as

$$a = \frac{\nu^{opt}}{(X^{opt})^b e^{cX^{opt}}}, \tag{2.3.10}$$

the straight line (2.3.7) can be described in the form

$$\nu - \nu^{opt} = \frac{\nu^{opt}(b + cX^{opt})}{X^{opt}}(X - X^{opt}). \tag{2.3.11}$$

Using relations (2.3.8, 2.3.10-2.3.11), we obtain

$$\begin{cases} \nu = \dfrac{\nu^{opt}\left(b+cX^{opt}\right)}{X^{opt}} \cdot X + \nu^{opt}\left(1-b-cX^{opt}\right), & 0 < X \le X^{opt} \\ \nu = \dfrac{\nu^{opt}}{\left(X^{opt}\right)^{b} e^{cX^{opt}}} X^{b} e^{cX} & , X^{opt} \le X < \infty \end{cases} . \quad (2.3.12)$$

Stage 2. Shift function (2.3.12) in order to make it pass the co-ordinate source

$$\begin{cases} \nu = \dfrac{\nu^{opt}\left(b+cX^{opt}\right)}{X^{opt}} \cdot X & , 0 < X \le X^{opt} \\ \nu = \dfrac{\nu^{opt}}{\left(X^{opt}\right)^{b} e^{cX^{opt}}} X^{b} e^{cX} - \nu^{opt}\left(1-b-cX^{opt}\right), & X^{opt} \le X < \infty \end{cases} . \quad (2.3.13)$$

By determining the extreme values of function (2.3.13) it can be well-recognized that the function obtains its minimum at two points $X = 0$ and $X = \infty$, and has the only maximum at point

$$X = X^{max} = -\frac{b}{c}. \quad (2.3.14)$$

Stage 3. The results obtained enable calculating values a, b and c:

$$a = \frac{\eta^{opt}}{\left(X^{opt}\right)^{b-1} e^{cX^{opt}}}, \quad (2.3.15)$$

$$b = \frac{X^{\max}}{X^{\max} - X^{opt}}, \quad (2.3.16)$$

$$c = -\frac{1}{X^{max} - X^{opt}}. \quad (2.3.17)$$

Thus, conclusions can be drawn that in order to determine production speeds (2.3.6) for a preset operation area W, one has to know in advance only three parameters: η^{opt}, X^{opt} and $X^{\max}$.

Using (2.3.10, 2.3.15-2.3.17), we can finally obtain function (2.3.6) in the form

$$\begin{cases} \nu = \eta^{opt} \cdot X, & 0 \le X \le X^{opt} \\ \nu = \lambda X^{b} e^{cX}, & X^{opt} < X < \infty \end{cases}, \quad (2.3.18)$$

where $\lambda = \eta^{opt}\left(\dfrac{e}{X^{opt}}\right)^{b-1}$.

Note that in terms of specific operation area Z and operation area W, function (2.3.18) is as follows:

$$\begin{cases} v = \eta^{opt} \cdot \dfrac{W}{Z} & , \quad Z^{opt} < Z < \infty \\ v = \lambda \cdot W \cdot Z^{-\frac{Z^{opt}}{Z^{opt} - Z^{max}}} exp\left\{ - \dfrac{Z^{opt} Z^{max}}{Z\left(Z^{opt} - Z^{max}\right)} \right\}, & 0 < Z \le Z^{opt} \end{cases} , \qquad (2.3.19)$$

where $Z^{\max} = \dfrac{W}{X^{\max}}$, η^{opt} is the maximal labor productivity of a routine processor, W and $Z = \dfrac{W}{X}$ are the operation and specific operation areas, correspondingly, and $\lambda = \eta^{opt}\left[e \cdot Z^{opt}\right]^{\frac{Z^{\max}}{Z^{opt} - Z^{\max}}}$.

Since the labor productivity (LP) is usually obtained by dividing the speed v by the number of processors, one can easily obtain

$$\begin{cases} LP = \eta^{opt} & , \quad 0 < X \le X^{opt} \\ LP = a \cdot X^{b-1} \cdot e^{cX} , & X^{opt} \le X < \infty \end{cases} \qquad (2.3.20)$$

and, later on, the activity – time duration

$$t = \frac{V}{v} \qquad (2.3.21)$$

satisfying

$$\begin{cases} t = \dfrac{V}{\eta^{opt} \cdot X} & , \quad 0 < X \le X^{opt} \\ t = \dfrac{V}{a \cdot X^{b} \cdot e^{cX}} , & X^{opt} \le X < \infty \end{cases} . \qquad (2.3.22)$$

2.3.2 Random labor productivities

However, a specific possibility has to be considered, namely, when the LP parameter is a random value. If the activity under consideration is processed under random disturbances, it is usually taken into account that even for the case of several identical processors the maximal LP-value of a routine processor has a normal distribution with the p.d.f.

$$p\left(\eta^{opt}\right) = \frac{N}{\sqrt{2\pi \cdot V\eta^{opt}}} \cdot exp\left\{ - \frac{\left[\eta^{opt} - \left(E\eta^{opt}\right)\right]^2}{2V\eta^{opt}} \right\}, \qquad (2.3.23)$$

where $E\eta^{opt}$ and $V\eta^{opt}$ are the mean and the variance values of η^{opt}, correspondingly; value N is obtained from an obvious relation

$$N \cdot \int_{\eta_a^{opt}}^{\eta_b^{opt}} p\left(\eta^{opt}\right) d\eta = 1, \qquad (2.3.24)$$

where η_a^{opt} and η_b^{opt} represent the lower and upper bounds, correspondingly, of the maximal labor productivity of a routine processor.

Thus, the activity – time duration t obtained from (2.3.22) becomes a random value, where parameter η^{opt} has a normally distributed p.d.f. Note that values η_a^{opt} and η_b^{opt} can be estimated by means of extensive statistical experimentation.

Values represented like (2.3.22) have been investigated [53] with the conclusion to be drawn that the random activity-time duration has a p.d.f.

$$p(t) = \frac{N\upsilon}{\sqrt{2\pi}} \cdot t^{-2} \cdot exp\left\{ -\frac{1}{2}\left(\frac{\upsilon}{t} - k\right)^2 \right\}, \qquad (2.3.25)$$

where $\upsilon = \frac{V}{\sqrt{D\eta^{opt}}}$ and $k = \frac{M\eta^{opt}}{\sqrt{D\eta^{opt}}}$. The corresponding probability function is as follows:

$$F(T) = \frac{N\upsilon}{\sqrt{2\pi}} \int_0^T t^{-2} \cdot exp\left\{ -\frac{1}{2}\left(\frac{\upsilon}{t} - k\right)^2 \right\} dt, \qquad (2.3.26)$$

which can be simplified to

$$F(T) = N\left[1 - \Phi\left(\frac{\upsilon}{T} - k\right)\right], \qquad (2.3.27)$$

where $\Phi(x) = \frac{1}{\sqrt{2\pi}} \int_0^x e^{-z^2/2} dz$.

Note that if the number of processors X is equal X^{opt}, the p.d.f. does not depend on values a, b and c. However, if $X \neq X^{opt}$, the p.d.f. function (2.3.25) becomes essentially more complicated.

The p.d.f. (2.3.25) is an asymmetric distribution with unimodal value

$$m_t = \frac{\upsilon k}{4}\left(\sqrt{1 + \frac{8}{k^2}} - 1\right), \qquad (2.3.28)$$

as well as approximate values of the mean

$$\mu_t \approx \frac{\upsilon}{k}\left(1 + \frac{1}{k^2}\right) \tag{2.3.29}$$

and variance

$$V_t \approx \frac{\upsilon^2}{k^4}\left(1 + \frac{8}{k^2}\right), \tag{2.3.30}$$

and is close to a beta-distribution p.d.f.

The following conclusions can be drawn from the results obtained:

1. For a broad spectrum of activities being processed by means of several identical resource units, the corresponding time – activity density functions prove to be asymmetric functions with finite upper and lower distribution limits. Those p.d.f.'s are close to a beta-distribution p.d.f.
2. Various assumptions in activity – time analysis (and in risk analysis as well !) center on determining a numerous "family" of beta-distributions with different versions - parameters α and β - of the general p.d.f. (2.1.3). Those versions may result in changing certain estimates for certain activities. At the same time, *they have practically no influence on the project as a whole.*
3. Thus, a general conclusion can be drawn that a random activity – time duration has a very high potential to be close to one of the beta-distribution probability density functions. The obtained theoretical grounds cover a broad variety of activities including the man-machine activities (with one processor) and semi-automated activities (with several processors).

§2.4 Some stable distribution laws close to β-distribution

We have shown that the β-distribution law can be used effectively to estimate the random duration of *one activity*. Moreover, such a conclusion can be drawn for a broad spectrum of organization systems. However, β-distribution becomes less effective to calculate *a fragment*, i.e., *a group of activities* entering an organization system. For example, calculation of a network with a deterministic structure is known to be reducible to computing the time of completion of the final network event or duration of the critical path. This purpose is commonly achieved by implementing Ford-Fulkerson algorithms [59-60] which can be easily run on computers. If the network is ordered so that $i < j$ for any activity (i, j) in the network, where i and j are, respectively, the initial and the end events of an arc, then the times of occurrence of network events in the order of their numbers ($j = 1,2,...,n$) are established using the following recurrent formula:

$$T_j = \max_i \{T_i + t(i, j)\}, \tag{2.4.1}$$

where $t(i, j)$ stands for the duration of activity (i, j); i runs the sequence of the numbers of the initial events of all arcs ending by arc j; T_j is the time of occurrence of the j-th event; and T_n is the length of the critical path of the entire network (T_{cr}).

In the stochastic network, $t(i, j)$ and, consequently, T_{cr} are random variables. Therefore, maximization and summarizing in the above formula are replaced by operations over the corresponding p.d.f. Therefore, the main task of methods facilitating calculations in stochastic networks, boils down to performing operations over p.d.f.'s, that is, to numerical calculations of the p.d.f.'s or their estimates. In numerous papers [67, 69, 92], attempts were made to carry out a probability-theoretical study of the laws of distribution of the durations of execution of both individual network fragments and the entire project as a whole.

Let us consider the laws of distribution that can characterize the duration of activity execution in such a network. Stability to the main operations over durations at the events and in the chains of the network will be used as the criterion. The stable laws arising at some point of the network retain their analytical form over some segment of the network. A distribution law stable to some operation can be the limiting law to which the resulting distribution tends under the infinite increase in the number of original random variables involved in the operation. Stated differently, for the law to be limiting (asymptotic), stability is the prerequisite. One or another form of the limiting law is defined, of course, by the properties of the initial distributions. At the same time, one can well-recognize that in real systems of network planning and control (NPC), traditional laws of distribution of execution of individual operations have already formed. In particular, the β-distribution with density (2.1.3)

$$B(p,q,x) = \begin{cases} x^{p-1}(1-x)^{q-1} / B(p,q) & \text{for } 0 \le x \le 1 \\ 0 & \text{for } x < 1,\ x > 1, \end{cases} \tag{2.4.2}$$

where $B(p,q)$ is the β-function, is accepted in some PERT-based NPC systems as the distribution of duration of activity execution. Its choice cannot be strictly substantiated, yet analysis of large volumes of statistical data and the fact that the general form of the β-distribution is defined by several (not many) factors argue for using the β-distribution as an *a priori* distribution. The experts responsible for execution of each activity must estimate the minimal ($a(i, j)$), maximal ($b(i, j)$), and most probable ($m(i, j)$) durations for activity (i, j).

As shown above, the β-distribution used in PERT systems was chosen so that the expectation $M(i, j)$ and variance $\sigma(i, j)$ of the time of execution for activity (i, j) satisfy

$$M(i, j) = [a(i, j) + 4m(i, j) + b(i, j)]/6,$$

$$\sigma^2(i,j) = \left[\left(b(i,j) - a(i,j)\right)^2\right] \Big/ 36 .$$

Since the β-distribution is an unstable law of distribution, one needs to determine laws of distribution such that, on one hand, they are close to β-distribution and, on the other hand, enable one to calculate the resulting function of distribution of the durations of fragments or entire network, such that it is reducible to algebraic operations over the parameters of these distribution laws which would replace involved calculations and laborious analysis. Laws stable to the main operations in the network models, that is, summarizing and maximization of the independent random variables (durations of individual activities), can be used as such.

2.4.1 Laws of distribution stable to operations of convolution

Let us consider the main operations of calculating network probabilities. In a stochastic network, the chain of activities following one after another (concatenation of arcs) can be replaced by one equivalent activity with the distribution of probabilities of the time of its execution equal to the p.d.f. of the sum of random variables, that is, the durations of executing the activities included in the chain. For example [139], it is known that upon summarizing independent random variables the p.d.f. density f_i ($i = 1,2,...,n$) of the summarized random variables is as follows:

$$f_{\sum(n)} = f_1^* f_2^* f_3^*,..., f_n^*,$$

where

$$f_i^* f_j^* = \int_{-\infty}^{\infty} f_i(z) f_j(x-z) dz = \int_{-\infty}^{\infty} f_i(x-z) f_j(z) dz .$$

This operation is also called the convolution of functions f_i^* and f_j^*. Since in the considered case the time of activity execution is a positive value and, consequently, f_i^* of the negative arguments equals zero, we obtain that relation

$$f_i^* f_j^* = \int_0^x f_i(z) f_j(x-z) dz = \int_0^x f_i(x-z) f_j(z) dz$$

holds. It is usually postulated that the times of carrying out individual activities are independent. Characteristic functions are used to investigate the operation of composition of random variables. The characteristic function of a random variable f_i^* is determined as the mathematical expectation

$$\left|f_\eta(t)\right| = \int_a^b f_\eta(x) e^{itx} dx$$

of random variable e^{itx}, where t is the real parameter. Interval (a,b) defines therefore the domain of definition of random variable η, because outside it the density of η equals zero. Then the characteristic function is, obviously, obtained by applying the Fourier transform to p.d.f. density $f_\eta(x)$:

$$\left|f_\eta(t)\right| = \int_{-\infty}^{\infty} f_\eta(x) e^{itx} dx .$$

The main advantage of characteristic functions lies in the fact that their characteristic functions are multiplied upon composition of random variables. Under certain assumptions regarding random variables, the counterparts of characteristic functions can be used alone with characteristic functions themselves: for integer random variables, the ξ-generating functions; for positive random variables, the Laplace ξ-transform of the corresponding densities of the distributions

$$\left|f_\xi^*(P)\right| = \int_0^{\infty} f_\xi(x) e^{-px} dx .$$

The Laplace transform is used also for analyzing distributions with distribution functions which exponentially approach unity for the argument tending to $+\infty$. The generating functions and Laplace transforms offer multiplicative properties (multiplied upon composition of distributions) and define uniquely the corresponding distributions.

As demonstrated above, the p.d.f. of the time of execution of all activities in a chain of the network is defined as the p.d.f. of the sum of independent random variables; therefore, the composition-stable laws of distribution play an ever increasing role in the theory of network models. A distribution is called *composition-stable* if for any $a_1 > 0$, b_1, $a_2 > 0$, and b_2 there will be $a > 0$ and b such that for all x

$$F(a_1 x + b_1) F(a_2 x + b_2) = F(ax + b).$$

As proved by Khinchin and Levy [112], the natural algorithms of characteristic functions that are stable to composition of distributions and only they admit representation

$$\ln f^*(t) = i\gamma t - C|t|^\alpha \{1 + i\beta t \omega(t,\alpha)/|t|\}, \qquad (2.4.3)$$

where α, β, γ and c are constants ($0 \le \alpha \le 2$, $-1 \le \beta \le 1$, $0 \le c$, and γ is any real number),

$$\begin{cases} \omega(t,\alpha) = \tan(\pi\alpha/2) & for \quad \alpha \neq 1 \\ \omega(t,\alpha) = 2\ln(t)/\pi & for \quad \alpha = 1 \end{cases}.$$

Here, α is called the characteristic parameter of a stable law. For $\alpha = 2$, the above relation boils down to the characteristic function of the normal law. If $\alpha = 1$ and $\beta = 0$, we obtain the characteristic function of the Cauchy law.

It can be well-recognized that the normal law is regarded the most important law stable to the operation of summarizing. By the central limit theorem, this distribution law has an asymptotic distribution for the sums of independent random variables under rather general assumptions about the p.d.f.'s of the random variables involved in summarizing. The summarized random variables may have different distributions, provided they are indefinitely small, and their number tends to infinity [157-158]. In the case of a positive random variable (consider, for example, the activity duration), this law, however, ceases to be accurate as it provides nonzero values of the probabilities for the negative argument. Thus, the normal law truncated for negative arguments is therefore unstable [112]. Besides the normal law, other composition-stable laws are known; they make up the class of infinite-variance laws depending on both parameters α and β. Of special interest is the positive definite law with p.d.f. density

$$f(x) = e^{-1/2x} \Big/ \sqrt{2\pi x^{3/2}} \quad for \quad x > 0, \tag{2.4.4}$$

which was used by Ringer [157-158] and obtained in the explicit form by Smirnov [112]. The characteristic function $f^*(t)$ of this law may be obtained from the general relation (2.4.3) for the composition-stable laws and $\alpha = 1/2$, $\beta = 1$, $\gamma = 0$, $c = 1$.

As a matter of fact, the distribution laws considered above, namely - the normal law, Cauchy law, and Smirnov law - exhaust the list of existing explicit composition-stable laws. In what follows, we denote by X_α all stable laws, where α is their characteristic parameter as represented by (2.4.3).

It is possible to demonstrate [69] that for $x \to \infty$ the p.d.f. becomes asymptotically close to

$$B(\alpha,\beta)/x^{\alpha+1}, \quad -1 \leq \beta \leq 1, \quad 0 < \varepsilon < 2, \tag{2.4.5}$$

where $B(\alpha,\beta)$ is independent of x.

2.4.2 <u>Distribution laws stable to operations of maximization</u>

Let us consider another basic operation used to simulate the stochastic network. Focus will be made on determination of the p.d.f. of the time of occurrence of some event. In classical network models, the random variable representing the time of occurrence of an event is equal to the maximal value among all times of completion of the activities entering the given event:

$$\eta_j = \max_i [\eta(i, j)],$$

where η_j stands for the time of occurrence of event j and $\eta(i, j)$ is the time of completion of the activity represented by the arc with initial event i and final event j. Since any activity (arc) exiting the given event (event) cannot start before the occurrence of that event, we take the instant of event occurrence as the origin for counting the duration of executing any subsequent chain of activities.

The p.d.f. of random variable η_j may be determined by multiplying the integral distributions of random variables $\eta(i, j)$, provided they are assumed to be independent:

$$F_{\eta(i,j)}(x) = \prod_i F_{\eta(i,j)}(x).$$

Therefore, at the events of the stochastic network we have the operation of multiplication of the integral distribution

$$F_{n\ \max}(x) = \prod_{i=1}^{n} F_i(x).$$

Now, we switch over to natural logarithms of both sides of the last equality:

$$\ln F_{n\ \max}(x) = \ln \sum_{i=1}^{n} F_i(x).$$

After differentiation, we obtain

$$f_{n\ \max}(x)/F_{n\ \max}(x) = \sum_{i=1}^{n} f_i(x)/F_i(x),$$

where $F_{n\ \max}(x)$ and $f_i(x)$ stand for appropriate p.d.f. densities.

If we introduce the function $\beta(x) = f(x)/F(x)$, then

$$\beta_{n\ \max}(x) = \sum_{i=1}^{n} \beta_i(x),$$

that is, upon maximization of the random variables, their characteristics $\beta_i(x)$ are summarized.

Now, one may represent integral distributions $F(x)$ in terms of $\beta(x)$:

$$\beta(x)dx = dF(x)/F(x).$$

Integrating both sides of the resulting equality from t to infinity yields in

$$\int_t^{\infty} \beta(x)dx = \ln F(x)\Big|_t^{\infty};$$

since $F(\infty)=1$, we obtain $F(t)=e^{-\gamma(t)}$, where $\gamma(t)=\int_t^{\infty}\beta(x)dx$.

Similar to $\beta_i(x)$, random variables $\gamma_i(x)$ are also summarized upon maximization. Conditions $F(0)=0$ and $F(\infty)=1$ suggest that - must be a positive decreasing function with $\gamma(0)=\infty$ and $\gamma(\infty)=0$. If we require that upon summarizing functions $\gamma_i(x)$ their form would be retained to within the linear transformation of the argument, then the distribution law will be stable to the operation of maximization. The simplest function of this kind would be as follows:

$$\gamma(x)=(\Theta/x)^{\nu} \quad (\Theta>0, \nu>0).$$

Then, we obtain a distribution with integral function

$$F(x)=\begin{cases} \exp(-\Theta/x)^{\nu} & \text{for } x>0 \\ 0 & \text{for } x<0 \end{cases} \tag{2.4.6}$$

and density

$$f(x)=\begin{cases} \nu\Theta^{\nu}\exp(-\Theta/x)^{\nu}/x^{\nu+1} & \text{for } x>0 \\ 0 & \text{for } x<0 \end{cases}. \tag{2.4.7}$$

Let us determine the mode X_m of this distribution:

$$f(x)=\nu\Theta^{\nu}\exp(-\Theta/x)^{\nu}\left[\nu\Theta^{\nu}/x^{\nu}-(\nu+1)\right]/x^{\nu+2}.$$

Hence,

$$X_m^{\nu}=\nu\Theta^{\nu}/(\nu+1), \quad X_m=\Theta[\nu/(\nu+1)]^{1/\nu}. \tag{2.4.8}$$

Therefore, mode X_m is proportional to parameter Θ. If random variables obeying such a law with identical parameters ν for all random variables are maximized, then we obtain the same distribution with parameter

$$\Theta_{n\max}^{\nu}=\sum_{i=1}^{n}\Theta_i^{m},$$

which stems from the fact that corresponding functions $\gamma_i(x)$ are summarized. Consequently, with regard to (2.4.8) the mode of the resulting distribution X_{mp} may be represented as

$$X_{mp} = \left[\nu \sum_{i=1}^{n} \Theta_i^m / (\nu + 1) \right]^{1/\nu} = \left[\sum_{i=1}^{n} \Theta_i^m \nu / (\nu + 1) \right]^{1/\nu},$$

that is, upon maximization of such random variables

$$X_{mp}^{\nu} = \sum_{i=1}^{n} X_{mi}^{\nu}$$

and for $\nu = 1$, in particular, the modes are summarized. It can be well-recognized that distribution (2.4.6) is limited from left:

$$f(x) = \begin{cases} \exp\left[\Theta/(x-\alpha)\right]^{\nu} & for \quad x \geq \alpha > 0 \\ 0 & for \quad x < \alpha \end{cases}.$$

If doing so, the entire curve and, correspondingly, the mode are just right-shifted by parameter α. It can be well-recognized [69] that the following stability condition is satisfied for law (2.4.6):

$$F(\alpha_1 x) F(\alpha_2 x) = F(\alpha x),$$

where $x \geq 0$ and $\alpha_1, \alpha_2, \alpha > 0$, that is, these distributions may differ in parameters Θ_i. In terms of [69], p.d.f. (2.4.6) is often referred to as the Frechét law and denoted by $\Phi_\nu(x)$.

The advantages of implementing the Frechét law as the p.d.f. of the duration of executing an activity from the network fragment are as follows:

1. Positive definiteness ($F(x) = 0$ for $x \leq 0$).
2. Simplicity of limiting from left by any value $\alpha \geq 0$ (the density function $f(x)$ has no discontinuity at truncation point $x = \alpha$, that is, $F(\alpha + 0) = 0$, where α can be treated as the minimal time required to execute the given activities).
3. Stability to the operation of maximizing random variables with different parameters Θ_i (consequently, different modes).
4. Simplicity of determining the new parameter $\Theta_{n\,\max}^{\nu}$ obtained as the result of maximization.

Therefore, if the p.d.f. of the time of executing all parallel activities between two events of the network is described using the given law with fixed parameter ν value, then they may be readily replaced by one activity distributed similarly and with parameter

$$\Theta_{n\,\max}^{\nu} = \sum_{i=1}^{n} \Theta_i,$$

which raises the question as to whether there exists a value $\nu = \lambda$ such that equality

$$f_\lambda(x/\Theta_1) * f_\lambda(x/\Theta_2) = f_\lambda(x/\Theta)$$

holds, where

$$f_\lambda(x/\Theta) = \lambda\Theta^\lambda \exp\left[-(\Theta/x)\right]^\lambda / x^{\lambda+1}, \quad x \ge 0,$$

and $*$ stands for the operation of convolution of functions. Therefore, it would be desirable to determine the value of parameter $\nu = \lambda$ such that the Frechét distribution law is stable to the operation of composition (convolution of the densities) of random variables. In doing so, different combinations of parallel and sequential connections of the arcs in the stochastic networks could be easily replaced by an equivalent arc, and determination of the equivalent p.d.f. of this arc would boil down to calculating the new value of Θ by means of arithmetic operations over the values of Θ_i for the arcs entering the transformed segment of the network. To investigate the composition of independent random variables obeying this distribution law, it is desirable to obtain the characteristic function or the Laplace transform of the density for this law. It can be well-recognized that such an approach, which, in principle, can provide the Laplace transform for the Frechét law with integer values of parameter $\nu = n$, is described in [69] and includes the following considerations:

1. For composition of identically distributed independent random variables obeying law Φ_ν ($0 < \nu < 2$), X_α with characteristic parameter $\alpha = \nu$ is the limiting law.
2. For maximization of identically distributed independent random variables obeying law X_α ($0 < \nu < 2$), Φ_ν with characteristic parameter $\nu = \alpha$ is the limiting law.

This conclusion stems from the fact that limiting laws satisfy the necessary and sufficient conditions for limitedness of themselves. Therefore, the above limiting laws are boundary laws for the distribution of the time of the critical path. If the number of parallel activities tends to infinity, we obtain the Φ_ν law. If the number of sequential activities tends to infinity, we obtain the composition-stable X_α law ($0 < \alpha < 2$). The parameters of these laws retain their values and are identical for the boundary laws ($\alpha = \nu$).

As demonstrated in [69], distribution laws X_α and Φ_ν under consideration can be approximated in the limit by sufficiently close functions. It can be well-recognized that behavior of the p.d.f. laws in infinity, that is, in the extremal zones of the distributions, is of utmost importance for stability to the operations of composition and maximization. We represent the density of the Φ_ν law as

$$f_\nu(x) = A\exp\left[-(\Theta/x)\right]^\nu / x^{\nu+1},$$

where $A = \nu\Theta^{\nu}$ is the constant independent of x. Here, $\lim_{x\to\infty} f_{\nu}(x)x^{\nu+1}/A = 1$ holds. Consequently, $f_{\nu}(x)$ behaves for $x \to \infty$ as $A/x^{\nu+1}$, that is, the following relationship holds

$$f_{\nu}(x) \approx A/x^{\nu+1} \text{ for } x \to \infty. \qquad (2.4.9)$$

With regard to (2.4.5), the asymptotic formulae of the densities of composition-stable X_{α} law for $x \to \infty$ are representable in the general form as

$$P_{\alpha}(x) \approx B/x^{\alpha+1} \ (-1 < \beta \le 1,\ \ 0 < \alpha < 2), \qquad (2.4.10)$$

where $B = \varphi(\alpha, \beta)$ is independent of x.

Comparison of (2.4.9) and (2.4.10) suggests that for $0 < \alpha = \nu < 2$ the densities of both laws Φ_{ν} and X_{α} at infinity behave in a similar way. Hence, we draw a similar conclusion that Φ_{ν} laws are approximately stable to composition and X_{α} laws are approximately stable to maximization and that they are "close".

2.4.3 Using stable distribution laws in applied calculations

On the basis of the above results, we can draw a conclusion that in stochastic networks the operation of maximization of random variables is of no less importance than the operation of composition of random variables. The main temporal characteristic - the length of the maximal path from the initial event i - is representable as the maximum of the random variables which are the lengths of all paths from the initial network event to the given event i. In the general case, these random variables are independent, which is the cause of main difficulties.

For approximate calculation of the temporal characteristics of the stochastic networks, it is possible to assume that the random times of event occurrences satisfy the Φ_{ν} law, and that addition to them of the random durations of activities does not modify the type of the distribution law. If the activity durations can be of the same order, then for a great number of network activities the form of the distribution curve for an individual activity can be regarded as being of no significance; it is only natural to assume *a priori* the same distribution with parameter Θ_{ij} for each activity (i, j).

By relying on the above considerations, it can be well-recognized, therefore, that the duration of any activity (i, j) satisfies the distribution law with integral function

$$\Phi_{ij}(t) = \exp\left[-(\Theta_{ij}/t)^{\nu}\right],\ \ 0 < t < \infty. \qquad (2.4.11)$$

The maximal and minimal estimates of an activity duration will be regarded as the respective quantiles of a probability close to unity and of a very small probability. The exponent ν which is taken to be the same for the entire network characterizes the level of uncertainty of the design because for the given distribution law the final instants of the degree ℓ exist only for $\ell > \nu$.

We assume that $\nu > 1$. Then, the expectation of the duration of activity (i, j) may be represented as follows:

$$m_{ij} = \Theta_{ij}\Gamma[(\nu - 1)/\nu], \tag{2.4.12}$$

where $\Gamma[\alpha] = \int_0^\infty xe^{-x}dx$. The value of Θ_{ij} can be determined from (2.4.12) using the known values of m_{ij} and ν or from the known value of the mode μ_{ij} from relation

$$\mu_{ij} = \Theta_{ij}[\nu/(\nu + 1)]^{1/\nu}. \tag{2.4.13}$$

Thus, our study results in obtaining an efficient method for calculating characteristics of distribution laws for durations of network fragments realization belonging to the sequential-parallel type.

1. The sequential subnetwork η can be replaced by one arc with parameter Θ_Σ:
   $\Theta_\Sigma = \sum_{(i,j)\in\eta} \Theta_{ij}$.
   In this context a sequential subnetwork designates the path passing through the events all of which - with the exception of the initial and the final events of the subnetwork - have precisely one entering arc and one leaving arc. The rule of summarizing for parameters Θ_{ij} stems from the property of summarizing the expectations and proportionality of the expectation to parameter Θ:
   $$m_\Sigma = \sum_{(i,j)\in\eta} m_{ij} = \Gamma[(\nu - 1)/\nu] \sum_{(i,j)\in\eta} \Theta_{ij} = \Gamma[(\nu - 1)/\nu]\Theta_\Sigma. \tag{2.4.14}$$
2. The parallel subnetwork γ can be replaced by one arc with parameter $\Theta_{\max}$:
   $$\Theta_{\max}(i, j) = \left[\sum_{(i,j)_k\in\gamma} \Theta^\nu(i, j)_k\right]^{1/\nu}. \tag{2.4.15}$$
   In this context a parallel subnetwork designates a set of arcs $(i, j)_k$ having identical boundary events.

The rule for calculating parameter $\Theta_{\max}$ of the equivalent arc of the parallel subnetwork stems from the rule of summarizing parameters $\Theta^\nu(i, j)_k$ where the operation of maximization is applied to independent random variables:

$$t(i, j) = \max_k\{t(i, j)_k\}.$$

The Martin algorithm [139] can be used to extract sequential and parallel subnetworks, but here the formulae of sequential and parallel reduction are much simpler because only one parameter Θ is involved. Consequently, an algorithm to represent the non- sequential-parallel networks as a hierarchical tree and then calculate them becomes quite feasible. The relation

$$t_j = \max_i \{t_i + t(i, j)\}, \quad i < j,$$

of the moment of occurrence of an event in the determinate ordered network offers another way to calculating the network probabilities from the estimate of parameter Θ. Here, the moment t_j of occurrence of the event i and the duration $t(i, j)$ of activity (i, j) are random variables. By assuming that random variables $[t_i + t(i, j)]$ are independent, we obtain

$$\Theta_j = \left(\sum_i \Theta_i + \Theta_{ij} \right)^{1/\nu}.$$

This method is similar to the Fulkerson-Clinger method [44, 60] because in fact the estimate of the expectation of the random variable $t_j \left[m_j = \Theta_j \Gamma((\nu - 1)/\nu) \right]$ is calculated. Its implementation is computer-friendly because – on the contrary to the Clinger method [44] – it requires no multiplication of the integral distribution functions $F_{ij}(x - c_i)$ and does not assume that t_i is the numerical estimate c_i calculated at the preceding step rather than a random variable. It can be well-recognized that in case $\nu > 2$ there exist finite variances which can also be calculated using Θ_i:

$$Dt_i = \Theta_i^2 \Gamma((\nu - 2)/\nu) - [\Theta_i \Gamma((\nu - 1)/\nu)]^2.$$

It seems that the best results may be obtained by calculating Θ in combination with the Meshkov method [67] where several most significant paths η_k (most lasting in expectation and least correlated with the rest of the paths) are preselected, and then the moments of the random variable

$$\eta_{\max} = \max_k \{\eta_k\}$$

are calculated as the estimates of the moments of the critical path.

Having estimated the moments, one can assume that the actual law of distribution of the probabilities of lengths of the critical path lies somewhere between the composition-stable and maximization-stable laws. If $\nu < 2$, than these are the above laws X_α and Φ_ν ($\alpha = \nu$). If $\nu \geq 2$, than these are the normal law and the law to which the normal law tends for the operation of maximization of independent and similarly distributed random variables. For example, consider a parallel subnetwork where the lengths $\xi(i)$ ($i = 1,2,...,n$) of all parallel arcs comply with the integral function

$$F_{\xi(i)}(x) = 1 - erf\left\{\sqrt{\alpha/2x}\right\}. \tag{2.4.16}$$

The execution time of such a subnetwork may be determined from the obvious relation

$$\eta_n = \max_{i=1,2,\ldots,n}\{\xi_i\}.$$

Let us consider the random variable

$$x = \lim_{n\to\infty}\max_{i=1,2,\ldots,n}\left\{\xi_i/n^2\right\} = \lim_{n\to\infty}\left\{\eta_n/n^2\right\}. \tag{2.4.17}$$

Since $F_{\xi/A}(x) = F_\xi(Ax)$, we obtain

$$F_\xi(x) = \lim_{n\to\infty}\left\{F_\xi\left(n^2x\right)\right\}^n = \lim_{n\to\infty}\left\{1 - erf\left[\sqrt{\alpha/2xn^2}\right]\right\}^n. \tag{2.4.18}$$

It can be well-recognized that for $y\to\infty$ the function $erf(y) \approx 2y/\sqrt{\pi}$. Since for $n\to 0$ in (2.4.18) the argument $\sqrt{\alpha/2xn^2}\to 0$, we obtain

$$F_x(x) = \lim_{n\to\infty}\left\{1 - \left[\sqrt{2\alpha/\pi x}\right]/n\right\}^n = \exp\left\{-\sqrt{2\alpha/\pi x}\right\}.$$

For a sufficiently great n in (2.4.17), we conclude that $\eta_n \approx n^2$; then, the distribution of the execution time of the parallel subnetwork boils down to

$$F_{\eta(n)}(x) \approx \exp\left\{-\left(2\alpha n^2/\pi x\right)^{1/2}\right\},$$

that is, we obtained the law $\Phi_{1/2}$ with parameter $\Theta_n = 2\alpha n^2/\pi$. Implementing relation (2.4.15) of parallel reduction brings us to

$$\Theta_{RD} = \left(\sum_{i=1}^{n}\Theta_i^{1/2}\right) = n^2\Theta,$$

because the lengths of all arcs are distributed identically. Consequently, if the calculations are based on

$$\Theta_n = 2\alpha/\pi, \tag{2.4.19}$$

then $\Theta_n = \Theta_{RD}$. Therefore, the calculated and theoretical results coincide for a sufficiently great number of arcs (without the *a priori* assumption that the arcs lengths are distributed according to the Φ_ν law).

2.4.4 <u>Closeness of stable laws to β-distribution</u>

We have already noted that β-distribution is the most popular one among the random laws of activity duration p.d.f. Let us compare the calculation of the arc length by using Φ_ν law as well as analyzing methods for estimating its parameters, with the law of β-distribution.

We compare normalized distributions over the interval $(0,1)$, which does not lead to loss of generality because linear transformation of the random variable enables passing to an arbitrary interval (a,b). To enable consideration over a finite interval, distribution Φ_ν is truncated from right for the unlikely great values. In doing so, the confidence coefficient $(1-p)$ was chosen to be close to unity, upon which the $(1-p)$-quantile of Φ_ν was equated to the right boundary (b) of the interval including the remaining part of the distribution:

$$b = F_\nu^{-1}(1-p),$$

where $p << 1$ and F_ν^{-1} is the function inverse to the integral distribution function

$$F_\nu(x) = \exp\left\{-(\Theta/x)^\nu\right\}.$$

Hence, we get the equation

$$b = \Theta/\{-\ln(1-p)\}^{1/\nu} .$$

For small p, we obtain $\ln(1-p) \approx p$ which in turn - because of the normalization $b=1$ - boils down to the appropriate relation

$$\Theta = p^{1/\nu}. \qquad (2.4.20)$$

We will superpose the modes and expectations of the compared distributions because these parameters vary according to the same linear transformation as the random variables upon passing to the distribution over an arbitrary interval. The mode $X_{m(\Phi)}$ retains its position upon truncation:

$$X_{m(\Phi)} = \Theta\{\nu/(\nu+1)\}^{1/\nu}. \qquad (2.4.21)$$

For the truncated distribution Φ_2, we derive the relation for expectation $m(\Phi)$:

$$m(\Phi) = R\int_0^1 \exp\left\{-(\Theta/x)^2\right\} dx/x^2 ,$$

where

$$R = 2\Theta^2 .$$

The obvious substitution $\omega = \Theta/x$ reduces this integral to

$$m(\Phi) = R\int_0^\infty \left(e^{-\omega}\right)^2 d\omega/\Theta .$$

The final result might be therefore represented as follows:

$$m(\Phi) = \Theta\sqrt{\pi}\left[1 - erf(\Theta)\right],$$

where $erf(x) = 2\int_0^x \left(e^{-\omega}\right)^2 d\omega/\sqrt{\pi}$. For small x, we obtain $erf(x) \approx 2/\sqrt{\pi}$. The density of the β-distribution over the interval $(0,1)$ is described as follows:

$$\varphi(x) = cx^{\alpha}(1-x)^{\gamma} ,$$

where c is a constant. Both mode $X_{m(\beta)}$ and expectation $m(\beta)$ of this distribution are expressed in terms of their parameters α and γ:

$$\begin{cases} X_{m(\beta)} = \dfrac{\alpha}{\alpha+\gamma} \\ m(\beta) = \dfrac{\alpha+1}{\alpha+\gamma+2} \end{cases} .$$

By choosing appropriate characteristics of the distributions $m(\beta) = m(\Phi)$ and $X_{m(\beta)} = X_{m(\Phi)}$, we solve the system of equations $\alpha/(\alpha+\gamma) = X_{m(\Phi)}$ and $(\alpha+\gamma)/(\alpha+\gamma+2) = m(\Phi)$ relative to parameters α and γ and, as a result, obtain

$$\begin{cases} \alpha = \dfrac{(1-2m(\Phi))X_{m(\Phi)}}{m(\Phi) - X_{m(\Phi)}} \\ \gamma = \dfrac{(1-2m(\Phi))(1-X_{m(\Phi)})}{m(\Phi) - X_{m(\Phi)}} \end{cases} . \tag{2.4.22}$$

We assign the value $p = 0.1$, which corresponds to the probability of appearance of the truncated values. Then, $\Theta \approx 0.32$, and we calculate $X_{m(\Phi)} \approx 0.26$ and $m(\Phi) \approx 0.37$.

Their substitution in (2.4.22) provides $\alpha \approx 0.64$ and $\gamma \approx 1.78$. We also compare variances σ_β^2 and σ_Φ^2. For the β-distribution,

$$\sigma_\beta^2 = \frac{(\alpha+1)(\gamma+1)}{(\alpha+\gamma+2)^2(\alpha+\gamma+3)}.$$

For the given values of parameters α and γ, we obtain $\sigma_\beta^2 = 0.042$. We derive the relation for the initial moment of the second order of the truncated distribution:

$$V_2 = R\int_0^1 \exp\left\{-(\Theta/x)^2\right\} dx/x.$$

Substitution $\omega = -(\Theta/x)^2$ reduces the formula of V_2 to

$$V_2 = \Theta^2 E_i\left(-\Theta^2\right),$$

where $E_i(y) = \int_{-\infty}^{-y} e^{\omega} d\omega/\omega > 0$.

Now, we calculate σ_Φ^2 from $\sigma_\Phi^2 = V_2 - m_\Phi^2$ and obtain that $V_2 \approx 0.184$ and $\sigma_\Phi^2 \approx 0.046$. Therefore, for the given accuracy of calculations we get $\sigma_\Phi^2 \approx \sigma_\beta^2$, that is, the variances virtually coincide. This comparison suggests that distributions Φ_2 are close to the β-distribution with parameters $\alpha \approx 0.64$ and $\gamma \approx 1.78$. Upon adjustment of these values, we obtain the β-distribution with parameters $\alpha = 1$ and $\gamma = 2$ and density $\varphi(x) = 12x(1-x)^2$ which is reducible by linear transformation $y = a + (b-a)x$ to distribution over an arbitrary interval (a,b) as developed for the two-estimate technique [67, 69, 73-74], where $\varphi(y) = 12(y-a)(b-y)^2/(b-a)^4$. Therefore, the final distribution can also be approximated by distribution Φ_2, which enables applying the two-estimate technique to define parameters of distribution Φ_2 from expert estimates.

2.4.5 Conclusions

1. The results obtained enable (2.4.6) to be recommended as the distribution of activity duration for stochastic network projects to be outlined in Chapters 5 and 8. Index $\Theta > 0$ varies here from activity to activity, whereas index $\nu > 0$, designating the uncertainty parameter for the *project as a whole*, remains constant. In the regarded application, value $\nu = 2$ is preferable.
2. Subnetwork Σ_1 consisting of n parallel activities (i, j) may be reduced to one resulting activity (2.4.15), whereas subnetwork Σ_2 consisting of n successive activities may be replaced by one aggregated activity (2.4.14). For $\nu = 2$, distribution

(2.4.6) can be regarded as the stable law for operations of both convolution and maximization.

3. If for activity (i, j) a two-estimate β-distribution with p.d.f.

$$\varphi_{ij}(x) = \frac{12}{(b_{ij} - a_{ij})^4}(x - a_{ij})(b_{ij} - x)^2 \tag{2.4.23}$$

is used in a real NPC system as the duration law, then p.d.f. (2.4.23) has to be reduced to the form (2.4.6) by taking into account (2.4.21). Keeping in mind $X_{m(\Phi)} = (2a_{ij} + b_{ij})/3$ and $\nu = 2$, we finally obtain here

$$\Theta_{ij} = \frac{\sqrt{1.5}}{3}(2a_{ij} + b_{ij}) \approx 0.4(2a_{ij} + b_{ij}). \tag{2.4.24}$$

4. The results outlined above can be implemented in practically all optimization models described below in Chapters 5-12 and 14-16. Those models are based on optimizing OS comprising elements (activities) with random probability durations laws. In our opinion, they all have to refer to the "β-family". This assertion stems from the fact that justification of implementing β-distribution p.d.f. as outlined in the above §§ 2.1-2.2, was based on <u>general assumptions</u> about activities' random durations. Those assumptions are valid for <u>practically any activity that might enter a man-machine OS</u>, and not only for random activities in project management.

Chapter 3. Control Problems in Multilevel Organization Systems

§3.1 Introduction

In the recent five decades extensive research has been undertaken in the area of on-line control for various multilevel organization systems under random disturbances [see, e.g., Babunashvili [4], Mesarovich [141], Golenko-Ginzburg [67-72, 93-96], Elsayed and Boucher [56-57], Golenko-Ginzburg and Sinuany-Stern [79, 170-171], Kusiak [130], Ben-Yair [9, 103], etc.]. It can be well-recognized that a modern organization system S usually consists of two parts: the functional part, aimed at performing a certain set of operations, and the control part, provided for realizing algorithms defined on the set of operations and ensuring the system's advancement towards a certain goal.

We can conditionally examine the functional part as an executive mechanism of the system. Every system's operation is, thus, performed and realized by that mechanism at a definite speed. Obviously, the functional part determines the dynamic properties of the system.

The presence of the control part in the system is determined by the need for a purposeful growth and development of the system's process, which, in turn, is assured by selfcoordination: the control part coordinates the work of all elements incorporated into the system's functional part.

Before formulating the problem of controlling the system, we must determine the basic parameters characterizing the routine process of local system's elements, as well as of the system as a whole.

One of the essential parameters for controlling an organization system is the volume of the system's program, expressed either in output units (items) or in cost. For most enterprises of this kind, the assortment of the output is practically stationary and will not tolerate substantial changes within time. This means that the goal function of such systems is either a vector with a relatively small number of components, or is ordinarily reduced to a general equivalent, usually expressed in cost terms. This enables a clear formalization of the system's control procedures as well as of the nature of control actions to be introduced.

A second essential parameter characterizing functioning of the system is the system's capacity - the amount of resources (financial, manpower, materials, etc.) at its disposal, which must be managed to the best advantage for achieving the goal set by the system. This parameter characterizes the system's ability to advance to its goal at a definite speed $v_{pl}(t)$, $0 \le t \le T_{pl}$, where T_{pl} stands for the duration of the plan period for the system, and thereby the time for carrying out a given volume of work.

§3.2 Basic parameters

Imagine the control of a production system S as a unit whose input is the plan assignment η, and the outcome is a coordination output or a control action φ. Note that the system's advance to its goal takes place under certain disturbances ψ affecting the system.

In the process of advancing towards the goal, inspections of the system's state are required at definite moments of time $t = t_1, t_2, \ldots,\ 0 \le t \le T_{pl}$ {we will henceforth call them inspection (control) or query moments}, to compare values corresponding to the true, actual curve reflecting movement towards the goal $V_f(t)$, which are random values, with values of the planned trajectory towards the goal $T_{pl}(t)$. Function $V_f(t)$, which characterizes the actual state of the system at every moment of time, is called the goal variable of the system.

As for the plan assignment η, keeping with the terminology of [4, 67, 94-96], we can see that it is represented in the same form

$$\eta = V_{pl}(t). \tag{3.2.1}$$

Here $V_{pl}(t)$, as shown above, is the planning trajectory of the system's advancement towards the goal; it can be determined thus:

$$V_{pl}(t) = \int_0^{T_{pl}} v_{pl}(R,t)dt, \tag{3.2.2}$$

where $v_{pl}(R,t)$ is assumed to be a certain plan dependence on the time of the system's speed towards the goal, and can be ensured by available resources R.

To develop an efficient control procedure, one must be able to devise a dependence between the system's speed in moving towards the goal and the amount of available resources, i.e., dependence

$$v(R,t) = f(R_t), \tag{3.2.3}$$

where R_t designates the capacity of the system's resources at moment t.

Here it proves convenient to introduce the concept of the so-called complex resource [4, 67, 103]. This can be defined as the totality of the minimum quantities of resources of different kinds essential and sufficient for doing the job, i.e., a closed set of operations formulated in a definite way. Denote the unit of the complex resource by r. One of the best examples of a complex resource unit is a standard building team which is able to carry out any construction work.

Let N_r denote the number of units (capacity) of complex resource r at moment t, then the total resources of the system can be formalized as

$$R = N_r \cdot r + R^{'} \ , \tag{3.2.4}$$

where $R^{'}$ stands for incomplete resources used in the system.

Let the dependence of speed v on resources R be linear. Then a change in the system's resources by ΔN_r units results in the change of the system's speed towards the goal by a certain value Δv. In such a case, the system's speed at any moment of time is expressed by the resources through relation

$$v(R,t) = \frac{N_r}{\Delta N_r} \cdot \Delta v \ . \tag{3.2.5}$$

Further, let f in relation $v(R,t) = f(R_t)$ not depend on time t.

From this it follows that with a plan capacity of a complex resource $N_r^{(pl)}$, the system's speed of advancing towards the goal v_{pl} ensures realization of the production program in the time planned, namely T_{pl}.

Notice that different speeds of the system may correspond to one and the same capacity of resource R, depending on the degree of intensification of production. In keeping with [4, 67-68, 94], we will henceforth call speed , corresponding to the maximal intensity of production, the *optimistic* one, while speed , corresponding to the minimal intensity, will be regarded to as *pessimistic*.

Let us examine more closely the concept of complex resources on the example of a production shop of a serial enterprise. For small size serial production mapped by the network model, we believe that the concept of complex resource refers to a set of minimal volumes of resources of the primary and secondary kind, capable of performing the elementary job in the network project. For production systems of serial and large-scale serial type with a steady listing of output and substantial cycles of processing (when there is a big list of parts and units), we can regard as the unit of the complex resource a set of raw, unfinished, and other materials, as well as the lathes and automatic production lines capable of turning out an individual item.

It is quite useful to employ the concept of complex resource since it allows using relations $v_{pl}(N_r) = v_{pl}$, $v_{opt}(N_r) = v_{opt}$, $v_{pes}(N_r) = v_{pes}$, and implementing them in the process of optimal production control if we have the corresponding standards and specifications.

Obviously, the minimal time T_I for reaching the goal (the planned volume of the system's program) must correspond to the maximal speed v_{opt} of the system's advancement towards the goal, satisfying relation $T_I < T_{pl}$. Correspondingly, there must also be the upper duration limit T_{II} of achieving the system's goal when moving with pessimistic speed v_{pes}, $T_{II} > T_{pl}$.

For discrete production, the capacity of complex resource R can change within the limits of $N_r^{\min}$ and $N_r^{\max}$. By definition, complex resource capacity $N_r^{\min} = 1$. In other words, when $R = 0$, the speed of advancing towards the goal equals zero, and when $R > N_r^{\max}$ there is no further increase in value $v(R,t)$; moreover, under certain circumstances this speed can even decrease.

Thus, resources of capacity R supplied to system S ensure completion of the system's program $V_{pl}(R,T_{pl})$ for the period $\left[0, T_{pl}\right]$.

§3.3 Planned trajectories

Depending on the degree of intensification of the system's process, the trajectory of the system's advancement to the goal can take the following forms:

a) *An optimistic trajectory of the system's movement to the goal* $V_{opt}(R,t)$, corresponding to the maximal intensification of available resources R. As pointed out above, inequality $T_{pl} \geq T_I$ must hold, with value T_I characterizing the moment when the system achieves the goal if work is done at speed $v_{opt} = \frac{dV_{opt}}{dt}$, and being the result of solving equation
$$V_{pl} = V_{opt}(R,T_I). \qquad (3.3.1)$$
Note that specific values of all the parameters regarded above (values T_{pl}, T_I, V_{pl}, R, etc.) are determined at the *planning stage*, i.e., the stage preceding both inspection and control. A similar conclusion holds for an analytical description of dependence $V_{opt}(R,t)$.

b) *A planned trajectory of the system's movement to the goal* $V_{pl}(R,t)$, corresponding to some average intensity of the production process (i.e., taking into account the effect of certain favorable and certain adverse factors). Here the obvious equality
$$V_{pl} = V_{pl}(R,T_{pl}) \qquad (3.3.2)$$
holds.

c) *A pessimistic trajectory of the system's movement to the goal* $V_{pes}(R,t)$ is characterized by the minimal degree of intensification of the course of the production process. Note that the corresponding minimal speed of the system's advancement towards the goal can depend both on the presence of exceptionally unfavorable circumstances and the system's control device (the system is purposefully transferred to the least intensive work).

If the production program is carried out at speed $\frac{dV_{pes}(R,t)}{dt}$, moment T_{II} when the system reaches the goal satisfies the obvious relation

$$T_{II} > T_{pl} > T_I \tag{3.3.3}$$

and may be calculated by solving equation

$$V_{pl} = V_{pes}(R, T_{II}). \tag{3.3.4}$$

It goes without saying that all the parameters and characteristics corresponding to trajectories $V_{pes}(R,t)$ and $V_{pl}(R,t)$ are also formulated at the stage preceding all control procedures.

An organization system can, thus, function with three rates: *planning* (normal), *optimistic* (tense) and *pessimistic* (not tense). Naturally, the "zero" non-working state, when no production program is drawn up at all, has also to be taken into account.

In the process of moving to a definite goal, the system must aim at optimizing the conditions in which the movement takes place, since repeated and lengthy work at utmost rates (at the system's maximal speed v_{opt} towards the goal) can exhaust and prematurely wear out the system. In other words, the planning time for achieving the goal $T_I < T_{pl} < T_{II}$ must be chosen so that it will ensure reaching the goal not later than at the due date T_{pl}, together with minimizing the expenses of carrying out the production process.

This, of course, by no means signifies that the system should not be allowed under any circumstances to work at the utmost rate accompanied by a certain overloading of its functional part. On the contrary, it is precisely the possibility of the system's functioning with the overload that constitutes the potential internal reserve of the system which must be used first of all in the process of removing possible discords between values $V_{pl}(R,t)$ and $V_f(R,t)$.

It should be pointed out that practically any organization system functions in a situation of numerous random influences, circumstances and interferences from the environment, such as illness among personnel, disruption of supplies of raw materials, equipment going out of commission accidentally, and so on.

The influence of such random factors must therefore be felt and reliably taken into account in the process of supervising the course of the system's production. At every routine query of the state of the system, objective conclusions must be drawn as to whether deviations $V_{pl}(R,t)$ from $V_f(R,t)$ can be explained by the disturbing influence of only random fluctuations, or whether the discord exceeds the permissible limits and one or another purposeful control action has to be introduced in the course of the system's functioning.

In this it is expedient to make use of the concept of the working cycle of a system t_c [4], which can be represented as the sum of two intervals of time: working time, and time for renewing the system. There is a certain minimal level of renewal time t_r, which must be determined in such a way that after the work of the system at the utmost rate (throughout time $t_w = t_c - t_r$) the time t_r would prove to be sufficient for no irreversible wearing out system S to take place. In each cycle, a certain possible potential increment to the goal variable can be introduced as follows:

$$\Delta V^{*}_{cycle} = \left(v_{opt} - v_{pl}\right) \cdot t_w , \qquad (3.3.5)$$

where $v_{opt} = \frac{1}{T_I} V_{pl}\left(R, T_{pl}\right)$ is the maximal value of the average speed in proceeding towards the goal, and $v_{pl} = \frac{1}{T_{pl}} V_{pl}\left(R, T_{pl}\right)$ is the planned speed for that movement.

In the process of the system's movement towards the goal, the number of remaining working cycles $k_c(t)$ may be determined from relation

$$k_c(t) = \frac{T_{pl} - t}{t} , \qquad (3.3.6)$$

where t stands for the current moment of time within the bounds of the period that the system functions. Then the remaining possible potential increment to the goal variable of the system may be finally represented as

$$\Delta V^{*} = \left(v_{opt} - v_{pl}\right) \cdot \frac{t_w}{t_c}\left(T_{pl} - t\right). \qquad (3.3.7)$$

The last equality means that if at the current moment of time t the deviation of the goal variable's value from the planned trajectory of advancement towards the goal does not exceed value ΔV^{*}, the deviation can be eliminated by mobilizing only inner reserves of the system, i.e., by local (internal) control. Otherwise, it can be done only by introducing a control action from outside into the process of the system's movement, i.e., a parametrical (external) control. This alters the resources of the system N_r, which, in turn, corrects speeds v_{opt}, v_{pes}, v_{pl}, as well as the corresponding terms for the system to achieve its goal.

Control of the system is effected by observing its goal variables at definite moments of time. Depending on the value of the goal variable's deviation from the planned trajectory, the control part of the system works out various purposeful control actions for compensating the deviations, and changing the structure of the system itself. In this, the strategy of querying the system should so be built that it will primarily ensure the system's achievement of the goal by attracting only its internal resources [4, 67-68, 94-95, 100].

The procedure proper for carrying out parametrical control actions for the system constitutes the second stage of the control process. Here there arises the need to obtain additional information on the state of the functional part of the system in order to effect such a redistribution of resources between its different elements that will ensure achieving the goal within the required planning period.

A conclusion, thus, can be drawn that if the system's advance to its goal proceeds normally, with no deviation from the planning trajectory exceeding value ΔV^* determined by (3.3.7), there is no need in restricting the query policy of the control. Even if in that situation the system's functional part undergoes certain changes, in most cases those changes are insignificant from the point of view of advance towards the goal.

On the other hand, we naturally have to supervise the state of the outlined above functional parameters in general, since most substantial changes in the functional part of the system can disrupt the planning terms for achieving the goal.

Note that in our practice we have mostly used three speeds with various levels of resource intensification. The number of possible speeds may be extended but the basic relations and definitions remain the same.

§3.4 Control actions by means of resource reallocation in organization systems

3.4.1 Introduction

As pointed out above, the system's labor productivity at a controlled installation depends on the volume of available resources. If the target, e.g., the volume of the production program, V_{pl}, is expressed as a general equivalent and the resources consumed by the installation can also be expressed uniformly (e.g., in units of a complex resource [68, 95]), the question of optimizing the models presents no substantial difficulty. It is far more complex and worthy of attention when the controlled installation consists of a group of elements E_i, $i = 1,2,...,n$, each of which contributes to the fulfillment of the production program and produces items of the same kind while consuming the same kind of resources.

We can regard as such elements, in particular, a group of sections functioning in parallel and equipped with practically the same machinery. Of course, different elements may differ from each other in size or capacity, as well as in labor productivity. Let us say that all the elements start functioning at the same initial time T_0 and complete work by the moment the planned period T_{pl} terminates. Let us further assume that for each element E_i, $i = 1,2,...,n$, we can determine a functional $V_i(R_i, t_i)$, expressing the volume of the outcome product manufactured by the element (in form of the general equivalent), depending on work time t_i and resources R_i at its disposal (also expressed in the form of the general equivalent).

Each of the elements E_i at moment T_0 is supplied with resources of capacity $R_i^{(0)}$, satisfying an obvious equality $\sum_{i=1}^{n} R_i^{(0)} = R$, where R stands for the total volume of resources available to the system control device (it is understood that the control has not changed throughout the planned period). Note that values $V_i(R_i, t_i)$ can be determined by means of a simulation model of a shop or a section, and, in principle, are not deterministic in nature. They can be expressed either as a table or a nomograph, or a quite complex analytical relationship, or a correlation of other type (non-linear, in principle). In the below sub-sections we will formulate and classify those problems.

3.4.2 Simplified resource allocation problems without synchronization

Let us consider a formalized statement of an optimization problem in resource redistribution.

Determine the optimal n-dimensional vector $\vec{R}^*$ for the values of resource capacities $\{R_1, R_2, ..., R_n\}$, supplied for elements E_i, to maximize

$$Max\, J = \underset{\{R_i^*\}}{Max} \left\{ \sum_{i=1}^{n} V_i\left(R_i, T_{pl} - t\right) \right\} \qquad (3.4.1)$$

subject to

$$\sum_{i=1}^{n} R_i = R. \qquad (3.4.2)$$

Optimization problem (3.4.1-3.4.2) can be solved by regular methods or by means of statistical optimization. Note that optimization techniques enter the simulation model as an optimization unit.

3.4.3 An example of the multilevel system's description

A group of problems to synchronize elements functioning in parallel in production systems of mass production-line type is an important particular case of solving optimization problems. Systems of this kind are multilevel and consist of a finite set of production flows (such as automatic production lines) which are also structured hierarchically. For any flow we can single out a subset of elements, or aggregates, at a certain hierarchical level into which semi-manufactured or raw materials are fed in from outside. It is assumed that each aggregate has an output bunker of either limited or unlimited capacity. When processing is accomplished, the semi-manufactured product (part) enters the aggregate's output bunker, which, as a rule, is of limited capacity. The external flows of raw materials for each aggregate are random variables, whose distribution functions are assumed to be known.

The set of aggregates has a subset of aggregates, from whose bunkers the processed parts are fed into the input (input bunker) of any one aggregate of the following level according to the technological flow. It is assumed that processed parts can be transferred from a certain level aggregate to the supreme level aggregate only in batches of pregiven volume, i.e., after a definite quantity ("transfer level") of processed parts has piled up in the output bunker of the preceding aggregate. Putting it another way, the work of any aggregate at any level, except for the first one, begins only when its input bunker contains at least one complete set of parts processed in the group of preceding aggregates.

If an input bunker is full, a batch's transfer to it is suspended and parts begin to pile up in the corresponding output bunker of the preceding level. If an output bunker is full, the corresponding aggregate becomes blocked, i.e., processing of parts in it is suspended until room is available in the output bunker. Under certain circumstances, such a blocking process can spread in the flow.

Parts are transferred from the second level to the third level similarly, and so on, until the finished product is achieved on a single aggregate at the top level.

3.4.4 Two-level optimization to synchronize the production process

Problems of analyzing and synthesizing on simulation models of a standard two-level module consisting of a set (group) of aggregates at the same level working in parallel and an aggregate at the supreme level whose input bunker receives processed parts from the group of aggregates of the preceding level, prove to be of particular interest. Any type of flow structure can be represented as a union of modules described through information inputs of the supply of batches from the level output bunkers to the input bunker of the intermediate level aggregate.

It is assumed that the productivity of any aggregate $A^{(s)}$ of the s-th level depends solely on vector $\vec{R}^{(s)}$ of primary resources. For the standard two-level module denote symbols $R_i^{(m)}$, $i = 1,2,...,\ell$, $m = 1,2,...,W$, for the primary resources available to aggregates $A_i^{(s-1)}$ supplying processed parts to the assembly aggregate $A^{(s)}$. Here m stands for the primary type resource, such as equipment, etc., W is the number of types of resources, i is the ordinal number of the aggregate at the $(s-1)$-th hierarchical level, while ℓ is the total amount of aggregates.

We are considering restrictions of type

$$\sum_{i=1}^{n} R_i^{(k)} \le R^{(k)}, \quad k = 1,2,...,W, \tag{3.4.3}$$

on variables $R_i^{(k)}$ throughout the time that the module functions. Here $R^{(k)}$, $k = 1,2,...,W$, is a constant value designating the total quantity of k-th type resources available.

The productivity $\omega_i^{(s-1)}$ of aggregate $A_i^{(s-1)}$ per unit of time is random, and the corresponding mathematical expectation is determined by

$$E\left[\omega_i^{(s-1)}\right]=\sum_{m=1}^{W}\alpha_i^{(m)}R_i^{(m)}\ , \qquad (3.4.4)$$

where values $\alpha_i^{(m)}$ are of constant nature and are considered to be known beforehand.

The purpose of control is to synchronize the work of the aggregates, since delay by even one aggregate in turning out produce by the pregiven due date can cause idleness of the corresponding aggregate at the next (receiving) level.

The optimization problem of synchronizing output by a group of aggregates at one level working in parallel, at a routine inspection moment t, is formalized as follows:

Determine optimal values $R_i^{(m)}$, $i=1,2,...,\ell$, $m=1,2,...,W$, maximizing the output product for an assembly aggregate

$$\sum_{i=1}^{\ell}\left\{V_f\left(A_i^{(s-1)}\right)+\left(T_{pl}-t\right)E\left[\omega_i^{(s-1)}\right]\right\} \qquad (3.4.5)$$

with synchronizing restrictions

$$\begin{cases} V_f\left(A_i^{(s-1)}\right)+\left(T_{pl}-t\right)E\left[\omega_i^{(s-1)}\right]=V_f\left(A_j^{(s-1)}\right)+\left(T_{pl}-t\right)E\left[\omega_j^{(s-1)}\right] \\ i\neq j \\ 1\leq i \\ j\leq \ell \end{cases} \qquad (3.4.6)$$

and resource restrictions

$$\begin{cases} \sum_{i=1}^{\ell}R_i^{(m)}\leq R^{(m)} \\ R_i^{(m)}\geq 0 \\ i=1,2,...,\ell \qquad , \\ m=1,2,...,W \end{cases} \qquad (3.4.7)$$

values $E\left[\omega_i^{(s-1)}\right]$ being determined by (3.4.4).

The problem of scheduling resource delivery moments to supply resources from the environment is of essential interest. Suppose, the system consists of n homogenous elements of various productivity which we have described above. The controlled installation's *secondary resources come from outside*, and we know the law by which resource supplies enter the system.

The mean value of resources entering the system equals the planned volume of consumption R_{pl}. As the resources enter, they are distributed among elements E_i, the distribution discipline being strictly formalized. Unlike functional $V_i(R_i, t)$, functional $V_i(R_i)$ is introduced, characterizing the volume of produce turned out by element E_i per unit of time when employing the R_i unit of resources.

It can be well-recognized that such a controlled installation depends mainly on the use of raw materials, while primary resources renewed in the process of their use (machines and mechanisms) are not limited by the installation.

As an additional system's ability, there could be the possibility of manufacturing by each element E_i with several intensities and with varying productivity for the invariable capacity of primary resource consumption and for different volumes of secondary resources.

3.4.5 *Inventory models*

In keeping with the theory of inventory models [175], signal levels of secondary resource reserves C_R must be controlled by the system periodically for each time unit (e.g., a day or a week). Value C_R is, thus a regulated characteristic.

We propose therefore the following formal statement for an optimization problem:

- It is required to minimize the signal level of secondary resource reserves

$$C_R^* = \min C_R \tag{3.4.8}$$

with restrictions

$$P\left\{\sum_{i=1}^{n} V_{fi}(T_{pl}) \geq V_{pl} \middle| C_R^*\right\} \geq 1 - \delta, \quad \delta > 0, \tag{3.4.9}$$

$$E\left\{R\left[T_0, T_{pl}\right] \middle| C_R^*\right\} > R_{pl} \tag{3.4.10}$$

We determine value $C_R^* = \min C_R$ by means of a simulation model, and for the sake of undertaking better approximation, the entire plan period $\left[T_0, T_{pl}\right]$ is divided into elementary subperiods.

It should be noted that the only way to solve optimization problem (3.4.8-3.4.10) is by implementing statistical simulation methods, subsequently determining the optimal value of C_R^* and carrying out the multiple "run" of the simulation model in each iterative loop, in order to calculate the frequency at which the system carries out the plan at each assured resource level. In the course of simulation, we simulate the submission of urgent demands for also adding secondary resources (if the actual presence of the resources proves to be less than the assured value), and simulate non-plan procedures for adding resources upon those demands.

It can be well-recognized that optimization problem (3.4.8-3.4.10) may be modified for the case of d secondary resources $\{R^{(1)}, R^{(2)}, ..., R^{(d)}\}$ and d signal levels $S_R^{(k)}$, $k = 1,2,...,d$, respectively. In this case, the optimization problem would become as follows:

- Determine d optimal values of signal reserve levels $C_R^{(k)}$, $k = 1,2,...,d$, to minimize the objective

$$J = \sum_{k=1}^{d} \rho_k C_R^{(k)} \tag{3.4.11}$$

with restrictions (3.4.9) and

$$E\left\{R^{(k)}\left\{T_0, T_{pl}\right\} \left| C_R^* \right.\right\} = R_{pl}^{(k)}, \quad k = 1,2,...,d, \tag{3.4.12}$$

where ρ_k, $k = 1,2,...,d$, are pregiven priority coefficients.

Note that though the external cycle in optimization problem (3.4.6-3.4.10) is usually implemented on the simulation model either by means of the dichotomy method [176] or other analogous ways of searching for the extremum, in case of problem (3.4.9, 3.4.11-3.4.12) performing the external cycle is more efficient by the directed random search method [176] in the d-dimensional space $C_R^{(k)}$. As to performing the internal cycle, optimizing problem (3.4.9, 3.4.11-3.4.12) makes no principal difference in the procedure of searching for the extremum, as compared with the case of one signal level.

Thus, we have formulated several resource optimization models for one- and for two-level production control problems. Inventory and synchronization models are also implemented within the global framework of a multilevel model. The fitness of all optimization problems can be assessed by means of a simulation model which comprises all optimization problems as a control device.

§3.5 Models of optimal probability control

3.5.1 Introduction

Probability control for industrial organization systems is mostly based on determining control actions ensuring pregiven reliability of carrying out the production program by the given deadline [56].

In the case of individual controlled installations under random disturbances, control actions boil down to adding resources from reserves of higher hierarchical levels. The corresponding volume of compensatory resources, ordinarily expressed in cost, can be determined on the basis of classical models of the theory of automatic control (see, e.g., [56]).

Such a strategy, quite efficient in the case of a controlled installation with a single or small-scale serial output, is inexpedient for the case of production-line serial output with several controlled elements (aggregates. automatic lines, etc.) functioning in parallel at one and the same hierarchical level. In the latter case, the choice and construction of the optimal strategy of probability control are based mainly on resource redistribution between the elements [95].

In this section we intend to formulate optimization probability control problems for production units working in parallel, including problems of optimal resource distribution.

3.5.2 The system's description

The formalized description of the corresponding models is as follows. It is assumed that a two-level production system S comprises k elements $A^{(i)}$, $i=1,2,...,k$, functioning in parallel at a certain hierarchical level in discrete time moments $t=0,1,2,...,T_{pl}$. The functioning of the elements results in the production output. Moreover, there is a $(k+1)$-th element A_0 at the supreme hierarchical level, called the control element.

It is assumed, further, that the output by the i-th element per time unit of time is a random variable $\xi^{(i)}$, independent of the outputs of other elements of the system.

The distribution laws of random variables [96]

$$P\left\{\xi^{(i)} \le x\right\} = \varphi_i\left(x,t,R^{(i)}\right), \quad i=1,2,...,k, \tag{3.5.1}$$

are pregiven. These are, generally speaking, time functions, as well as increasing functions of a certain generalized complex resource $R^{(i)}$, called the control parameter. Further on, parameter $R^{(i)}$ will be understood as the capacity of primary resources being used only by the i-th element and renewed periodically in the course of operations.

Let us introduce a random vector variable as follows:

$$\vec{x}_{t+1} = \vec{x}_t + \vec{\xi}_t, \quad t=0,1,...,T, \quad T<T_{pl},$$

where

$$\vec{\xi}_t = \left\{\xi_t^{(1)}, \xi_t^{(2)}, \dots, \xi_t^{(k)}\right\},$$

$$P\left\{\xi_t^{(i)} \le x\right\} = \varphi_i\left(x,t,R_t^{(i)}\right).$$

Note that a vector-function $\vec{R}_t = \left\{R_t^{(1)}, R_t^{(2)}, \dots, R_t^{(k)}\right\}$ actually determines parametrical control actions of a control element A_0 in the course of manufacturing at moment t. Further, we will call the final set of vector-functions $\vec{R}_t$, $t = 0,1,\dots,T_{pl}-1$, *the control strategy,* and value $x_T^{(i)}$ will denote the total output product of the i-th element. For every control strategy adopted, i.e., the resource distribution vector $\vec{R}_t$, relation

$$x_T^{(i)} = \sum_{t=0}^{T-1} \xi_t^{(i)}$$

holds. Without losing the generality, we can determine $\vec{x}_0 = 0$.

Each element $A^{(i)}$, $i = 1,2,\dots,k$, is destined to comply with plan term $T_{pl}^{(i)}$ and plan $V_{pl}^{(i)}$, i.e., the directive total output of the i-th element for period $\left[0, T_{pl}^{(i)}\right]$. The purpose of the system is to carry out its plan for each of the elements.

Let us examine vector $\vec{V}_{pl} = \left\{V_{pl}^{(1)}, V_{pl}^{(2)}, \dots, V_{pl}^{(k)}\right\}$ in greater detail. For each $\vec{V}_{pl}$ we can determine probability $P\left\{\vec{x}_T \ge \vec{V}_{pl}\right\}$ and $\vec{x}_T \ge \vec{V}_{pl}$ when, and only when, $x_T^{(i)} \ge V_{pl}^{(i)}$ for any $i = 1,2,\dots,k$. Suppose a stationary case of elements functioning takes place, i.e., relations

$$\varphi_i\left(x, t, R^{(i)}\right) = \varphi_i\left(x, R^{(i)}\right), \quad i = 1,2,\dots,k,$$

hold.

Let the plan periods of all elements be equal $T_{pl}^{(i)} = T_{pl}$, $i = 1,2,\dots,k$, and time unit $\Delta t << T_{pl}$. Then by applying the central limit theorem, we obtain the distribution function of the total output of the i-th element at moment T_{pl} in the form of

$$P\left\{x_{T_{pl}}^{(i)} \ge V_{pl}^{(i)}\right\} = 1 - \Phi\left[\frac{V_{pl}^{(i)} - E^{(i)}}{\sigma^{(i)}}\right], \qquad (3.5.2)$$

where

$$E^{(i)} = \sum_{t=0}^{T_{pl}-1} E\left[\xi_t^{(i)}\right], \quad \delta^{(i)} = \sqrt{\sum_{t=0}^{T_{pl}-1} V\left[\xi_t^{(i)}\right]},$$

($E\left[\xi_t^{(i)}\right]$, $V\left[\xi_t^{(i)}\right]$ being the mathematical expectation and the variance of random variable $\xi_t^{(i)}$, respectively), and

$$\Phi(x) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{x} \exp\left(-\frac{1}{2} u^2 \right) du .$$

According to our assumption, control element A_0 also functions at discrete time moments τ (at inspection moments), but with a step $\Delta\tau > \Delta t$. At an inspection moment, the control element receives full information of the state of the system, and, if necessary, redistributes resources between the elements. Moreover, if an open model of the production system is constructed, element A_0 provides coordination inputs and transmits information inputs to the higher control device.

3.5.3 Resource control strategies

Now let us formulate several problems for controlling resources of system $S = \left\{ A^{(i)}, A_0 \right\}$ (or controlling random process $\vec{x}_t$):

I. Determine a control strategy (a dynamic resource distribution vector) $\vec{R}_t$ maximizing the probability of system S 's completing the plan:

$$P_S\left(\vec{V}_{pl}\right) = P\left\{ \vec{x}_{T_{pl}} \geq \vec{V}_{pl} \right\} \Rightarrow \max \qquad (3.5.3)$$

and satisfying conditions:

a) the total resources are limited,

$$\left(\vec{R}_\tau, \vec{e}\right) \leq R^*, \quad \tau = 0,1,\ldots,T_{pl} - 1, \qquad (3.5.4)$$

(symbol $\vec{e}$ stands for a unit vector and the parentheses signify the scalar product of vectors), and

b) $\vec{R}_t$'s components are non-negative,

$$\vec{R}_\tau \geq 0, \quad t = 0,1,\ldots,T_{pl} - 1. \qquad (3.5.5)$$

Here R^* denotes the total volume of primary resources at the system's disposal.

II. Determine a control strategy $\vec{R}_t$ minimizing the total resources in system S :

$$P_S\left(\vec{V}_{pl}\right) = \sum_{\tau=0}^{T_{pl}-1} \left(\vec{R}_\tau, \vec{e}\right) \Rightarrow \min \qquad (3.5.6)$$

and satisfying both conditions

$$P\left\{ \vec{x}_{T_{pl}} \geq \vec{V}_{pl} \right\} \geq p_{pl} \qquad (3.5.7)$$

(the probability that the system will meet the plan's deadline must not be below the given probability p_{pl}) and non-negativity conditions (3.5.5).

III. Determine a control strategy $\vec{R}_t$ satisfying conditions (3.5.4-3.5.5, 3.5.7) of Problems I and II, and minimizing time T_{pl} for system S to complete the planned production volume $\vec{V}_{pl}$.

3.5.4 A generalized model

In particular, we will examine a shop with a mass production line consisting of N flows functioning in parallel, their structure being close to that considered above, in §3.4.

Each of the N flows involves several automatic lines and an assembly section. Let us introduce symbolic according to which a flow with a variable index n consists of s_n automatic lines $G_n^{(i)}$, $i=1,2,...,s_n$, $n=1,2,...,N$, and an assembly section M_n. Considering that there is also an input bunker (a raw material store) A_n for the n-th flow in the shop and an output bunker B_n before the assembly section M_n, the structure of the flow can be presented as a two-phase service system with an input flow P_n^* for the supply of raw materials, bunkers $A_n^{(i)}$ and $B_n^{(i)}$ before and after each service channel, respectively, (at the first phase there are s_n parallel service channels), and an output flow P_n^{**} of finished products, $n=1,2,...,N$ (see Fig. 3.1).

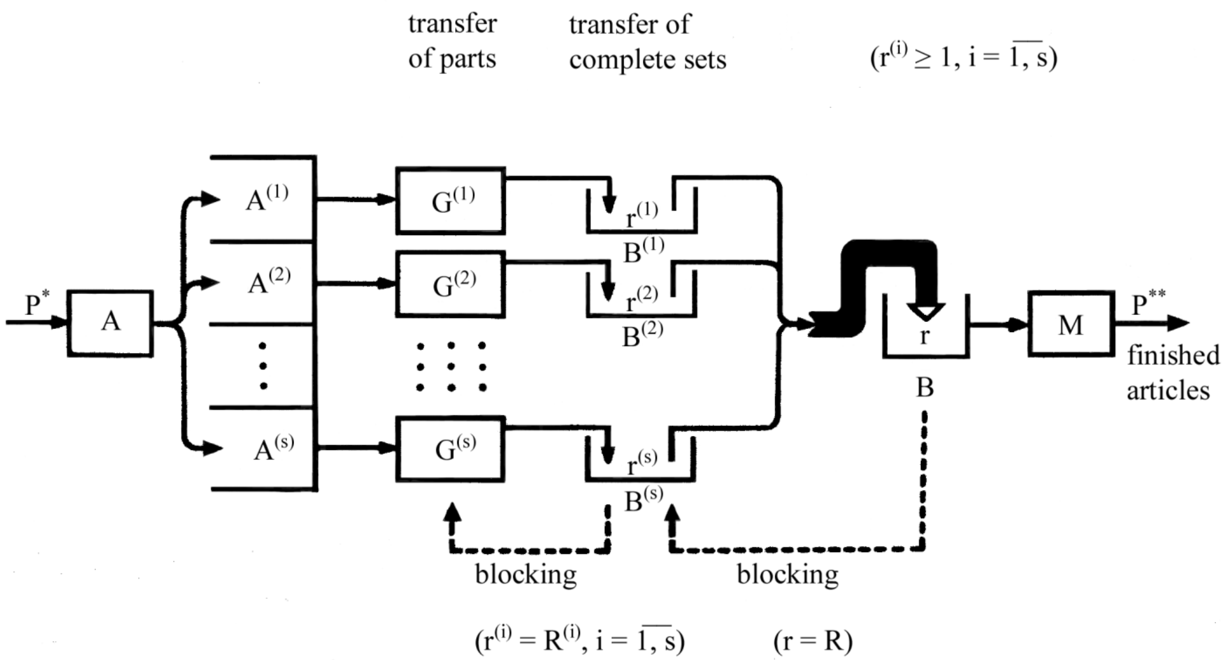

Figure 3.1. A formalized scheme of control actions for a FMS

The mass production shop as a whole is represented as an N-channel service system which includes N two-phase flow models. Parts are fed into bunker B_n from automatic lines $G_n^{(i)}$ of one and the same n-th flow at moment t only if each of the bunkers $B_n^{(i)}$ at the output of the corresponding automatic line $G_n^{(i)}$ has at least one complete set of finished parts made on the line at that moment of time.

In other words, sets of output products are supplied from the output bunker $B_n^{(i)}$, corresponding to automatic line $G_n^{(i)}$, into the assembly section bunker at moment t only if the number of parts $r_n^{(i)}$ in each bunker $B_n^{(i)}$ is no less than the corresponding complete set's volume $K_n^{(i)}$. If at a certain moment no set can be completed on at least one line, processed parts pile up in bunkers $B_n^{(i)}$ at the output of all the automatic lines $G_n^{(i)}$, till $r_n^{(i)} \geq K_n^{(i)}$, $i = 1,2,...,s_n$.

We will assume that the capacity of bunkers $B_n^{(i)}$ is also limited. For the sake of convenience, let us omit index n and consider only one standard flow. For that flow denote the current volumes of parts in bunkers $B^{(i)}$ by symbols $r^{(i)}$, $i = 1,2,...,s$, and their permissible limit capacities by $R^{(i)}$.

If the current volume r of the exit bunker B (R is the bunker's limit capacity) overflows, or if the automatic lines $G^{(i)}$ are not working synchronically, parts will pile up in bunkers $B^{(i)}$. This can lead further to an overflow of individual $B^{(i)}$ and as a result will block (stop) the work of corresponding line $G^{(i)}$, as it is shown on Fig. 3.1.

In a general case, the incoming flow of raw material R^* is a discrete random process, characterized by moments θ_ℓ of feed-in, and by random variables $L(\theta_\ell)$, the amount of raw materials supplied at those moments of time.

We will also consider productivity $X^{(i)}$ of each automatic line $G^{(i)}$ as a random variable, with known distribution function $F_X^{(i)}$. Mathematical expectations $E\left[X^{(i)}\right]$ and variances $V\left[X^{(i)}\right]$ of values $X^{(i)}$ are pregiven deterministic functions of primary resources (control parameters) $R_1^{(i)}, R_2^{(i)}, ..., R_m^{(i)}$, used for the i-th automatic line.

The actual productivity of line $G^{(i)}$ at moment of time t will be determined as a random variable $Z^{(i)}$, equal to the minimum of its productivity $X^{(i)}$ and of the volume of input bunker $A^{(i)}$:

$$Z^{(i)}(t) = \min\left\{ X^{(i)}(t), P^{(i)}(t)\right\}, \quad i = 1,2,...,s. \qquad (3.5.8)$$

The total actual production of the i-th line in plan period $\left[\, 0, T_{pl} \,\right]$ equals

$$x_{T_{pl}}^{(i)} = \sum_{t=0}^{T_{pl}-1} Z^{(i)}(t), \quad i = 1,2,...,s. \qquad (3.5.9)$$

Formalization of the shop's work allows us to state the basic problems of optimal control.

For the case of a mass production line, the direct problem of optimal control, maximizing control reliability, is as follows:

Determine a set of vectors $\vec{R}^{(i)} = \left\{ R_1^{(i)}, R_2^{(i)}, \ldots, R_m^{(i)} \right\}$, $i = 1,2,\ldots,s$, maximizing goal function

$$\psi\left(\vec{R}\right) = \underset{\{\vec{R}_i\}}{Max} \underset{1 \le i \le s}{Min} \left\{ 1 - \Phi\left[\frac{E^{(i)} - W^{(i)}}{\sigma^{(i)}} \right] \right\} \tag{3.5.10}$$

with restrictions

$$\sum_{i=1}^{S} R_j^{(i)} \le R_j^*, \quad j = 1,2,\ldots,m, \tag{3.5.11}$$

$$R_j^{(i)} \ge 0, \quad i = 1,2,\ldots,s, \quad j = 1,2,\ldots,m. \tag{3.5.12}$$

Here $E^{(i)}$ is the planned shift quota for the i-th automatic line and

$$\begin{cases} W^{(i)} = \sum\limits_{t=0}^{T_{pl}-1} E\left[Z^{(i)}(t) \right] \\ \sigma^{(i)} = \sqrt{\sum\limits_{t=0}^{T_{pl}-1} V\left[Z^{(i)}(t) \right]} \end{cases}. \tag{3.5.13}$$

It can be well-recognized that the inverse optimization control problem for a case of homogeneous resources $(m = 1)$ would be follows:

Determine values of homogeneous resources $R^{(1)}$, $R^{(2)}$, … , $R^{(S)}$ applied to corresponding lines $G^{(i)}$ to minimize the sum

$$R_S = \sum_{i=1}^{S} R^{(i)} \tag{3.5.14}$$

with restrictions

$$\begin{cases} \psi\left(R^{(i)}\right) = 1 - \Phi\left[\dfrac{E^{(i)} - W^{(i)}}{\sigma^{(i)}} \right] \ge p_{pl} \\ R^{(i)} \ge 0, \quad i = 1,2,\ldots,s \end{cases}, \tag{3.5.15}$$

probability p_{pl} being fixed in advance and close to one, and $\Phi(x) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{x} e^{-\frac{y^2}{2}} dy$.

3.5.5 The simulation model

An integrated model to simulate functioning and various control procedures for a mass production line shop has been developed in [68, 96]. Optimization problems of maximizing the reliability of control and minimizing primary resources have been successfully solved by its use. These are the stages of the model's work:

After simulating model parameters "drawn" or given beforehand (specific productivity of automatic lines, parameters of the distribution laws for feeding raw materials into the flow, data on the specialization of equipment, primary resources, etc.), "draw" the random number of resource units out of production and in working order in the routine elementary period of time, - a day, for instance. Then determine the deficiency of resources on the automatic lines and in the assembly section, and distribute reserve resources according to the priorities given beforehand, or by solving inverse problem (3.5.14-3.5.15). After that, to simulate characteristics for the automatic lines as well as for the assembly section, "draw" random durations of processing the parts, or the productivity per unit of time.

Simulating the work of the shift boils down to sequentially activating the "drawing" generators of the automatic lines and the assembly section productivities, taking into account the contents of the input bunkers A_n, $n = 1,2,...,N$, and recalculating their current volumes, as well as the output bunkers B_n. Fix the current output of the lines and sections, idleness and incomplete production, the completion of the parts for assembly and compare the current time value with inspection moment t_k and the moment of the shift's end T_{pl}.

At inspection moments t_k, determined either beforehand or by means of the theoretical grounds worked out in [68, 94-96], simulate control actions: resource redistribution according to the solution of the direct or inverse control problems, and reset according to the resources renewed, the generators of random productivity of the subdivisions of the shop flows. The resources are redistributed for the three-level system according to the shop's hierarchical structure: production line - flow - shop.

At first, solve the problem of maximal synchronization for the automatic lines of each flow, a direct problem of redistributing resources (3.5.10-3.5.12), complying with corrections for partially completing the volume of work by inspection moments t_k. If the optimal value ψ_{opt} of probability of fulfilling the plan by the line (the value obtained by formula (3.5.10)) is greater than the given p_{pl}, the redistribution at the automatic line level is completed, and forecasting plan fulfillment for the assembly section (the shop as a whole) is subsequently carried out.

If the forecast probability ψ_M of fulfilling the assembly plan is less than the given $p_{M\,pl}$, go again to the unit controlling the resources of the automatic line and solve the inverse problem of redistributing resources, in order to seek additional resources for the assembly section. If the resources supplied by the automatic line prove sufficient, the distribution at the second level, that of the flow, is over. Otherwise, seek additional resources at the third control level, that of the shop, in order to obtain surplus resources from other flows.

In the same way, transfer resources from the assembly section to the automatic lines in cases $\psi_{opt} < p_{pl}$ and $\psi_M > \psi_{M_{pl}}$. After obtaining additional resources (if they are less than the requested) once again solve the direct problem of redistribution for the automatic line.

We, thus, determine an iterative interlevel process of distributing resources which ends after a full review of the elements at all levels on shop scale, each iteration step solving an optimization control problem, direct or inverse.

Besides inspection moments t_k, the so-called emergency moments, moments of blockage t_{bl}, must also be taken into account (see Fig. 3.1). Upon request in emergency moments, the resources are redistributed from blocked automatic lines to other elements of the flow in order to increase the productivity of the flow and unload intermediary bunkers.

Concerning the direct and inverse problems of optimal resource control, the latter boil down to either maximizing the automatic lines' probability of fulfilling their tasks for the shift, or minimizing the total quantity of primary resources used. The direct problem is based on smoothing the forecast probabilities of fulfilling the plan by various production sections. The iterative step-by-step smoothing algorithm is based on the existence of linear dependences in the form

$$\begin{cases} E\left[X^{(i)}\right] = a_i R^{(i)},\ i = 1,2,...,s \\ \sigma\left[X^{(i)}\right] = b_i R^{(i)},\ a_i, b_i = const \end{cases}, \qquad (3.5.16)$$

The solution of the direct optimization problem, thus, takes productivity variations for various groups of automatic lines explicitly into account. The solution of the inverse problem boils down in practice to applying an analogous step-by-step algorithm with a given fixed probability of fulfilling the plan by all the automatic lines.

In our view, such a “run” of the course of production on a simulation model, combined with the control actions described above, makes a choice of an optimal control strategy possible. Such a strategy can be realized in combination with the mathematical models of FMS described in [57, 68].

Thus, we have formulated and suggested algorithmic solutions for various probability control production models for one and several hierarchical levels. The considered models are imbedded in a generalized production model including both optimization blocks and a simulation model.

§3.6 Generalized production control model with complex resources

3.6.1 Introduction

Assume that we are faced with controlling an organization system (manufacturing, constructing, etc.) which can produce the outcome product with several different speeds under random disturbances. Those speeds include:

1. The maximal (optimistic) speed v_{opt} which offers the utmost intensity of the production process;
2. The minimal (pessimistic) speed v_{pes} which corresponds to the minimal intensity of the production process;
3. The planned speed v_{pl} which lies between the outlined above boundary production rates.

Note that $\bar{v}_{opt} > \bar{v}_{pl} > \bar{v}_{pes}$ holds. To manufacture the product, the system requires resources R (manpower, machines, etc.), which can be evaluated in complex items, e.g., in standard teams of pregiven structure. Such a resource is called a complex one and can be evaluated in ordinal numbers. Call $V_{opt}(R,t)$, $V_{pes}(R,t)$ and $V_{pl}(R,t)$ the average manufactured product to be produced at moment t, on condition that the system has worked only with the optimistic, pessimistic and planned speed, correspondingly, within the period $[0,t]$ throughout. Call $V_f(R,t)$ the random outcome product observed at inspection point t. In papers [4, 67, 94-96] we have introduced two different control strategies in order to determine routine control points t_i^* (Strategy I) or t_i^{**} (Strategy II). In order to describe those strategies more definitely, let us introduce some additional notations:

T_{pl} - the due date of the system;

T_I - the average date to reach the system's goal, if only optimistic, i.e., the highest speed, is actually used throughout;

V_{pl} - the system's target;

ΔT - the least permissible time span between two adjacent control points (to force convergence).

3.6.2 Strategies

Two different strategies [4, 67, 94, 103] will be imbedded in the control policy:

Strategy I

It is assumed that in the case of the most unfavorable circumstances, which result in the minimal production intensity, the output within a certain interval $[t_1,t_2]$, $t_2 > t_1$, $0 \le t_1 < t_2 \le T_{pl}$, may not increase at all, i.e., will satisfy $V_f(t_1) = V_f(t_2)$.

Strategy II

It is assumed that even in the case of the minimal production intensity, the output within any interval $[t_1, t_2]$, $t_2 > t_1$, $0 \le t_1 < t_2 \le T_{pl}$, will increase by no less than $(t_2 - t_1) \cdot v_{pes}$, i.e., relation $V_f(t_2) \ge V_f(t_1) + (t_2 - t_1) \cdot v_{pes}$ holds.

3.6.3 Internal control actions with both strategies

Let us describe possible control actions (determining next control point, introducing a proper speed) for both strategies in greater detail [103]:

Strategy I

A. If at a routine control point t_i^*, $i \ge 1$, relation

$$V_f(R, t_i) > V_{pl}(R, t_i) \tag{3.6.1}$$

holds, the next, $(i+1)$-th control point t_{i+1}^* satisfies

$$V_f(R, t_i^*) = V_{opt}(R, t_{i+1}^* - T_{pl} + T_I), \tag{3.6.2}$$

and the system does not change its production speed.

B. If relation

$$V_{pl}(R, t_i^*) > V_f(R, t_i^*) > V_{opt}(R, t_i^* - T_{pl} + T_I) + \Delta V \tag{3.6.3}$$

holds, where $\Delta V > 0$ is a permissible threshold error (given beforehand), the maximal speed has to be introduced. The next control point t_{i+1}^* satisfies (3.6.2).

C. If at a routine control point t_i^*

$$\left| V_f(R, t_i^*) - V_{opt}(R, t_i^* - T_{pl} + T_I) \right| < \Delta V \tag{3.6.4}$$

holds, system S applies the maximal speed, while the next control point $t_{i+1}^* = t_i^* + \Delta T$ is determined by using the time span ΔT.

D. If inequality

$$V_f(R, t_i^*) < V_{opt}(R, t_i^* - T_{pl} + T_I) - \Delta V \tag{3.6.5}$$

holds, additional resources have to be introduced since the system is unable to reach its goal at moment T_{pl}, even by using the maximal speed $V_{opt}(R, t)$ throughout the remaining time. In this case, the higher hierarchical level has to be applied to obtain help with resources of volume δ_R. Value δ_R can be determined from relation

$$V_{opt}(R + \delta_R, T_{pl} - t_i^*) = V_{pl}(R, T_{pl}) - V_f(R, t_i^*). \tag{3.6.6}$$

E. If in the process of the system's functioning, value t_i^* is so close to value T_{pl} that inequality

$$T_{pl} - t_i^* < \varepsilon_T \tag{3.6.7}$$

holds, where $\varepsilon_T > 0$ is a permissible error (given beforehand), while the output product is inspected at moment T_{pl}.

Strategy II

By applying this strategy, the following situations can be pointed out:

A. If at a routine control point t_i^{**}, $i \geq 1$, relation

$$V_{pes}\left(R,\, T_{pl}\right) + V_f\left(R,\, t_i^{**}\right) - V_{pes}\left(R,\, t_i^{**}\right) \geq V_{pl}\left(R,\, T_{pl}\right) \tag{3.6.8}$$

holds, we assume $t_{i+1}^{**} = T_{pl}$, and the minimal speed $v_{pes}\left(R,t\right)$ has to be introduced up to T_{pl}.

B. If relations

$$\begin{cases} V_{pes}\left(R,\, T_{pl}\right) + V_f\left(R,\, t_i^{**}\right) - V_{pes}\left(R,\, t_i^{**}\right) < V_{pl}\left(R,\, T_{pl}\right) \\ V_f\left(R,\, t_i^{**}\right) \geq V_{pl}\left(R,\, t_i^{**}\right) \end{cases} \tag{3.6.9}$$

hold, the routine inspection moment t_{i+1}^{**} is determined by

$$V_{opt}\left(R,\, t_{i+1}^{**} - T_{pl} + T_I\right) = V_{pes}\left(R,\, t_{i+1}^{**}\right) + V_f\left(R,\, t_i^{**}\right) - V_{pes}\left(R,\, t_i^{**}\right), \tag{3.6.10}$$

and system S continues functioning with the planned speed without introducing any other control actions.

C. If at control moment t_i^{**} relation (3.6.3) holds, the situation is similar to *Case B* for *Strategy I*, with the difference that the next control moment t_{i+1}^{**} is determined by using (3.6.10), instead of (3.6.2).

D. This case is equivalent to *Case C* for *Strategy I*.

E. This case is equivalent to *Case D* for *Strategy I*.

F. If trajectories $V_{opt}\left(R,t\right)$, $V_{pes}\left(R,t\right)$ and $V_{pl}\left(R,t\right)$ are straight lines

$$\begin{cases} V_{pl}\left(R,t\right) = \dfrac{t}{T_{pl}} V_{pl}\left(R,T_{pl}\right), \\ V_{opt}\left(R,t\right) = v_{opt}\left(R,t\right) \cdot t, \\ V_{pes}\left(R,t\right) = v_{pes}\left(R,t\right) \cdot t, \end{cases} \tag{3.6.11}$$

it is convenient to illustrate the control actions graphically, as presented on Fig. 3.2.

3.6.4 Graphical illustration

If on Fig. 3.2 we jointly construct diagrams $V_{opt}(R,t)$, $V_{pes}(R,t)$ and $V_{pl}(R,t)$ as straight lines $AB = V_{opt}(R,t)$, $CE = V^*(R,t)$, $AJ = V_{pes}(R,t)$, $LC \| AJ$, point C having coordinates $\left[T_{pl}, V_{pl}(R,T_{pl})\right]$, then rectangle *AGCF* is domain of definition of the set of all possible states of the system in the process of advancing to the target V_{pl} within the planning horizon $\left[0, T_{pl}\right]$. It can be well-recognized that rectangle *AGCF* is subdivided by the straight lines into several areas. As the criterion for classifying those areas, let us examine: why do the points representing the actual state of the system at the control moments fall into them? Then, proceeding from the criterion chosen, we can single out the following three areas:

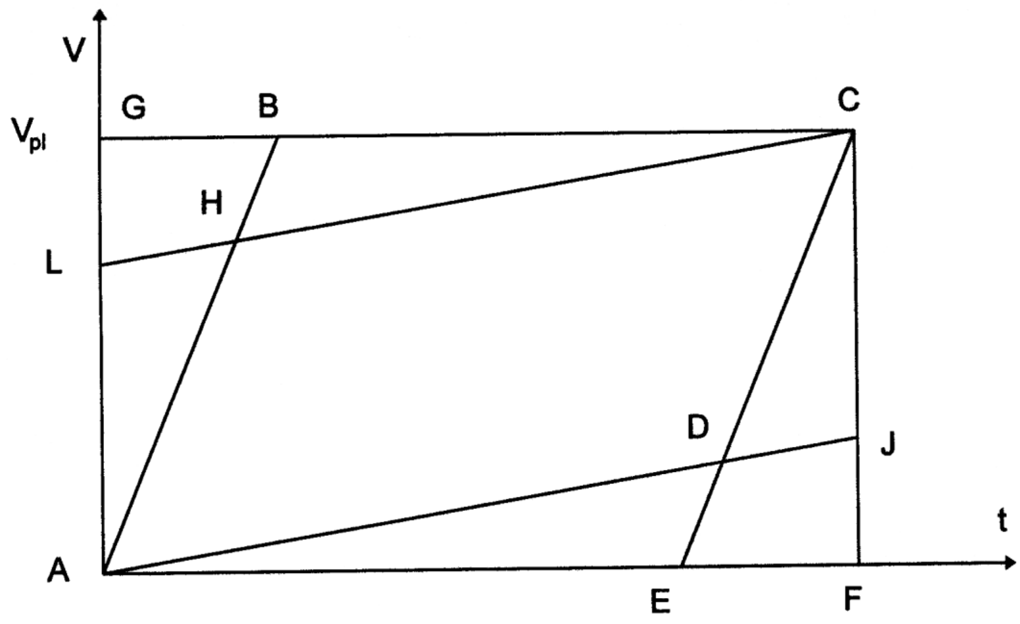

Figure 3.2. *Decision-making areas for on-line internal and external control*

Region (area) ABCJ. Point Q, representing the current state of the system, can fall into this region both with a change in the speed of its movement towards the goal within boundaries $v_{pes}(R,t) \leq v_f(R,t) \leq v_{opt}(R,t)$ and as a result of the influence on the system by short-term positive and negative disturbances.

Region AJF. Point Q can fall into this region only due to negative disturbances influencing the system that hamper the normal course of the production process. In such conditions, the average actual speed $\bar{v}_f(R,t)$ of the system's movement towards the goal satisfies $\bar{v}_f(R,t) < v_{pes}(R,t)$.

Region AGB. Point Q can fall into this area due to factors favoring the process of the system's movement towards the goal, determining the average speed of movement to the goal $\bar{v}_f(R,t) > v_{opt}(R,t)$.

But if we regard as criterions for the regions the control actions necessary to be introduced in the system's work in order to meet on time the planned volume of the production program, then the regions indicated must be described as follows:

Region ALCE. By altering the speed of the system's movement within bounds $v_{pes}(R,t) \le v_f(R,t) \le v_{opt}(R,t)$, the goal can be reached from any point of the region. Local control action on the system at moment t_i is in this case no more than an indication of the need to adopt a new speed $v'(R,t_i)$ for carrying out the production program, where

$$v'(R,\, t_i) \;=\; v_f(R,\, t_i) + \frac{V_{pl}(R,\, t_i) - V_f(R,\, t_i)}{T_{pl} - t_i}. \tag{3.6.12}$$

In other words, condition

$$V_{pl}(R,\, t_i) - V_f(R,\, t_i) \;\le\; \Delta V^* \tag{3.6.13}$$

holds, where value ΔV^* is determined according to (3.6.11). If we denote the change in the complex resource of the system as a result of performing external control actions obtained from the routine control by symbol δ_R, relation $\delta_R = 0$ is obviously satisfied in region *ALCE*, and there is no need to alter resource R.

3.6.5 External control actions

Region ECF. If point Q, representing the current state of the system, falls into this region, the system's further advance even at maximum speed $v_{opt}(R,t)$ towards the goal cannot ensure meeting the target at point C. In other words, inequality

$$v_{opt}(R,\, t) \;=\; v_f(R,\, t_i) + \frac{V_{pl}(R,\, t_i) - V_f(R,\, t_i)}{T_{pl} - t_i} \tag{3.6.14}$$

holds. This, in turn, means that

$$V_{pl}(R,\, t_i) - V_f(R,\, t_i) \;>\; \Delta V^*, \tag{3.6.15}$$

and as has already been indicated, the system can achieve goal C in this case only by introducing a corresponding external control action by way of changing the total resources of the system by value $\delta_R \ne 0$. Here δ_R can be determined from the following production situations:

A. When it is possible to transfer system S to the optimistic speed, value δ_R can be obtained from (3.6.11) as follows:

$$v_{opt}(R+\delta_R)(T_{pl} - t_i) = V_{pl}(R,\, T_{pl}) - V_f(R,\, t_i) \tag{3.6.16}$$

with restriction

$$R + \delta_R \;\le\; N_r^{\max}, \tag{3.6.17}$$

where $N_r^{\max}$ is the maximal capacity of complex resource items.

B. When system S continues working with the planned speed, value δ_R can be determined from relations (3.6.5) and (3.6.11) in the form

$$R = \frac{V_{pl}(R, t_i) - V_f(R, t_i)}{(T_{pl} - t_i)\Delta v} r \quad (3.6.18)$$

subject to (3.6.17).

By symbol Δv we mean a certain "specific" increment in the system's speed of movement, achieved as a result of altering its resources by one unit of complex resource ΔN_R.

Region LCG. When the point representing the current state of the system falls into this region, it means that in this case, the system, moving the further even at minimum speed $v_{pes}(R,t)$, will reach the goal ahead of time T_{pl}. Here $V_{pl}(R,t_i) < V_f(R,t_i)$, and

$$v_{pes}(R, t) > v_f(R, t) + \frac{V_{pl}(R, t_i) - V_f(R, t_i)}{T_{pl} - t_i}. \quad (3.6.19)$$

If now a part of resources of the system can be released for working outside the system, it will be obviously not prevent the system from reaching its goal by T_{pl}. The released resources δ_R^* can be determined as follows:

A. If the system continues working with the planned speed we apply relation (3.6.20) analogous to (3.6.18), and

$$\delta_R^* = \frac{V_f(R, t_i) - V_{pl}(R, t_i)}{(T_{pl} - t_i)\Delta v} r \quad (3.6.20)$$

subject to

$$R - \delta_R \geq N_r^{\min}, \quad (3.6.21)$$

where $N_r^{\min}$ is the minimal capacity of complex resource items to operate the system.

B. When the system is transferred to the pessimistic or to the optimistic speed, the quantity of released resources can be obtained from equations (3.6.22) or (3.6.23), respectively,

$$v_{pes}(R - \delta_R)(T_{pl} - t_i) = V_{pl}(R, T_{pl}) - V_f(R, t_i), \quad (3.6.22)$$

$$v_{opt}(R - \delta_R)(T_{pl} - t_i) = V_{pl}(R, T_{pl}) - V_f(R, t_i), \quad (3.6.23)$$

subject to (3.6.21).

Thus, from the different situations considered, connected with the system's movement to the goal, the corresponding conclusions can be drawn, regarding the need to change the movement's speed on the basis of internal control, and using possible external control actions by correcting the value of resources supplying the system.

3.6.6 Hierarchical external control actions

We now take up a more detailed analysis of external control actions in hierarchical production systems [4, 67, 103]. Let symbol B_i^j denote an element with an i-th ordinal number at the j-th hierarchical level of system S. As has been pointed out above, any external control, if we disregard cases of known preterm realization of the production program, is linked to additional resources introduced into B_i^j which must be obtained from a certain element B_ℓ^{j-1} at the supreme level (for instance, at the expense of a centralized reserve of resources of element B_ℓ^{j-1}). An alternative way of finding the required volume of additional resources in these conditions is to redistribute resources among co-subordinate elements (let them be m). It must be noted that the procedure for redistributing resources also presupposes a review of certain plan characteristics, in particular, values $V_{pl}(R,T_{pl})$.

When it is necessary to introduce an external control action for element B_i^j, the control of that element results in determining necessary requirements for resources with respect to the corresponding element B_ℓ^{j-1} at the supreme hierarchical level in order to ensure the required intensity of the movement to the goal for all B_i^j, $i = 1,2,...,m$. However, before beginning to develop an algorithm for determining an adequate amount of additional resources, the structure of the resources must be examined somewhat more attentively; in several cases, the resources have quite specific properties, and in our view this is still insufficiently taken into account in contemporary control systems.

In order to ensure that element B_i^j reaches the goal at the given intensity, a definite set of resources must be available (the complex resource of the system), the system's speed of movement to the goal acquiring zero value if even one component of the set is lacking. Each component of the complex resource determines the speed and other possible qualities of the process of carrying out the production program, the set of such characteristics, for its fact, being determined by the technology for carrying out jobs by element B_i^j.

Generally speaking, the production program can be carried out with various resources, differing from each other in definite qualities of carrying out operations (speed, for instance).

We will estimate the quality degree of the complex resource's suitability to the goal by a certain value μ, $0 \le \mu \le 1$, and denote ε, $0 \le \varepsilon \le 1$, for the minimum threshold value of this degree. Then the requirement of the quality degree of a complex resource will be expressed by inequality $\mu \ge \varepsilon$.

The unit of the complex resource r can be represented in the form of $\vec{r} = (r_1,...,r_n)$, where r_k, $k = 1,2,...,n$, is the quantity of the k-th resource of the complex resource unit.

The quality degree of the k-th component of the complex resource will be denoted by symbol μ_k, $k = 1,2,...,n$.

The threshold requirements of the quality degree for an individual component of the complex resource will be denoted by ε_k, $k = 1,2,...,n$. The requirements of the quality degrees of the complex resource's components are thus expressed by inequalities $0 \le \varepsilon_k \le \mu_k \le 1$.

The need to implement external control actions arises when a production situation appears in element B_i^j that presents new requirements of the quality degrees ε_k^* for certain components of the complex resource. The corresponding element B_ℓ^{j-1} is then faced with the problem of optimizing a subset of values $\mu_k^* \ge \varepsilon_k^*$ within the corresponding components.

For solving this problem, we will take advantage of a convenient characteristic of difficulty in reaching the goal

$$d = 1 - \prod_{k=1}^{n} \left(1 - d_k\right), \tag{3.6.24}$$

where d_k is a partial difficulty for the k-th component of the complex resource, which can be determined by formula

$$d_k = \frac{\varepsilon_k}{\mu_k} \cdot \frac{1-\mu_k}{1-\varepsilon_k}. \tag{3.6.25}$$

Values d and d_k are within interval $[0,1]$, difficulty d becoming critical ($d = 1$) if for at least one k, $k = 1,2,...,n$, equality $\mu_k = \varepsilon_k$ holds.

It is known that improving the quality of any kind of resource is accompanied by an increase in expenditure for it, the expenditure, generally speaking, increasing non-linearly. Let $f_k\left(\mu_k^* - \mu_k\right)$ denote the value of expenditure caused by a given increase in the quality of the k-th component over threshold value ε_k, $\mu_k^* - \mu_k > 0$. Then the total expenditures per unit of the complex resource with given μ_k^*, $k = 1,2,...,n$, at fixed requirements ε_k^*, $\mu_k^* \ge \varepsilon_k^*$, are expressed by value

$$\sum_{k=1}^{n} f_k\left(\mu_k^* - \mu_k\right). \tag{3.6.26}$$

If we consider that element B_ℓ^{j-1} has means at its disposal singled out for the requirements of element B_i^j, and that the volume of those means adds up to A_i^j, in this case choosing new values μ_k^*, $k = 1,2,...,n$, requires solving the relatively simple problem of non-linear programming, namely,

$$\min d \,, \tag{3.6.27}$$

with restrictions

$$\begin{cases} \sum_{k=1}^{n} f_k\left(\mu_k^* - \mu_k\right) \leq A_i^j \\ \mu_k^* \geq \mu_k \end{cases} . \tag{3.6.28}$$

Given practically one restriction, even at a large value of n, it is not very difficult to solve the problem of non-linear programming (3.6.27-3.6.28).

Thus, having chosen qualitative characteristics of the complex resource, element B_ℓ^{j-1} must pass on to determining the corresponding quantity characteristics. The problem of evaluating the optimal set of quantities for each type of resources χ_k, $k = 1,2,...,n$, can be solved as a minimax problem

$$\underset{\chi_k}{Max}\,\underset{k}{Min}\,\frac{\chi_k}{r_k}\,, \tag{3.6.29}$$

the cost restriction taking the form, for example, of

$$\sum_{k=1}^{n} C_k \chi_k \leq A_i^j \,, \tag{3.6.30}$$

where χ_k is the quantity of the k-th type of resource units,

C_k is the cost per unit of the resource, and

A_i^j is the total cost restriction for element B_i^j.

After singling out the corresponding quantities of resources for element B_i^j, the problem transfers to local control actions, whose principles have been considered above.

An integrated flow-chart for both internal and external control is presented on Fig. 3.3.

We have, thus, considered the case of application of external control actions for supplementing resources without redistributing them. If there is a need to redistribute the resources among co-subordinate elements (subsystems) of one and the same hierarchical level of system S, other optimization problems arise.

§3.7 New quality concepts for multilevel organization systems

3.7.1 Introduction

Man-machine OS are characterized nowadays by increasing both the systems' complexity and the number of their hierarchical levels as well as by various random disturbances which affect the systems' realization. Man-machine OS are usually managed by decision-makers on different hierarchical levels. Decision-making is usually carried out on the basis of periodical systems' inspection in control points and is also characterized by a variety of optimization problems [68, 78-79] which are solved at control points in order to determine control actions to speed up the system's progress and to increase the system's reliability value. In the material under consideration we intend to show some applications of results described above, in §§3.1-3.6, on the example of two- and three-level flexible manufacturing systems.

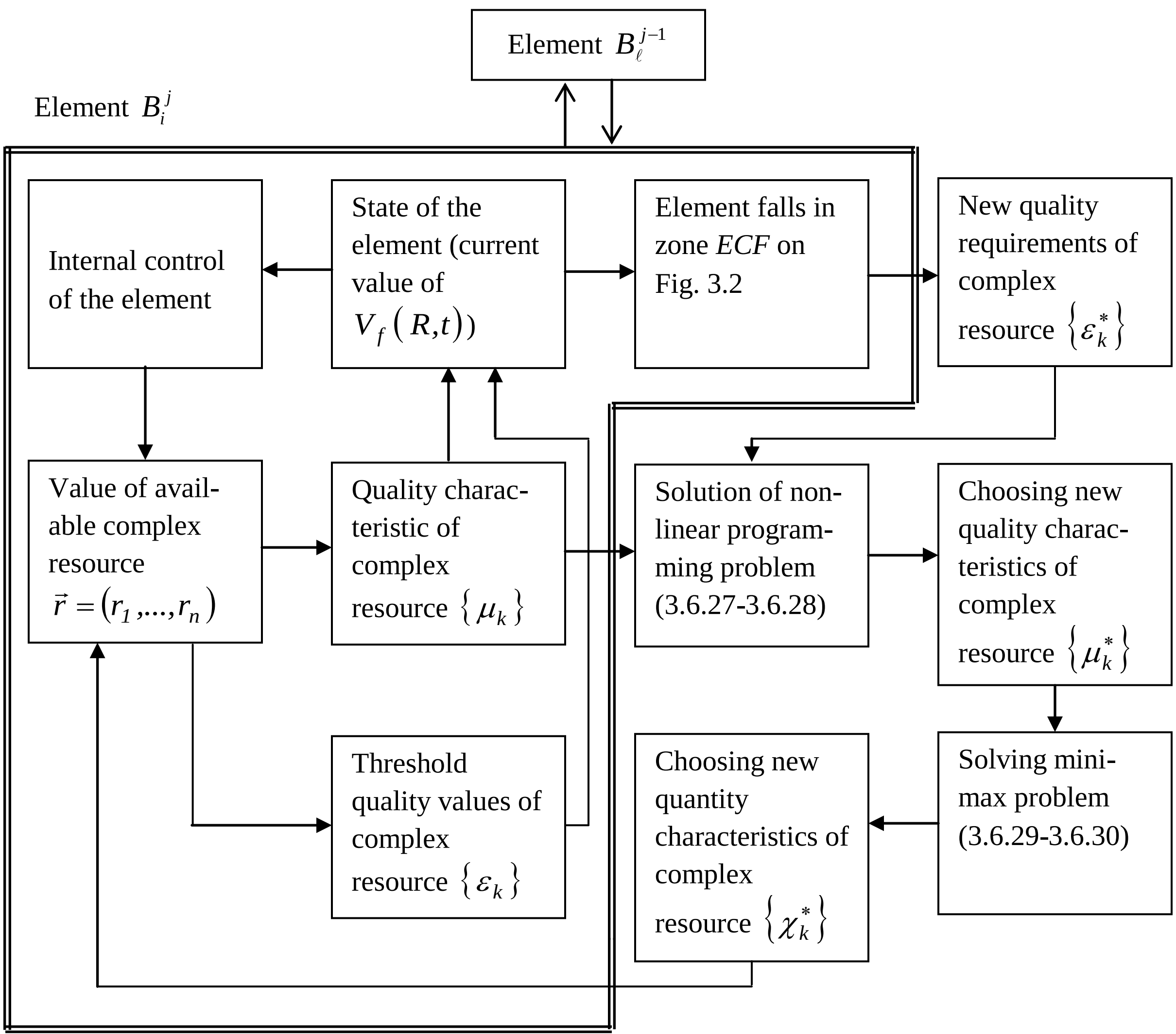

Figure 3.3. Flow chart for external and internal control of a multi-level system

3.7.2 Notation

Let us introduce the following terms:

S - flexible OS;

$\vec{R}$ - vector of resource capacities at the disposal of the system (personnel, various renewable resources, budget values, etc.);

$\vec{V}^{pl}$ - vector of the planned program (the system's target);

T^{pl} - the due date (the planning horizon);

$\vec{V}^{f}(t)$ - the actual state of the system observed at moment t, $0 \le t \le T^{pl}$ (a random value);

Q - the system's objective;

p^* - the least permissible probability of meeting the system's target on time (pregiven).

To simplify the problem, consider a two-level OS with a control device at the upper level and n elements E_i, $1 \le i \le n$, at the lower level. Elements E_i may represent production units entering a section, etc. Additional terms have to be introduced as follows:

$\vec{R}_i$ - vector of resource capacities assigned to the i-th element;

$\vec{V}_i$ - vector of the planned program assigned to the i-th element;

$v_{ij}(\vec{R}_i)$ - the j-th speed of element E_i, $1 \le j \le m$ (a random value with density function $f_{ij}(v)$ depending parametrically on vector $\vec{R}_i$);

m - number of possible speeds;

t_{ik} - the k-th inspection (control) point of element E_i, $k = 0, 1, \dots, N_i$;

N_i - the number of control points of the i-th element;

s_{ik} - the index of the speed chosen by the decision-maker at the control point t_{ik} (the lower level);

$ICA^{(i)}$ - internal control action to speed up the element E_i (by introducing a higher speed v_{ij});

ICA - internal control action for the two-level system (by reallocating resources $\vec{R}$

or re-distributing target $\vec{V}$ among elements E_i, $1 \le i \le n$);

ECA - external control action for the two-level system (by supporting the system with additional resources $\Delta\vec{R}$);

T_r - the r-th emergency point, $r = 1, 2, \ldots$.

3.7.3 The problem

The optimization problem to be solved at each emergency point T_r when it is anticipated that a certain element cannot meet its local target on time, may be formulated as follows:

$$\underset{\{\vec{V}_i,\, \vec{R}_i,\, t_{ik},\, s_{ik}\}}{Opt} Q, \tag{3.7.1}$$

subject to

$$\sum_{i=1}^{n} \vec{R}_i = \vec{R}, \tag{3.7.2}$$

$$\sum_{i=1}^{n} \vec{V}_i = \vec{V}^{pl}, \tag{3.7.3}$$

$$\Pr \left\{ \vec{V}^f \left(T^{pl} \right) \ge \vec{V}^{pl} \middle| ICA \right\} \ge p^*, \tag{3.7.4}$$

$$\Pr \left\{ \vec{V}_i^f \left(T^{pl} \right) \ge \vec{V}_i \middle| ICA^{(i)} \right\} \ge 1 - \varepsilon, \quad 1 \le i \le n, \tag{3.7.5}$$

where ε is close to zero. Restriction (3.7.4) means, that the system is able to meet its target on time by using only its internal reserves $\vec{R}_i$, namely, by supporting the slower elements on the account of the faster ones. Restriction (3.7.5) means, that each i-th element is able to accomplish its local program $\vec{V}_i$ on time by using the element's reserves, e.g. introducing higher speed v_{ij}, $s_{ik} = j$, at control points t_{ik}.

3.7.4 Quality estimates

We suggest the following quality estimates [100]:

A. Case of one-level OS

1. The system's *stability value* q^* satisfies

$$q^* = \Pr \left\{ \vec{V}^f \left(T^{pl} \right) \ge \vec{V}^{pl} \right\}, \tag{3.7.6}$$

and defines the system's ability of meeting the due date on time without any internal control actions, i.e., by choosing at moment $t = 0$ *speed* v_j *which will be used within the planning horizon without inspection points.*

2. The system's *internal ability value* q^{**} satisfies

$$q^{**} = \Pr\left\{\vec{V}^{f}\left(T^{pl}\right) \geq \vec{V}^{pl} \middle| ICA\right\} \geq p^{*}, \tag{3.7.7}$$

and defines the system's ability of meeting the due date on time by introducing only internal control actions, i.e., by changing periodically, at control points t_k*, the speed* v_j*,* $j = s_k$*, in order to speed up the system*. If $q^{**} < p^{*}$ holds, external control actions have to be introduced.

3. The system's *external ability value* q^{***} satisfies

$$q^{***} = \Pr\left\{\vec{V}^{f}\left(T^{pl}\right) \geq \vec{V}^{pl} \middle| \Delta\vec{R},\, ICA\right\} \geq p^{*}, \tag{3.7.8}$$

which means that *to meet the deadline on time both external and internal control actions have to be applied.* Note that the need of ECA results *in a low system's quality*. Thus, value q^{***} cannot be regarded as a high quality estimate.

B. Case of a two-level system

Stability value q^{*} is defined by (3.7.6) taking into account that at moment $t = 0$, both vectors $\vec{V}^{pl}$ and $\vec{R}$ will be redistributed among the elements. For all of them at moment $t = 0$ speeds v_{ij} will be determined which will not undergo any changes within the planning horizon $\left[0, T_{pl}\right]$.

The system's *internal ability value* q^{**} satisfies (3.7.7). Note, that for a two-level system S ICA results not only in changing the elements' speeds at the lower level, but *mainly* in re-allocating periodically the remaining $\vec{R}$ and $\vec{V}^{pl}$ at emergency points T_r among the system elements E_i, $1 \leq i \leq n$. In case $q^{**} < p^{*}$ the external ability value q^{***} is determined by (3.7.8), where $\Delta\vec{R}$ is the *minimal additional reserve* which enables $q^{***} > p^{*}$.

3.7.5 *A three-level OS*

Consider a three-level production system: factory (company) – section – production unit. At the upper level the optimal problem is solved at each emergency point T_r and results in minimizing value C - the cost of hiring and maintaining the vector of resource capacities $\vec{R}$. Additional terms have to be introduced:

S_g - the g-th section subordinated to the company, $1 \leq g \leq e$;

e - number of sections;

E_{ig} - the i-th element - production unit at the lower level - subordinated to the g-th section, $1 \leq i \leq n_g$;

n_g - number of production units subordinated to the g-th section;

$\vec{R}_g, \vec{V}_g$ - resource and program vectors assigned to S_g;

$\vec{R}_{ig}, \vec{V}_{ig}$ - resource and program vectors assigned to E_{ig};

t_{kig} - the k-th control point for the element E_{ig};

$s_{kig} = j$ - the speed introduced for E_{ig} at $t = t_{kig}$;

v_{jig} - the j-th speed of element E_{ig};

ICA_s - internal control action for system S;

ICA_g - internal control action for section S_f;

ICA_{ig} - internal control action for element E_{ig}.

The problem is as follows:

$$\underset{\{\vec{R},\vec{R}_g,\vec{V}_g,\vec{R}_{ig},\vec{V}_{ig},t_{kig},s_{kig}\}}{Min} C \tag{3.7.9}$$

subject to

$$\Pr\left\{\vec{V}^{f}\left(T^{pl}\right) \geq \vec{V} \middle| ICA_s\right\} \geq p^*, \tag{3.7.10}$$

$$\sum_g \vec{R}_g = \vec{R}, \tag{3.7.11}$$

$$\sum_g \vec{V}_g = \vec{V}^{pl}, \tag{3.7.12}$$

$$\sum_{i=1}^{n_g} \vec{R}_{ig} = \vec{R}_g, \tag{3.7.13}$$

$$\sum_{i=1}^{n_g} \vec{V}_{ig} = \vec{V}_g \tag{3.7.14}$$

$$t_{k+1,ig} - t_{kig} \geq \Delta_{ig}, \tag{3.7.15}$$

where Δ_{ig} is the minimal time span introduced in order to force convergence.

The general problem (3.7.9-3.7.15) can be subdivided into hierarchical models as follows:

Regulation Model at the Upper Level at emergency points T_r:

$$\underset{\{\vec{R},\vec{R}_g,\vec{V}_g\}}{Min}\ C \qquad (3.7.16)$$

subject to (3.7.10-3.7.12) and

$$\underset{\{\vec{R}_g,\vec{V}_g\}}{Max}\left[\underset{g}{Min}\left[Pr\left\{\vec{V}_g^{\,f}\left(T^{pl}\right) \geq \vec{V}_g \,\middle|\, ICA_g\right\}\right]\right] \geq p^* \qquad (3.7.17)$$

Regulation Model at the Section Level:

$$\underset{\{\vec{R}_{ig},\vec{V}_{ig}\}}{Max}\left[\underset{i}{Min}\left[Pr\left\{\vec{V}_{ig}^{\,f}\left(T^{pl}\right) \geq \vec{V}_{ig} \,\middle|\, ICA_{ig}\right\}\right]\right] \qquad (3.7.18)$$

subject to (3.7.13-3.7.14).

On-Line Control Model at the Production Unit Level

$$\underset{\{t_{kig},s_{kig}\}}{Max}\ N_{ig} \qquad (3.7.19)$$

and

$$\underset{\{t_{kig},s_{kig}\}}{Max}\ Pr\left\{\vec{V}_{ig}^{\,f}\left(T^{pl}\right) \geq \vec{V}_{ig}\right\} \qquad (3.7.20)$$

subject to (3.7.15).

Note that $ICA_s = \bigcup_g \{ICA_g\}$, while $ICA_g = \bigcup_i \{ICA_{ig}\}$. Here ICA_g is determined by (3.7.3-3.7.5) subject to (3.7.11-3.7.12), while control actions ICA_{ig} are determined by various heuristic rules. Those actions result in determining control points t_{kig} and speeds $j = s_{kig}$.

For a three-level system value q^{**} is defined by (3.7.7).

Thus, we suggest using values q^* and q^{**} as the main quality estimates for OS. It can be well-recognized that relation $q^{**} > q^*$ always holds. As to value q^{***}, it depends on $\Delta\vec{R}$ and, thus, cannot serve as a quality estimate.

Chapter 4. Models for Determining Inspection Moments in Multilevel Organization Systems

§4.1 Introduction

Controlling production process is an activity aimed at fulfillment of the production program (plans, filling orders) for the entire list of products. In order to carry out this task, the management must make a timely evaluation of fulfilling the program, watch the tendency of production to deviate from the planning rate and direct the resources at its disposal towards eliminating those deviations.

In many fields of production, e.g., petrochemistry, sugar refining, etc., accounting the amount of intermediate and finished products is automated, and the personnel can at any time know the figures characterizing the course of production. However, in fields like automobiles, metallurgy, construction, high technologies, and certain others, it is quite difficult to evaluate how the production program is proceeding.

Every operation for showing the actual fulfillment of a program and controlling delivery time for each type of products calls for taking stock of the finished product both dispatched and in storage, keeping count of all process stock, and the state of the means of production. This is an expensive operation, often calling for suspension of the production process. It is therefore desirable that this be done as rarely as possible, but without missing the moment when the tendency to deviate develops into jeopardizing the output of finished products.

Let us examine the process of controlling the work of production system S of a single-goal type, the volume of the production program being expressed in the form of a general equivalent - in output units (items) or in cost. For production programs turning out several important types of output products, inspections for each type simultaneously have to be undertaken.

In the previous chapter we have pointed out that the functional dependence of the course of carrying out production program $V_{pl}(R,t)$ on time t is the planning trajectory satisfying $V_{pl}(R,T_{pl}) = V_{pl}$.

Within the planning horizon, when advancing towards the goal, it is then necessary to compare the true (actual) values $V_f(R,t_i)$ - random values - with those calculated for the planned trajectory $V_{pl}(R,t_i)$ at definite moments t_i. The latter have to be determined beforehand. The corresponding control actions thereby ensure that the system will reach its goal at the pregiven due date T_{pl}.

The planned trajectory $T_{pl}(R,t)$ corresponds, as has been outlined in Chapter 3, to a certain temporary estimation of the duration of that moment T_{pl}. Besides, in the process of inspecting the work of the system, it is essential to use concepts introduced before, such as $V_{opt}(R,t)$, $V_{pes}(R,t)$, and $V_f(R,t)$. Note that actually speaking, all those trajectories are realized under random disturbances and are determined on the basis of average values.

§4.2 Determining inspection moments

Let us consider the method for determining inspection points [67, 71] by using the basic system's characteristics V_{pl}, T_{pl}, T_I, T_{II}. We will call the corresponding procedure, which is illustrated on Fig. 4.1, Strategy I. First, we shift trajectory $V_{opt}(R,t)$ parallel to itself in such a way that the end of the trajectory $V_{opt}(R,t)$ coincides with point $(V_{pl};T_{pl})$. Henceforth, this shifted trajectory will be denoted the symbol $V_{opt}(R,t-T_{pl}+T_I)=V^*(R,t)$. At the intersection of the abscissa axis and line $V^*(R,t)$, we obtain point t_1. It is easy to see that even if the system has not functioned at all by moment t_1 and has not advanced towards its goal (in other words, $V_f(R,t_1)=0$), there obviously still exists the probability, differing from zero, that beginning with that moment of time t_1, and employing the extreme possibilities of the system's functional part, we can still reach the goal by moment T_{pl}. Thus, moment t_1 can be considered as the extreme permissible time for the first inspection of the system. Physically, this consists in the following:

- If the first inspection is made later than time t_1 and reveals the presence of an unfavorable situation in the system, since the highest speed of the system's advancement towards the goal is determined by trajectory $V_{opt}(R,t)$, the due date T_{pl} can under no circumstances be assured.

It can be well-recognized that the expression for estimating the time t_1 of the first inspection of the system will appear as follows:

$$t_1 = T_{pl} - T_I . \tag{4.2.1}$$

Thus, in order to assure that the system achieves goal T_{pl} by the deadline, the first inspection must be made at a moment of time within the span of $0=t_0<t\le t_1$. Upon reaching the set time t_1 and having made the inspection, the system receives information as to the dynamics of the course of fulfilling the production program by comparing values $V_f(R,t_1)$ and $V_{pl}(R,t_1)$.

Based on the information obtained, if it shows a deviation between the plan and the actual course of the production process, local or parametrical control actions which have been introduced in the previous chapter, must be initiated to eliminate the deviation. Further, drawing a line through point $(t_1,V_f(R,t_1))$ parallel to the abscissa till its intersection with the shifted curve $V^*(R,t)$, we obtain a point with abscissa t_2, determining, according to the reasons submitted, the limit value of the moment for the second inspection of the system.

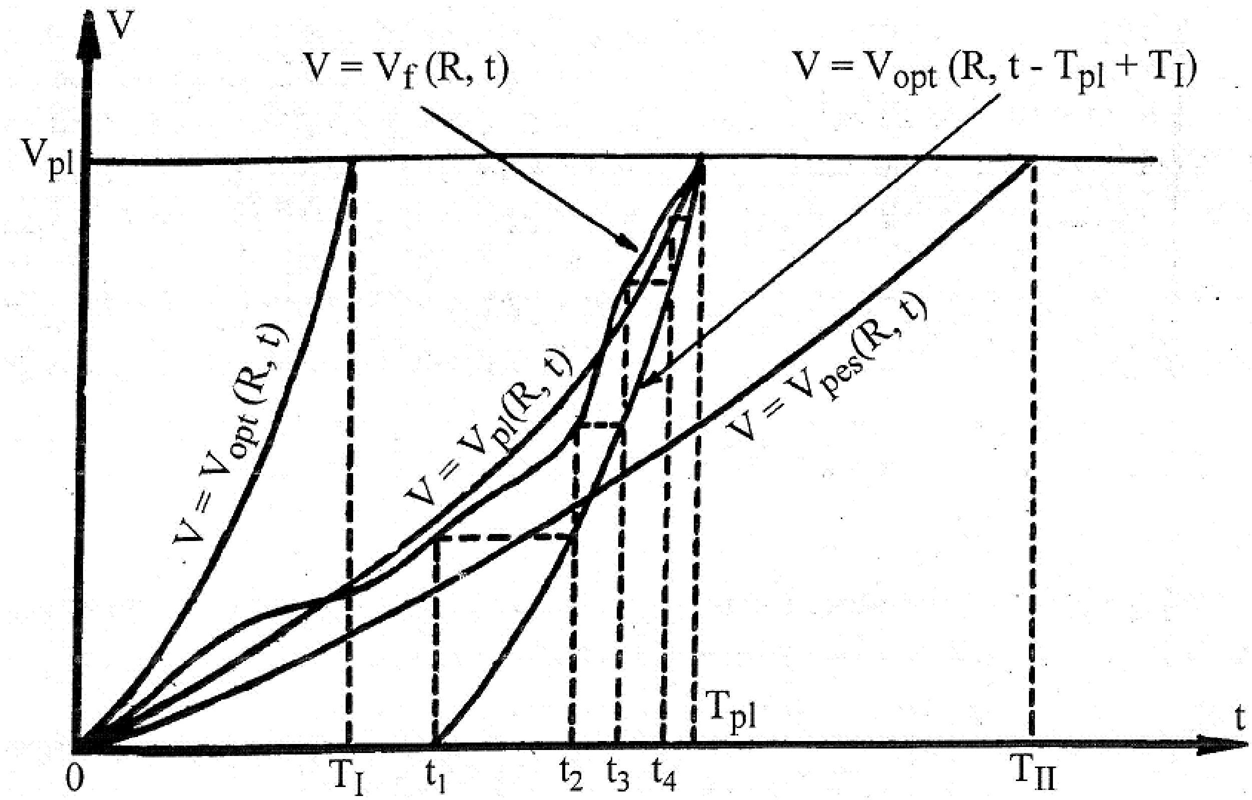

Figure 4.1. *Obtaining inspection moments by Strategy I*

The next inspection moments are determined analogously. If for value $V_f(R,t_i)$, $t_i \geq t_1$, relation

$$V_f(R,t_i) > V_{opt}(R, t_i - T_{pl} + T_I) \tag{4.2.2}$$

holds, the next $(i+1)$-th inspection moment t_{i+1} is determined by solving equation

$$V_f(R,t_i) = V_{opt}(R, t_{i+1} - T_{pl} + T_I). \tag{4.2.3}$$

The method described may have several modifications. In particular, the moment of the first inspection of the system can be determined on the basis of Strategy II (see Fig. 4.2) and employing relation

$$V_{pes}(R,t_1) = V_{opt}(R, t_1 - T_{pl} + T_I) \tag{4.2.4}$$

In a general case, we can allow the not quite obvious assumption that the speed of the system's advancement to the goal can under no circumstances be less than the pessimistic one. In the same way, following inspection moments t_i, $i > 1$, are devised, as shown on Fig. 4.2, by using relation

$$V_{pes}(R,t_{i+1})+V_f(R,t_i)-V_{pes}(R,t_i)=V_{opt}(R,t_{i+1}-T_{pl}+T_I) \quad (4.2.5)$$

Among the methods described above for determining inspection moments, there are, thus, two methodological approaches. By the first, under the worst of circumstances, the system will not increase the volume of the production program already achieved; i.e., it advances to its goal at speed $\frac{dV}{dt}=0$. However, when adopting the second approach, the system in a similar situation does continue advancing to its goal, by maintaining the pessimistic trajectory $V_{pes}(R,t)$ at that; the system functions at the minimal rate.

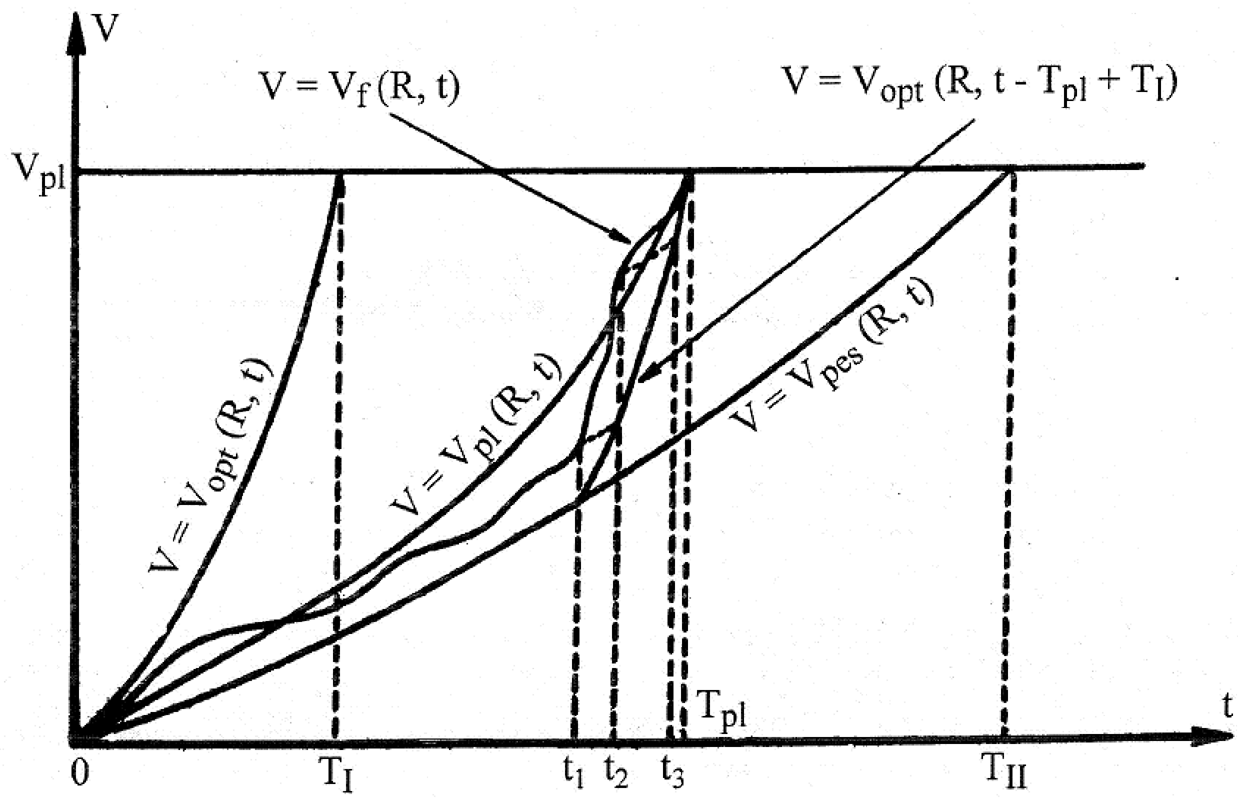

Figure 4.2. <u>Obtaining inspection moments by Strategy II</u>

Between those two extreme approaches there may be intermediate assumptions, namely, the following method can be suggested for determining moments t_i, $i=1,2,...$. We will denote terms t_i^* and t_i^{**}, $i=1,2,...$, respectively, for moments to inspect the system, obtained by applying relations (4.2.2-4.2.3) or (4.2.4-4.2.5), respectively. Note that usually inequality $t_i^{**}>t_i^*$ holds, and the number of inspection moments by using the first of the methods exceeds the number when applying relations (4.2.4-4.2.5), as will be shown below.

Under these conditions, the values of the inspection moments would be estimated by applying relation

$$t_i = \alpha t_i^* + (1-\alpha) t_i^{**}, \quad (4.2.6)$$

where α is either the value of a random variable equidistributed in interval $[0,1]$, or established by experts as the specific weight of one of the strategies.

Along with that, a fundamentally different approach can be taken, based on the fact that at each routine inspection, Strategy I is chosen with probability α, or Strategy II with probability $(1-\alpha)$. Note that, in principle, we can employ elements of adaptation and self-instruction in the process of performing such a method, with α being calculated as a function of the ordinal number of inspection i, $\alpha = \alpha_i$, where $0 < \alpha_i < 1$.

The strategy for determining inspection moments for the system is no more, no less than a result of applying the principle of randomization to the procedure of supervising the course of production, like randomizing preference rules in the scheduling theory [68]. A sort of simulation of the heuristic action of an experienced dispatcher checking the work of the system is effected, with the right of interference in the course of the production process.

To our opinion, it is also promising to synthesize methods of forecasting and modeling at varying intensities which consists in the following:

- The evaluation of the routine inspection moment can be determined as follows:

$$t_{i+1} = \sum_{k=1}^{4} \beta_k \cdot t_{i+1,k}, \quad (4.2.7)$$

where $t_{i+1,1}$ is the root of equation (4.2.3);

$t_{i+1,2}$ is the root of equation (4.2.5);

$t_{i+1,3}$ is the root of equation

$$V_{opt}(R, t_{i+1,3} - T_{pl} + T_I) = F(R, t_{i+1,3}), \quad (4.2.8)$$

where $F(R,t)$ stands for the extrapolation polynomial constructed on the basis of dynamic series $F(R,t_1),...,F(R,t_i)$ in points $t_1,...,t_i$; and

$t_{i+1,4}$ is the root of equation

$$V_{opt}(R, t_{i+1,4} - T_{pl} + T_I) = V_f(R, t_i) + (t_{i+1,4} - t_i) \cdot \frac{1}{T_{pl}} V_{pl}(R, T_{pl}). \quad (4.2.9)$$

Equation (4.2.9) is used on the assumption that during period $\left[t_i, T_{pl}\right]$, system S will function at planned intensity.

As for weight coefficients β_k, $k = 1,...,4$, they are defined either experimentally or by normalization of values γ_k of a random variable, uniformly distributed in the interval $[0,1]$.

The main deficiency of the methods described above is that they do not use statistical information accumulated in the process of modeling the system's work.

We have already underscored that the actual trajectory of the system's movement to the goal $V_f(R,t)$ is random in nature. Depending on the actual form and shape of the trajectory, one and the same volume V_{pl} of the production program can therefore be produced during some interval of actual time $\left[t_0, T_f\right]$. It can be well-recognized that three different cases may be distinguished here:

1. Case $T_{pl} < T_f$ arises when:
   i) the system is uncontrollable; obviously, for such a system there is no sense in the inspection procedure, and the corresponding degenerate situation will no longer concern us; or
   ii) when even if the system is controlled, condition $T_f \le T_{pl}$ is not assured. In this case, at some step of the inspection process we will surely land on the shifted trajectory $V^*(R,t)$. Since the probability of the system's strict movement along trajectory $V^*(R,t)$ for any length is small, the task of the control unit in this situation boils down to ensuring the least delay in fully completing the volume of jobs (production program) planned by deadline T_{pl}.
2. When complying with inequality $T_I < T_f < T_{pl}$, a final number of steps is obviously required for inspecting the system, to verify its advance towards the goal.
3. Lastly, when $T_f = T_{pl}$, the sequence of all inspection points has a convergence limit T_{pl}, and the approach process takes place, strictly speaking, in an infinite number of steps. However, since practice ordinarily requires accomplishing value V_{pl} by the deadline T_{pl} at only a preset accuracy $T_{pl} \pm \Delta T_{pl}$, the task in this case boils down to falling into region $\left[T_{pl} - \Delta T_{pl}, T_{pl} + \Delta T_{pl}\right]$, which will be reached, unlike convergence to T_{pl}, in a finite number of steps of inspecting the system.

However, we must bear in mind the circumstance to be demonstrated later, that when performing local control actions at inspection moments t_i, the system's speed towards its goal can be subject to alternations. Then the situation can be illustrated by Fig. 4.3 for quite a widespread case, when trajectories $V_{opt}(R,t)$ and $V_{pes}(R,t)$ are given as straight lines.

Assume that trajectories $V_{opt}(R,t)$ and $V_{pes}(R,t)$ are as follows:

$$\begin{cases} V_{opt}(R,t) = tg\,\nu \cdot t \\ tg\,\nu = V_{pl}/T_I \end{cases}, \tag{4.2.10}$$

$$\begin{cases} V_{pes}(R,t) = tg\varphi \cdot t \\ tg\varphi = V_{pl} / T_{II} \end{cases}, \quad (4.2.11)$$

Denote as the average speed:

$$tg\alpha_i = \frac{V_{pl}(R,T_{pl}) - V_f(R,t_i)}{T_{pl} - t_i}, \quad (4.2.12)$$

at which it is necessary to move, beginning at moment t_i in order to complete the production program by moment T_{pl}.

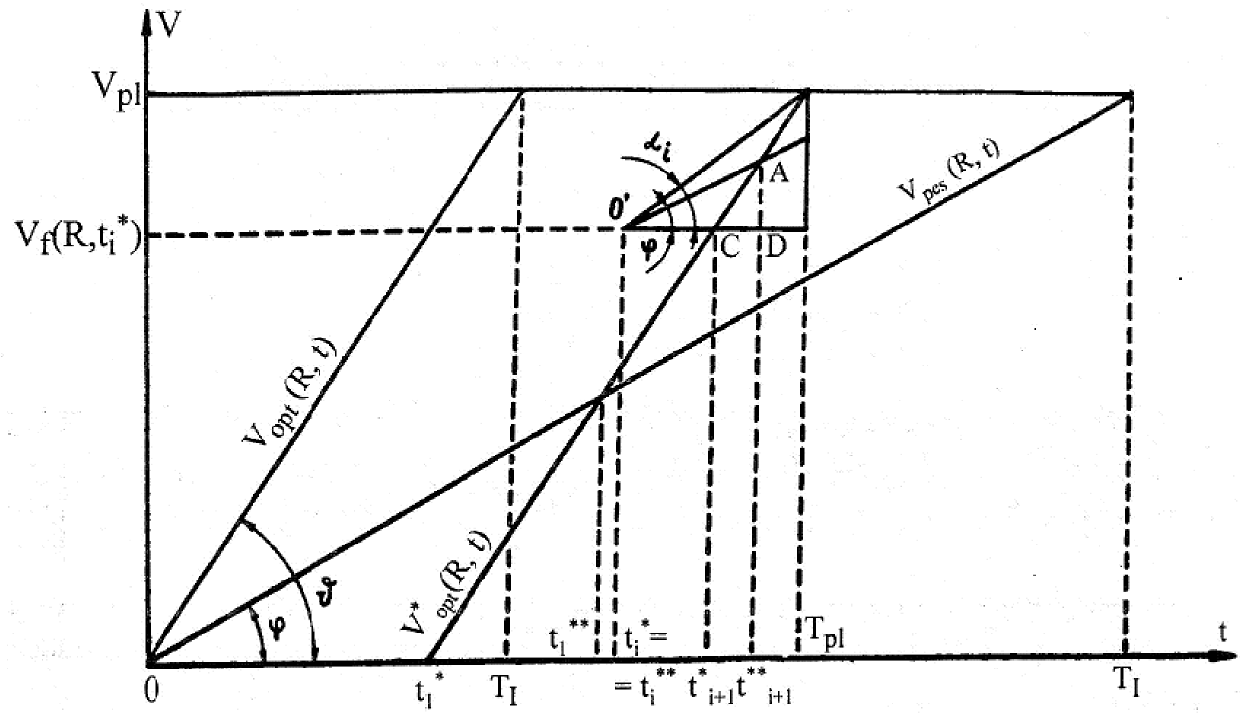

Figure 4.3. *A modified algorithm to determine inspection moments*

Employing relations (4.2.2-4.2.3, 4.2.10, 4.2.12), we obtain for the case of a straight line relation

$$t_{i+1}^{*} = T_{pl} - \left(T_{pl} - t_i^{*}\right)\frac{tg\alpha_i}{tg\nu}. \quad (4.2.13)$$

By applying recurrent relation (4.2.13) $(i-1)$ times, we can obtain the following relation for the limit inspection moment

$$t_{i+1}^{*} = T_{pl} - \left(T_{pl} - t_1^{*}\right)\frac{\prod_{k=1}^{i} tg\,\alpha_k}{(tg\,\nu)^i}. \tag{4.2.14}$$

It can be well-recognized that implementation of (4.2.14) depends substantially on the earliest time for reaching the goal $T_I = T_{pl} - t_1^{*}$, and on the corresponding maximum average speed of advancement towards the goal $tg\,\nu = v_{opt} = \frac{V_{pl}}{T_I}$.

§4.3 Determining the limit inspection moments

Let us determine the limit inspection moments using relation (4.2.11). Examination of triangle CAD on Fig. 4.3 reveals that:

$$\Delta V\left(t_{i+1}^{**}\right) = \left(t_{i+1}^{**} - t_{i+1}^{*}\right)\cdot tg\,\nu, \tag{4.3.1}$$

while examination of triangle $O^{'}AD$ leads to

$$\Delta V\left(t_{i+1}^{**}\right) = \left(t_{i+1}^{*} - t_{i}^{*}\right)\cdot tg\,\varphi. \tag{4.3.2}$$

By equating the right parts of these relations and making simple transformations, we obtain

$$t_{i+1}^{**} = \frac{t_{i+1}^{*}\cdot\frac{tg\,\nu}{tg\,\varphi} - t_i^{*}}{\frac{tg\,\nu}{tg\,\varphi} - 1}, \tag{4.3.3}$$

where t_{i+1}^{*} is the limit value of the $(i+1)$-th inspection moment for the case of zero value of the lower boundary of speed of carrying out the production program (Strategy I); t_i^{*} stands for the moment of the i-th inspection obtained under the same assumptions as t_{i+1}^{*}. Symbol t_{i+1}^{**} signifies the limit value of the $(i+1)$-th inspection moment for the case of the lower boundary v_{pes} of the system's speed in moving towards the goal (Strategy II), and values $tg\,\nu$ and $tg\,\varphi$ are determined by using relations (4.2.10-4.2.11).

Transforming (4.3.3) with consideration of equality , we obtain the following estimate for the limit moment of inspecting the system:

$$t_{i+1}^{**} = \frac{tg\,\nu - tg\,\alpha_i}{tg\,\nu - tg\,\varphi}\cdot T_{pl} + \frac{tg\,\alpha_i - tg\,\varphi}{tg\,\nu - tg\,\varphi}\cdot t_i^{**}. \tag{4.3.4}$$

§4.4 Determining the number of inspection moments

Consider the number of inspections of system S to be obtained for the case of relations (4.2.10) and $V_{pl}(R,t)=\frac{V_{pl}}{T_{pl}}\cdot t$ with pregiven values T_{pl} and $V_{pl}(R,T_{pl})=V_{pl}$. Assume that the actual course of the system's movement towards the goal coincides with the planned one, and in the process of optimizing the query frequency we apply Strategy I. It can be well-recognized [71] that under these conditions, the relation for determining the $(i+1)$-th limit inspection moment would become as follows:

$$t_{i+1}^* = t_1^* + \frac{V_f(R,t_i^*)}{V_{pl}}\cdot(T_{pl}-t_1^*), \tag{4.4.1}$$

where $V_f(R,t_i^*)=V_{pl}(R,t_i^*)$ represents the actual state of system S at moment of time t_i^*, t_1^* being the first limit inspection point. Taking into account the obvious relation

$$\frac{V_f(R,t_i^*)}{V_{pl}} = \frac{t_i^*}{T_{pl}}, \tag{4.4.2}$$

we can transform (4.4.1) into

$$t_{i+1}^* = t_1^* + \left(1-\frac{t_1^*}{T_{pl}}\right)\cdot t_i^*. \tag{4.4.3}$$

With the last expression, we can easily obtain the relation for the value of the $(i+1)$-th inspection moment:

$$t_{i+1}^* - t_i^* = \left(1-\frac{t_1^*}{T_{pl}}\right)\cdot(t_i^*-t_{i-1}^*). \tag{4.4.4}$$

The latter relation, obviously, may be also written as

$$t_{i+1}^* - t_i^* = \left[1-\frac{t_1^*}{T_{pl}}\right]^i\cdot t_1^*. \tag{4.4.5}$$

Further, in view of equality

$$\sum_{i=0}^{\infty}(t_{i+1}^*-t_i^*) = T_{pl}$$

transform (4.4.5) into

$$T_{pl} = \sum_{i=0}^{\infty} \left[1 - \frac{t_1^*}{T_{pl}}\right]^i \cdot t_1^* . \qquad (4.4.6)$$

Since $t_1^* < T_{pl}$, it is simple to see that the right part of (4.4.6) is a convergent series. Now representing the regarded equality as

$$T_{pl} = t_1^* \cdot \sum_{i=0}^{n} \left[1 - \frac{t_1^*}{T_{pl}}\right]^i + t_1^* \cdot \sum_{i=n+1}^{\infty} \left[1 - \frac{t_1^*}{T_{pl}}\right]^i , \qquad (4.4.7)$$

we can see that value

$$Q_n = t_1^* \cdot \sum_{i=n+1}^{\infty} \left[1 - \frac{t_1^*}{T_{pl}}\right]^i \qquad (4.4.8)$$

represents the remainder of the convergent series examined. Note that the accuracy error in fulfilling production program ΔT_{pl} is the parameter restricting the value of remainder Q_n, and $Q_n \le \Delta T_{pl}$. Meanwhile, the method for choosing value ΔT_{pl}, stemming from certain general conditions of the system's functioning, will be examined below.

Given a certain upper boundary Q_n equal to ΔT_{pl}, we can calculate the corresponding number of inspections n. Rewrite (4.4.7) as

$$T_{pl} = t_1^* + t_1^* \cdot \sum_{i=0}^{n} \left[1 - \frac{t_1^*}{T_{pl}}\right]^i + \Delta T_{pl} , \qquad (4.4.9)$$

and note that the member under the sign of the sum in the right part of the equality represents in fact the sum of a decreasing geometric progression. The latter can therefore be rewritten as

$$t_1^* \cdot \sum_{i=0}^{n} \left[1 - \frac{t_1^*}{T_{pl}}\right]^i = T_{pl} \cdot \left\{ \left[1 - \frac{t_1^*}{T_{pl}}\right] - \left[1 - \frac{t_1^*}{T_{pl}}\right]^{n+1} \right\}.$$

Substituting the last expression into (4.4.9) and implementing simple transformations, we obtain relation

$$\frac{\Delta T_{pl}}{T_{pl}} = \left[1 - \frac{t_1^*}{T_{pl}}\right]^{n+1} , \qquad (4.4.10)$$

from which we easily derive

$$n = \frac{\ln \frac{\Delta T_{pl}}{T_{pl}}}{\ln\left[1 - \frac{t_1^*}{T_{pl}}\right]} - 1. \qquad (4.4.11)$$

Taking into account the fact that $T_{pl} - t_1^* = T_I$ is the earliest moment for the system to reach the goal, the final expression for determining the number n of inspections required for checking advance of the system with parameters T_{pl}, T_I, and ΔT_{pl} towards its goal, boils down to

$$n = \frac{\ln \frac{\Delta T_{pl}}{T_I}}{\ln \frac{T_I}{T_{pl}}}. \qquad (4.4.12)$$

To develop the above estimate, we assumed values T_I, T_{II}, T_{pl}, and ΔT_{pl} being invariable and deterministic. However, when actual systems function, these values may be affected by a large number of random influences. They can be corrected and usually undergo alternation at inspection moments. Moreover, due to the implementation of parametric control actions, values T_I, T_{II}, T_{pl}, and ΔT_{pl}, and in some cases V_{pl} too, can change, which, in turn, results in correcting the corresponding inspection moment t_i for system S.

§4.5 Controlling multilevel organization systems by means of periodical inspections

4.5.1 Notation

Let us present the following terms [93]:

- S - hierarchical organization system;
- T_{pl} - final due date (planning horizon);
- n - the number of control points within the planning horizon for controlling a certain element entering hierarchical system S;
- V_{pl} - production plan (target amount) of the element;
- $\bar{v}_c(t)$ - the average speed to reach the element's target V_{pl} on time, $1 \le c \le d$; all speeds are sorted in ascending order;
- d - the number of possible speeds;
- $v_{pl}(t)$ - the planned speed function which on the average assures that the target V_{pl} will be reached by the due date T_{pl};
- T_I - the minimal average completion time of the goal, V_{pl}, if only the highest speed,

$v_{opt}(t)$, is actually used throughout;

$v_b^*(t)$ - the lower boundary value for speed $v_b(t)$, $1 \le b \le c$;

T_{pl}^* - the average completion time of the goal if only speed $v_{pl}(t)$ *with its minimal boundary rate, i.e., speed* $v_{pl}^*(t)$, is actually used throughout the planning horizon;

ΔT_{pl} - predetermined value of the closeness to the due date (to force convergence).

4.5.2 The strategies

Two different strategies - Strategy I and Strategy II, which have been introduced in the above §4.2, are used in the model.

According to Strategy I, $v_b^*(t) = 0$ for all speeds $v_b(t)$, i.e., the lower boundary rate equals zero for all speeds. Strategy II is based on the alternative assumption that even in the case of most unfavorable circumstances value $v_{pl}^*(t) > 0$ for any $t \in \left[0, T_{pl}\right]$. The following assumptions are also implemented in the model:

A. All speeds $v_c(t)$ are independent of time.

B. The time of inspecting the element is negligibly small.

The problem is to determine the number of inspection points within the planning horizon on condition that the process is on target, i.e., the system's element advances with the planned speed. As outlined above and in [93], value n satisfies

- for the case of implementing Strategy I:

$$n = \frac{\ln\left(\Delta T_{pl} / T_I\right)}{\ln\left(T_I / T_{pl}\right)}; \tag{4.5.1}$$

- for the case of implementing Strategy II:

$$n = \frac{\ln\left\{\dfrac{T_I \cdot \left(T_{pl}^* - T_{pl}\right)}{\Delta T_{pl} \cdot \left(T_{pl}^* - T_I\right)}\right\}}{\ln\left\{\dfrac{T_{pl} \cdot \left(T_{pl}^* - T_I\right)}{T_I \cdot \left(T_{pl}^* - T_{pl}\right)}\right\}}. \tag{4.5.2}$$

It can be well-recognized that implementing Strategy II results in decreasing the number of inspection points n, i.e., value n calculated by (4.5.2) would always prove to be less than that of (4.5.1).

4.5.3 Case of hierarchical organization systems

As an example of an organization system consider a large-size network project with a source node A and a sink node B, correspondingly. The project itself may be an element entering a large hierarchical project management (PM) system. Denote henceforth the system's top element as an element of zero ranking; decreasing the element's ranking results in increasing its rank number, i.e., an element of k-th rank subordinates to an element of $(k-1)$-th rank.

Assume that each activity entering the network project obtains the rank with number k while the enlarged network system regarded as an enlarged activity (A,B) obtains the rank with number $(k-1)$. From the other side, each elementary activity entering the project can be regarded as a subsystem of the k-th ranking belonging to the large PM organization system. The meaning of the term "subsystem" is as follows: there exists a subordinated controlled subsystem which is governed by a certain algorithm. The latter optimizes the objective of the subsystem which, in turn, is an element of the large system's objective.

Assume that the network project's critical path comprises m activities. Choose from those activities the one with the minimal number of inspection points (i.e., with the minimal number of preventive inspections) determined by (4.5.1) or (4.5.2). Without losing generality let us assume relation (4.5.1). Let the chosen activity be of number i. Thus, relation

$$n_i = \min_{1\le r\le m} n_r = \min_{1\le r\le m} \frac{\ln \dfrac{\Delta T_{pl}^{(r)}}{T_I^{(r)}}}{\ln \dfrac{T_I^{(r)}}{T_{pl}^{(r)}}} \tag{4.5.3}$$

holds, with evident relations

$$\Delta T_{pl}^{(r)} < T_I^{(r)} < T_{pl}^{(r)} . \tag{4.5.4}$$

If each r-th activity, $1 \le r \le m$, belonging to the critical path, is inspected n_i times, then the minimal number of inspections for the critical path is as follows

$$m \cdot n_i = m \cdot \min_{1\le r\le m} n_r , \tag{4.5.5}$$

where n_i denotes the minimal number of control points necessary to inspect the critical path with the assumed level of detalization $\Delta T_{pl}^{(i)}$ to be predetermined for the element of rank k.

Consider the network project as one activity (A,B) with certain parameters T_{pl}, T_I and ΔT_{pl}. Let those parameters depend on parameters of the i-th activity as follows

$$\begin{cases} T_{pl} = j_1 \cdot T_{pl}^{(i)} \\ T_I = j_1 \cdot T_I^{(i)} \\ \Delta T_{pl} = j_1 \cdot \Delta T_{pl}^{(i)} \end{cases}, \tag{4.5.6}$$

where j_1, j_2, j_3 are arbitrary positive values larger than 1. Assume that relation

$$j_1 \geq j_2 \geq j_3 \tag{4.5.7}$$

holds. Introduce a reasonable (from the point of a hierarchical structure) assumption as follows: diminishing the number of the rank results in reducing the number of inspection points. Assume that

$$n^* < n_i, \tag{4.5.8}$$

where n^* represents the number of inspections required to control the enlarged activity (A,B), taking into account its ranking with number $(k-1)$ and using relation (4.5.6). To satisfy (4.5.8), together with evident relations

$$j_1 \cdot T_{pl}^{(i)} \geq j_2 \cdot T_I^{(i)} \geq j_3 \cdot \Delta T_{pl}^{(i)}, \tag{4.5.9}$$

one has to implement certain constraints for values j_1, j_2 and j_3.

Implement evident transformations

$$\begin{aligned} n^* &= \frac{\ln \dfrac{j_3 \cdot \Delta T_{pl}^{(i)}}{j_2 \cdot T_I^{(i)}}}{\ln \dfrac{j_2 \cdot T_I^{(i)}}{j_1 \cdot T_{pl}^{(i)}}} = \frac{\ln \dfrac{j_3}{j_2} + \ln \dfrac{\Delta T_{pl}^{(i)}}{T_I^{(i)}}}{\ln \dfrac{j_2}{j_1} + \ln \dfrac{T_I^{(i)}}{T_{pl}^{(i)}}} \leq \frac{\ln \dfrac{j_3}{j_2}}{\ln \dfrac{j_2}{j_1}} + \frac{\ln \dfrac{\Delta T_{pl}^{(i)}}{T_I^{(i)}}}{\ln \dfrac{T_I^{(i)}}{T_{pl}^{(i)}}} = \frac{\ln \dfrac{j_3}{j_2}}{\ln \dfrac{j_2}{j_1}} + \min_{1 \leq r \leq m} n_r \leq \\ &\leq \frac{\ln \dfrac{j_3}{j_2}}{\ln \dfrac{j_2}{j_1}} + m \cdot \min_{1 \leq r \leq m} n_r = \frac{\ln \dfrac{j_3}{j_2}}{\ln \dfrac{j_2}{j_1}} + (m-1) \cdot \min_{1 \leq r \leq m} n_r + \min_{1 \leq r \leq m} n_r . \end{aligned} \tag{4.5.10}$$

From (4.5.10) it can be well-recognized that, in order to satisfy $n^* \leq \min\limits_{1 \leq r \leq m} n_r$, it is sufficient to satisfy

$$\frac{\ln\frac{j_3}{j_2}}{\ln\frac{j_2}{j_1}}+(m-1)\cdot\min_{1\le r\le m} n_r \le 0\,. \tag{4.5.11}$$

The latter inequality may be rewritten as follows:

$$\ln\frac{j_3}{j_2} \ge (m-1)\cdot\min_{1\le r\le m} n_r \,\ln\frac{j_1}{j_2}\,. \tag{4.5.12}$$

Taking into account

$$\min_{1\le r\le m} n_r = \frac{\ln\left\{\frac{\Delta T_{pl}^{(i)}}{T_I^{(i)}}\right\}}{\ln\left\{\frac{T_I^{(i)}}{T_{pl}^{(i)}}\right\}} \tag{4.5.13}$$

and carrying our evident transformations, we obtain *the first sufficient condition* as follows:

$$\frac{j_3}{j_2} \ge \left(\frac{\Delta T_{pl}^{(i)}}{T_I^{(i)}}\right)^{m-1}. \tag{4.5.14}$$

From the other side, taking into account the previously imposed constraint $T_I \ge \Delta T_{pl}$, i.e., $j_2 \cdot T_I^{(i)} \ge j_3 \cdot \Delta T_{pl}^{(i)}$, we obtain $\frac{j_3}{j_2} \ge \frac{T_I^{(i)}}{\Delta T_{pl}^{(i)}}$.

Thus, we obtain

$$\left(\frac{\Delta T_{pl}^{(i)}}{T_I^{(i)}}\right)^{m-1} \le \frac{j_3}{j_2} \le \frac{T_I^{(i)}}{\Delta T_{pl}^{(i)}}\,. \tag{4.5.15}$$

Using (4.5.11) and undertaking similar transformations, we obtain another sufficient condition

$$\frac{T_I^{(i)}}{T_{pl}^{(i)}} \le \frac{j_2}{j_1} \le \left(\frac{T_{pl}^{(i)}}{T_I^{(i)}}\right)^{\frac{1}{m-1}}. \tag{4.5.16}$$

It can be demonstrated [93] that the developed sufficient conditions are at the same time the necessary ones. Thus, we finally obtain that in order to satisfy $n^* \le \min_{1\le r\le m} n_r$, the following both sufficient and necessary conditions regarding values j_1, j_2 and j_3, $1 \le r \le m$, have to be honored:

$$\begin{cases} \left(\dfrac{\Delta T_{pl}^{(i)}}{T_I^{(i)}}\right)^{m-1} \le \dfrac{j_3}{j_2} \le \dfrac{T_I^{(i)}}{\Delta T_{pl}^{(i)}} \\ \dfrac{T_I^{(i)}}{T_{pl}^{(i)}} \le \dfrac{j_2}{j_1} \le \left(\dfrac{T_{pl}^{(i)}}{T_I^{(i)}}\right)^{\frac{1}{m-1}} \quad . \end{cases} \tag{4.5.17}$$

Note that values j_1, j_2 and j_3 are essential parameters for the element with ranking $(k-1)$ entering the large PM system.

Those relations are essential for the hierarchical control system and can be used for undertaking synthesis of optimization in multilevel organization systems.

§4.6 On-line models for determining control points for the case of random disturbances

4.6.1 Introduction

We have outlined above, in §§4.1-4.4, some analytical estimates for on-line inspection points within the planning horizon when controlling production systems with due date T_{pl} and target amount V_{pl}. Three production speeds (rates) are considered - the optimistic speed, the planned speed and the pessimistic speed. When using an optimistic speed the target can be reached at moment T_I while applying the pessimistic speed increases the time duration up to T_{II}, $T_{II} > T_I$. Two basic strategies are introduced for determining inspection moments:

Strategy I is based on the concept that in the worst case, due to certain breakdowns in the system, within certain subintervals the system's output does not increase.

Strategy II is used for cases when an assumption can be drawn as follows: the system always increases its output with the minimal rate equal to the pessimistic speed.

As has been pointed out before, organization systems' parameters are affected by various kinds of random influences, circumstances, and interferences, whose appearance causes a need to provide probability intervals for changing possible values, both of parameters T_I and T_{II} - intervals $[T_I', T_I'']$ and $[T_{II}', T_{II}'']$, respectively. Here two fundamentally different cases arise:

1. The lengths of conditional intervals $[T_I', T_I'']$ and $[T_{II}', T_{II}'']$ are close to the time unit for undertaking on-line control. In this case, we will assume that the estimate is given precisely, i.e., optimistic and pessimistic trajectories $V_{opt}(R,t)$ and $V_{pes}(R,t)$ are

deterministic functions. In such a situation, all the deliberations outlined in §4.2 remain in force.

2. The lengths of intervals $[T_I^{\prime}, T_I^{\prime\prime}]$ and $[T_{II}^{\prime}, T_{II}^{\prime\prime}]$ are substantial as compared with that of the time unit interval. In this case, it is obviously necessary to know the distribution functions of random variables T_I and T_{II} in the given interval. At the same time, we can help nothing that the requirement to know the distribution functions of various parameters characterizing the process of carrying out the production program is a too rigid condition, taking into account the high degree of uncertainty. It is therefore natural to postulate the most general distribution, one, for instance, like the beta distribution which has been successfully applied for developing the probability model of activity durations determined by two estimates (see Chapter 2).

We suggest a beta distribution of random values T_I or T_{II} with preset boundary values given in the form of intervals $[T_I^{\prime}, T_I^{\prime\prime}]$ or $[T_{II}^{\prime}, T_{II}^{\prime\prime}]$ with the density distribution

$$p(t) = \frac{12}{(a_t - b_t)^4}(t - a_t)(b_t - t)^2, \qquad (4.6.1)$$

where a_t is the lower boundary estimate of duration T_I (or T_{II}) for carrying out the production program;

b_t is the upper boundary estimate of the duration.

Distribution (4.6.1) corresponds to mathematical expectation

$$E(t) = \frac{2b_t + 3a_t}{5} \qquad (4.6.2)$$

and variance

$$V(t) = 0.04 \cdot (b_t - a_t)^2. \qquad (4.6.3)$$

Such a distribution law reflects the work of an actual system to a large degree []. After each inspection, all the time estimates are corrected on the basis of the information obtained. Postulating distribution of T_I or T_{II} by (4.6.1) we face the need to consider random changes of values $tg\vartheta$ and $tg\varphi$ (see §4.2) and, correspondingly, $V_{opt}(R,t)$ and $V_{pes}(R,t)$, step by step in the course of carrying out an on-line control.

4.6.2 Models for limit inspection moments

In this case, the limit values of inspection moments t_{i+1}^{*} in §4.2, $i \geq 1$, within the frame of applying *Strategy I* will be determined by the intersection point of a straight line passing through a point with coordinates t_i^* and $V_f\left(R,t_i^*\right)$, and parallel to the abscissa axis, with a corrected trajectory $V_{opt_i}\left(R,t\right)=V_i^*\left(R,t\right)$, whose form is determined at the i-th step of the inspection (see Figs. 4.1-4.2 in §§4.1-4.4).

Thus, several relations outlined in §4.2 require transformation, namely:

- relation (4.2.14) in §4.2 for this case takes the form

$$t_{i+1}^* = T_{pl} - \left(T_{pl} - t_i^*\right)\frac{tg\,\alpha_i}{tg\,\vartheta_i}, \qquad (4.6.4)$$

where $tg\,\vartheta_i$ is a certain average optimistic speed of system S's movement to the goal, determined as a result of correcting the limit possibilities of the system after the i-th inspection moment.

- appropriate changes will also take place in (4.2.15), which in this case may be re-written as

$$t_{i+1}^* = T_{pl} - \left(T_{pl} - t_i^*\right)\cdot\prod_{k=1}^{i}\frac{tg\,\alpha_k}{tg\,\vartheta_k}. \qquad (4.6.5)$$

When applying *Strategy II*, the time estimates are also subject to random influences and have to be corrected at the inspection moments. The expression for the limit inspection moment in this case will be (see Fig. 4.3 in §4.2) as follows:

$$t_{i+1}^{**} = \frac{tg\,\vartheta_i - tg\,\alpha_i}{tg\,\vartheta_i - tg\,\varphi_i}T_{pl} + \frac{tg\,\alpha_i - tg\,\varphi_i}{tg\,\vartheta_i - tg\,\varphi_i}t_i^{**}, \qquad (4.6.6)$$

where $tg\varphi_i$ is an average pessimistic speed characterizing the revised minimal possibilities of the system.

Let us denote

$$\frac{tg\,\vartheta_i - tg\,\alpha_i}{tg\,\vartheta_i - tg\,\varphi_i} = A_i; \quad \frac{tg\,\alpha_i - tg\,\varphi_i}{tg\,\vartheta_i - tg\,\varphi_i} = B_i, \qquad (4.6.7)$$

and present (4.6.6) as follows:

$$t_{i+1}^{**} = A_i\,T_{pl} + B_i\,t_i^{**}. \qquad (4.6.8)$$

Note the obvious equality $A_i + B_i = 1$.

Applying (4.6.4) and (4.6.6), we obtain the expression for the value of the inspection step for both cases

$$t_{i+1}^{*} - t_i^{*} = \left(T_{pl} - t_i^{*}\right)\left(1 - \frac{tg\,\alpha_i}{tg\,\vartheta_i}\right), \tag{4.6.9}$$

$$t_{i+1}^{**} - t_i^{**} = \left(T_{pl} - t_i^{**}\right)\frac{tg\,\vartheta_i - tg\,\alpha_i}{tg\,\vartheta_i - tg\,\varphi_i}. \tag{4.6.10}$$

Comparing (4.6.9) and (4.6.10), and equating t_i^{*} to t_i^{**}, we can show the validity of inequality

$$t_{i+1}^{**} - t_i^{**} > t_{i+1}^{*} - t_i^{*} \tag{4.6.11}$$

uniformly for all i's. Indeed, from inequality

$$\frac{tg\,\vartheta_i - tg\,\alpha_i}{tg\,\vartheta_i - tg\,\varphi_i} > \frac{tg\,\vartheta_i - tg\,\alpha_i}{tg\,\vartheta_i} \tag{4.6.12}$$

follows relation

$$A_i > 1 - \frac{tg\,\alpha_i}{tg\,\vartheta_i}, \tag{4.6.13}$$

which obviously proves our assertion.

4.6.3 Comparison of strategies for determining inspection points

The outlined above assertion stipulates that it is preferable to determine the inspection step according to *Strategy II*, rather than *Strategy I*, since a change of the strategy practically does not change the probability of the system's reaching the goal V_{pl} by moment T_{pl}. At the same time, as can easily be seen by (4.6.11), using *Strategy II* reduces the inspection frequency. This, in turn, results in reducing control expenses without decreasing the efficiency of the control itself [106].

Since values T_I and T_{II} fluctuate randomly, it is necessary to introduce appropriate correctives in the formula for determining the limit inspection moment.

There is the obvious danger that when determining a new optimistic speed, trajectory $V_i^{*}(R,t)$ will pass to the left of point $V_f\left(R,t_i^{*}\right)$. This testifies to a disruption of the plan time limit for performing the entire production program.

Indeed, by shifting trajectory $V^{*}(R,t)$ in parallel to itself by a certain value Δt so that it comes somewhat to the right of point $V_f\left(R,t_i^{*}\right)$, we find that the term for completing the production program of system S shifts by ΔT_I, in spite of the fact that the system's limit possibilities are employed. In order to forestall this, the inspection point must be shifted to the left.

Taking into account that parameter T_I is characterized by a density distribution function (4.6.1) with a finite variance, we can apply the Chebyshev inequality

$$P\left\{\left| T_I - E\left(T_I\right)\right| \le \Delta T_I\right\} \ge \frac{V\left(T_I\right)}{\left(\Delta T_I\right)^2}. \qquad (4.6.14)$$

Consequently, the value of the shift of limit inspection moment t^*_{i+1} at the $(i+1)$-th step is determined by formula

$$\Delta T_I \ge \sqrt{\frac{V\left(T_I\right)}{P\left\{\left| T_I - E\left(T_I\right)\right| \le \Delta T_I\right\}}}. \qquad (4.6.15)$$

Here, $P_I = P\left\{\left| T_I - E\left(T_I\right)\right| \le \Delta T_I\right\}$ is the probability that the optimistic time estimate taken at a certain $(i+1)$-th step will not differ from the mathematical expectation of the optimistic time estimate expressed by (4.6.1) by more than ΔT_I. Obviously, we must preset the value of probability p_I in order to determine the value of ΔT_I.

In the same way, we take into account the correction with regard to a random fluctuation of the pessimistic estimate; in relation (4.6.15) we need only to substitute T_I for T_{II}.

Consequently, a summary correction for determining the limit inspection moment t^{**}_{i+1} will equal

$$\Delta T = \Delta T_I + \Delta T_{II}. \qquad (4.6.16)$$

Then relation (4.6.8) will take the form

$$t^{**}_{i+1} = A_i T_{pl} + B_i t^{**}_i - \Delta T. \qquad (4.6.17)$$

We should know that a restriction is imposed on value ΔT, determined by the restrictions for the control system. One substantial restriction is the reliability of the control, characterized by the given probability p_{pl} that the target amount V_{pl} will be done by moment T_{pl}. This, in fact, corresponds to setting permissible boundaries for ΔT_{pl} to deviate from T_{pl}. In other words, in view of equality (4.6.16), the following restriction holds:

$$\Delta T_I + \Delta T_{II} \le \Delta T_{pl}. \qquad (4.6.18)$$

If at a given ΔT_{pl} and proceeding from the considerations examined, we should choose p_I and correspondingly determine ΔT_I, we can choose the optimal value of ΔT_{II}

by (4.6.18). The limit value can be determined in this case with the help of equality $\Delta T_{II} = \Delta T_{pl} - \Delta T_{I}$.

Having thus determined the value of ΔT_{II}, and knowing the value of $V(T_{II})$, we can determine probability

$$p_{II} = P\{|T_{II} - E(T_{II})| \leq \Delta T_{II}\} \geq \frac{V(T_{II})}{(\Delta T_{II})^2} \tag{4.6.19}$$

from the Chebyshev inequality.

Knowing probability p_{II}, we can obtain the corresponding quantile $W_{p_{II}}$. It determines a certain average speed that can be considered as the lower limit value of speed in order to obtain the subsequent inspection moments on the basis of the previous ones.

Chapter 5. Trade-off in Organization Systems to Determine Quality Estimates

§5.1 Introduction

In recent years the concept of system approach to large-scale organization systems (OS) has been developed and outlined in several publications (see, e.g., [4, 9, 26, 28, 38, 45, 109, 140]). In the system approach, an OS is viewed as an operation system, i.e., a collection of people, resources, and information that is intended to perform a specific function, or reach a predetermined objective [140]. Within the last three decades various analytical and simulation tools have been suggested to be used in the design, planning, organizing, and controlling those systems. Many of them, e.g., project planning and management systems, function under random disturbances.

Most developed techniques enable judging the system by three important criteria: cost, timeliness and quality. The latter criterion is usually applied for products and services, which have to be of a certain quality level and yet not overpriced. There is, thus, always a trade-off between quality and cost. An overwhelming number of publications deal with quality control which involves control of design products and services, control of incoming materials, control of work in process, and the final inspection and testing of completed products and services. It can be well-recognized that to implement quality concepts one needs to use utility concepts. Thus, in recent years, the utility theory has been developed.

Every operation for showing the actual fulfillment of a program and controlling delivery time for each type of products calls for taking stock of the finished product both dispatched and in storage, keeping count of all process stock, and the state of the means of production. This is an expensive operation, often calling for suspension of the production process. It is therefore desirable that this be done as rarely as possible, but without missing the moment when the tendency to deviate develops into jeopardizing the output of finished products.

Note that the use and importance of utility theory in various branches of operation management, e.g., in project management [168], has been outlined in recent years in various publications [38, 125-127, 161]. The developed utility theory techniques can be classified as follows:

- single-goal utility techniques;
- multi-criteria utility models and methods.

5.1.1 Single-goal utility models

The techniques outlined below refer mostly to decision-making in order to choose between a host of activities. These are referred to as actions (or strategies), and each results usually in a pay-off or outcome. Should decision-makers know the pay-off associated with each action, they would be able to choose the action with the largest pay-off. Most situations, however, are characterized by incomplete information, so for a given action, it is necessary to enumerate all probable outcomes together with their consequences and probabilities. The degree of information and understanding that the decision maker has about a particular situation, determines mostly how the underlying problem can be approached and resolved.

Two persons, faced with the same set of alternatives and conditions, are likely to arrive at very different decisions regarding the most appropriate course of action to be undertaken. What is optimal for the first person may not even be an attractive alternative for the second one. Judgment, risk, and experience work together to influence attitudes and choice preferences.

Implicit in any decision-making process is the need to construct, either formally or informally, a preference order so that alternatives can be ranked and the final choice made. Thus, a profit-maximization rule has to be determined. Note that in more complex situations where factors other than profit maximization or cost minimization apply, it may be desirable to explore the decision maker's preference structure in an explicit fashion, and to attempt to construct a preference ordering directly. Important classes of techniques that work by eliciting preference information from the decision maker are predicated on what is known as utility theory. This, in turn, is based on the premise that the preference structure can be represented by a real-valued function called a utility function. Once such a function is constructed, selection of the final alternative should be relatively simple. In the absence of uncertainty, an alternative with the highest utility would represent the preferred solution. For the case where outcomes are subject to uncertainty, the appropriate choice would correspond to that which attains the highest expected utility. Thus, the decision maker is faced with two basic problems involving judgment:

1. How to quantify (or measure) utility for various pay-offs.
2. How to quantify judgments concerning the probability of the occurrence of each possible outcome or event.

Assuming the presence of uncertainty, when a decision maker is repeatedly faced with the same problem, experience often leads to a strategy that provides, on average, the best results over the long run. In technical terms, such a strategy is one that maximizes expected monetary value (EMV). Notationally, let A be a particular action with possible outcomes $j = 1, 2, ..., n$. Also, let p_j be the probability of realizing outcome j with corresponding pay-off or return x_j. The expected monetary value of A is then calculated by using the expected utility maximization model

$$EMV(A) = \sum_{j=1}^{n} p_j x_j . \quad (5.1.1)$$

It can be well-recognized that in order to undertake proper decision-making, one has to implement utility measures for each alternative active under consideration. In [168] a number of reasonable axioms which refer to a single goal in decision-making (Axioms 1 and 2), to multiple goals with acceptable trade-off relations and to multiple goals which are not substitutable, are outlined. The axioms are as follows:

1. Ordering. For two alternatives A_1 and A_2, one of the following must be true: the person either prefers A_1 to A_2, A_2 to A_1, or is indifferent between them.

2. Transitivity. The person's evaluation of alternatives is transitive: if he prefers A_1 to A_2, and A_2 to A_3, then he prefers A_1 to A_3.

3. Continuity. If A_1 is preferred to A_2, and A_2 to A_3, there exists a unique probability p, $0<p<1$, such that the person is indifferent between outcome A_2 with certainty, or receiving A_1 with probability p and A_3 with probability $(1-p)$. In other words, there exists a certainty equivalent to any gamble.

4. Independence. If A_1 is preferred to A_2, and A_3 is some other prospect, a gamble with A_1 and A_3 as outcomes will be preferred to a gamble with A_2 and A_3 as outcomes, if the probability of A_1 and A_2 occurring is the same in both cases.

These axioms relate to choices among both certain and uncertain outcomes. That is, if a person conforms to the four axioms, a utility function (sometimes referred to as "value" function) can be derived that expresses his preferences for both certain outcomes and the choices in a risky situation. In essence, they are equivalent to assuming that the decision maker is rational and consistent in his preferences, and implies the following expected utility theorem:

- Given a decision maker whose preferences satisfy the four axioms, there exists a function U, called a utility function, that associates a single real number or utility index with all risky prospects faced by the decision maker. This function has the following properties:

1. If the risky prospect A_1 is preferred to A_2 (designated as $A_1 \succ A_2$), the utility index of A_1 will be greater than that of A_2 [i.e., $U(A_1) > U(A_2)$]. Conversely, $U(A_1) > U(A_2)$ implies that A_1 is preferred to A_2.

2. If A is the risky prospect with a set of outcomes $\{\theta\}$ distributed according to the probability density function $p\{\theta\}$, the utility of A is equal to the statistically expected utility of A; that is,

$$U(A) = EU(A). \tag{5.1.2}$$

If $p\{\theta\}$ is discrete,

$$U(A) = \sum_{\theta} U(\theta)p(\theta), \tag{5.1.3}$$

and if $p\{\theta\}$ is continuous,

$$U(A) = \int_{-\infty}^{\infty} U(\theta)p(\theta)d(\theta). \tag{5.1.4}$$

As these equations indicate, only the first moment (i.e., the mean or expected value) of utility is relevant to choice. Honoring the Bernoulli's principle, the variance or other higher moments of utility are irrelevant; the expected value takes full account of all the moments (mean, variance, skewness, etc.) of the probability distribution $p\{\theta\}$ of outcomes.

3. Uniqueness of the function is defined only up to a positive linear transformation. Given a utility function U, any other function U^* such that

$$U^* = aU + b, \quad a > 0 \tag{5.1.5}$$

for scalars a and b, will serve as well as the original function. Thus utility is measured on an arbitrary scale and is a relative measure.

The outlined above approach provides a mechanism for ranking risky prospects in order of preference, the most preferred prospect being the one with the highest utility. Hence two concepts are involved: degree of preference (or utility) and degree of belief (or probability).

Utility functions must be assessed separately for each decision maker. To be of use, utility values (i.e., subjective preferences) must be assigned to all possible outcomes for the problem at hand. Usually, a frame of reference is defined whose lower and upper bounds represent the worst and the best possible outcomes, respectively. In many circumstances, outcomes are non-monetary in nature. For example [168], while selecting a portable computer, the decision maker might consider such factors as speed, memory, display quality, and weight. It is possible to assign utility values to these outcomes; however, in most business- related problems, a monetary consequence is of major importance.

In the general case, we are given a set of m alternatives $A=\{A_1, A_2, \dots, A_m\}$, where each alternative may result in one of n outcomes or "states of nature". Call these $\theta_1, \theta_2, \dots, \theta_n$, and denote x_{ij} as the consequence realized if θ_j results when alternative i is selected. Also, let $p_j(\theta_j)$ be the probability that the state of nature θ_j occurs. Then, from (5.1.3) we can compute the expected utility of alternative A_i as follows:

$$U(A_i) = \sum_{j=1}^{n} p_j(\theta_j)U(x_{ij}), \quad i = 1,2,\dots,m, \tag{5.1.6}$$

where $x_{ij} \equiv x_{ij}(\theta_j)$ is an implicit function of θ_j. For the deterministic case where $n = 1$, implying that only one outcome is possible, (5.1.6) reduces to $U(A_i) = U(x_i)$.

5.1.2 Multi-criteria utility models and methods

The multi-criteria aspect of decision analysis appears because outcomes have to be evaluated in terms of several objectives (also called goals). These are stated in terms of properties, either desirable or undesirable, that determine the decision maker's preferences for the outcomes. As an example [168], for design of an automobile, the various multi-criteria objectives must be to:

(1) minimize production costs;

(2) minimize fuel consumption;

(3) minimize air pollution, and

(4) maximize safety.

The purpose of the value model is to take the outcomes of the system model, determine the degree to which they satisfy each of the objectives, and then make the necessary trade-offs to arrive at a ranking for the alternatives that correctly express the preferences of the decision maker.

The multi-attribute utility theory (MAUT) [125-127, 168] is usually applied to projects and suggests developing a hierarchy of objectives and sub-objectives with the lowest members of the hierarchy called attributes. Each attribute should represent a significant criterion in the decision-making process and should be quantified. The set of attributes should satisfy the following requirements:

1. Completeness. The set of attributes should characterize all the factors to be considered in the decision-making process.
2. Importance. Each attribute should represent a significant criterion in the decision-making process, in the sense that it has the potential for affecting the preference ordering of the alternatives under consideration.
3. Measurability. Each attribute should be capable of being objectively or subjectively quantified. Technically, this requires the possibility to establish a utility function for each attribute.
4. Familiarity. Each attribute should be understandable to the decision-maker in the sense that the latter should be able to identify preferences for different states.
5. Non-redundancy. No two attributes should measure the same criterion, a situation that would result in double counting.
6. Independence. The value model should be structured so that changes within certain limits in the state of one attribute should not affect the preference ordering for states

of another attribute or the preference ordering for gambles over the states of another attribute.

Once attributes have been assigned to all objectives and attribute states have been determined for all possible outcomes, it is necessary to aggregate the states by constructing a single unit of measurement that will accurately represent the decision maker's preference ordering for the outcomes. This can be achieved by specifying weights for each attribute or criterion.

If the set of attributes satisfies the requirements listed above, it is possible to formulate a mathematical function called a multi-attribute utility function that will assign numbers, called outcome utilities, to each outcome state. In general, the utility $U(x) = U(x_1, x_2, \ldots, x_N)$, of any combination of outcomes $(x_1, x_2, \ldots, x_N)$ for N attributes can be expressed either as:

(1) an additive, or;

(2) a multiplicative function of the individual attribute utility functions $U_1(x_1)$, $U_2(x_2)$,..., $U_N(x_N)$, provided that each pair of attributes is:

1. Preferentially independent of its complement; that is, the preference order of consequences for any pair of attributes does not depend on the levels at which the other attributes are held.
2. Utility independent of its complement; that is, the conditional preference for lotteries (probabilistic trade-offs) involving only changes in the levels for any pair of attributes, does not depend on the levels at which the other attributes are held.

To illustrate *Condition 1*, suppose that four attributes for a given project are: profitability, time-to-market, technical risk, and commercial success. Preferential independence means that if we judge technological risk, for example, to be more important than profitability, this relationship should remain true regardless of whether the level of profitability is high, low, or somewhere in between; and also regardless of the value of other attributes.

The second condition, namely utility independence, means that if we are deciding on the preference ordering (ranking) for probabilistic trade-offs between, for example, technological risk and time-to-market, this can be done regardless of the value of profitability.

It is necessary to verify that these two conditions are valid, or more correctly, to test and identify the bounds of their validity. The mathematical notation used to describe the model is therefore as follows [125]:

x_i - state of the i-th attribute;

x_i^0 - least preferred state to be considered of the i-th attribute;

x_i^* - most preferred state to be considered of the i-th attribute;

x - vector $(x_1, x_2, \ldots, x_N)$ of attribute states characterizing a specific outcome;

x^0 - outcome constructed from the least preferred states of all attributes; $x^0 = \left(x_1^0, x_2^0, \ldots, x_N^0\right)$;

x^* - outcome constructed from the most preferred states of all attributes; $x^* = \left(x_1^*, x_2^*, \ldots, x_N^*\right)$;

$\left(x_i, \overline{x}_i^0\right)$ - outcome in which all attributes except for the i-th attribute are at their least preferred state;

$U_i(x_i)$ - utility function associated with the i-th attribute;

$U(x)$ - utility function associated with outcome x;

k_i - scaling constant for the i-th attribute; $k_i = U\left(x_i^*, \overline{x}_i^0\right)$;

k - master scaling constant.

If the two independence conditions hold, $U(x)$ assumes the following multiplicative form:

$$U(x) = \frac{1}{k}\left\{\prod_{i=1}^{N}\left[1 + k \cdot k_i U_i(x_i)\right] - 1\right\}, \quad (5.1.7)$$

where the master scaling constant k is determined from the equation $1 + k = \prod_i (1 + k \cdot k_i)$. If $\sum_i k_i > 1$, then $-1 < k < 0$; if $\sum_i k_i < 1$, then $k > 0$; if $\sum_i k_i = 1$, then $k = 0$ and (5.1.7) reduces to the additive form:

$$U(x) = \sum_{i=1}^{N} k_i U_i(x_i). \quad (5.1.8)$$

Since utility is a relative measure as shown above, the underlying theory permits the arbitrary assignment of $U_i\left(x_i^0\right) = 0$ and $U_i\left(x_i^*\right) = 1$; that is, the worst outcome for each attribute is given a utility value of 0, while the best outcome is given a utility value of 1. The actual shape of the utility function depends on the decision maker's subjective judgment as to the relative desirability of possible outcomes. A point-wise approximation of this function can be obtained by asking a series of lottery-type questions such as the following: "For attribute i, what certain outcome, x_i, would be equally desirable as realizing the highest outcome with probability p, and the lowest outcome with probability $(1 - p)$?".

Thus, a conclusion can be drawn that the most important MAUT stages comprise:

- a pairwise preferential judgment [150] based on experts which is usually carried out in the form of a trade-off between a couple of attributes;

- assigning a scaling constant for each attribute which is pregiven by experts as well;
- determining the project's utility function in MAUT which depends on the decision-maker's subjective judgment on the relative desirability of possible attributes' outcomes. This can be achieved in the form of various interview questions being addressed to experts;
- the attributes are usually ranked in ascending order of importance as they progress from their worst to their best states (values).

The most important stage of MAUT is to rank the alternatives. This may be accomplished by using the multi-attribute utility function to calculate outcome utilities for each alternative under consideration. If two or more alternatives appear to be close to rank, their sensitivity to both the scaling constants and utility functions should be examined.

Multi-attribute utility theory can be applied in situation when the state of an attribute may be uncertain. "Completion time of a task", "reliability of a subassembly", and "useful life of the system" are common examples of attributes whose states may take on different values with known or unknown probabilities. In these cases, x_i is really a random variable, so it is more appropriate to compute the expected utility of a particular outcome. For the additive model, this can be implemented by means of the following equation:

$$E[U(x)] = \sum_{i=1}^{N} \left[k_i \int_{-\infty}^{\infty} U_i(x_i) f_i(x_i) d(x_i) \right], \tag{5.1.9}$$

where $f_i(x_i)$ represents the probability density function associated with attribute i, and $E[\bullet]$ stands for the expectation operator [127].

Ranking alternatives is often carried out on the basis of the analytic hierarchy process (AHP) which has been first outlined in [161] and provides a multiple-criteria methodology for evaluating alternatives.

Typical applications of the AHP can be found in portfolio selection, transportation planning, manufacturing systems design, artificial intelligence, etc. The advantages of the AHP lie in its ability to structure a complex, multi-person, multi-attribute problem hierarchically, and then to investigate each level of the hierarchy separately, combining the results as the analysis progresses. Pairwise comparisons of the factors (which, depending on the context, may be alternatives, attributes, or criteria) are undertaken using a scale indicating the strength with which one factor dominates another with respect to a high-level factor. This scaling process can then be translated into priority weights or scores for ranking the alternatives.

Like MAUT, the AHP starts with a hierarchy of objectives. The top of the hierarchy provides the analytic focus in terms of a problem statement. At the next level, the major considerations are defined in broad terms. This is usually followed by a listing of the criteria for each of the foregoing considerations. Depending on how much detail is called for in the model, each criterion may then be broken down into individual parameters, whose values are either estimated or determined by measurement or experimentation. The bottom level of the hierarchy comprises the alternatives or scenarios underlying the problem.

From analyzing the utility theory techniques in operation management, the following conclusions may be drawn:

1. Utility theory techniques can be used [150, 168] from the point of choosing new competitive goals to be reached. Those goals in combination with monetary policies usually apply to new technical devices to be designed and created.
2. The existing utility theory models and methods are not applicable to operation management (organization) systems which are usually governing and monitoring the process <u>of the systems' functioning</u>, since all MAUT models are restricted to market competitive problems alone. Thus, nowadays, the existing utility theory is mostly restricted to analyse the competitive quality of the organization systems' outcome products. The theory, however, does not deal with the quality of the systems' functioning, i.e., with organization systems in their entirety. This may result in heavy financial losses, e.g., when excellent project objectives are achieved by a badly organized project's realization.

§5.2 Trade-off optimization models

In the last five decades, the research literature on trade-off optimization models to determine a compromise between certain parameters in organization systems is practically restricted to project management systems. Various time – cost trade-off models have been developed by Arisawa and Elmaghraby [2], Arsham [3], Deckro and Hebert [47], Howard [117], Kelley [128], Peck and Scherer [152], Moder et al. [142], Moore et al. [145], Hillier and Lieberman [116], Menipaz [140], Nandi and Dutta [148], Golenko-Ginzburg [70], Gonik [109], de Coster [46], Chase and Aquilano [41], Panagiotakopoulos [151], Shtub [168], Siemens [169], Laslo [131], etc. Those publications usually investigate a compromise between time and cost parameters. Such a compromise may be implemented by means of stating and solving certain optimization problems.

5.2.1 <u>Deterministic time-cost trade-off procedures</u>

A variety of publications is related to deterministic network projects (in the form of a graph $G(N,A)$ comprising nodes $i \in N$ and activities $(i,j) \subset A$ leaving node i and entering node j) with deterministic activity durations. For any activity (i,j) entering the network project $G(N,A)$, it is assumed that:

- the corresponding activity duration t_{ij} depends parametrically on the budget c_{ij} assigned to that activity, and
- the budget value c_{ij} satisfies
  $c_{ij\ min} \le c_{ij} \le c_{ij\ max}$,
  where $c_{ij\ min}$ stands for the minimal budget capable of operating activity (i, j), and $c_{ij\ max}$ is the maximal budget required to operate activity (i, j). Both values $c_{ij\ min}$ and $c_{ij\ max}$ are pregiven beforehand.

Note that in case $c_{ij} > c_{ij\,\max}$ additional value $c_{ij} - c_{ij\,\max}$ is redundant. Thus, function $t_{ij} = f_{ij}(c_{ij})$ can be implemented for any $(i, j) \in A \subset G(N, A)$. The main objective of the time – cost trade-off procedure is to consider the relationship between the project duration and the total project costs.

Time constraints arise in a number of ways. First, the customer might contractually require a scheduled completion time for the project. Then, the original time constraint might change after the project has started, requiring new project planning. These amendments arise because of changes in the customer's plans; or, when delays occur in the early stages of a project, the new expected completion time of the project may be too late. The most interesting time constraint application arises when one asks for the project schedule that minimizes total project costs, direct plus indirect altogether. This is equivalent to the schedule that just balances the (indirect) marginal value of time saved (in completing the project one time unit earlier) against the (direct) marginal cost of saving it. This situation occurs frequently, for example, in the major overhaul of large systems, such as chemical plants, paper machines, aircraft, etc. Here the value of time saved is very high, and furthermore it is known quite accurately. In such application, the crux of the problem amounts to developing a procedure to establish the minimum (marginal) cost of saving time. This assumes, of course, that some jobs may be carried out faster if more resources are allocated to them. The resources may be manpower, machinery, and / or materials. It is usually assumed [109, 117, 144-145, 173] that these resources can be measured and estimated, reduced to monetary units, and summarized as a direct cost per unit time.

Thus, the main purpose of the time – cost trade-off can be stated as the development of a procedure to determine activity schedules to reduce the project duration time with a minimum increase in the project direct costs, by buying time along the critical path(-s) where it can be obtained at the least cost.

The development of the Critical Path Method (CPM) time – cost trade-off procedure is based on a number of definitions which are outlined below and represented in Fig. 5.1.

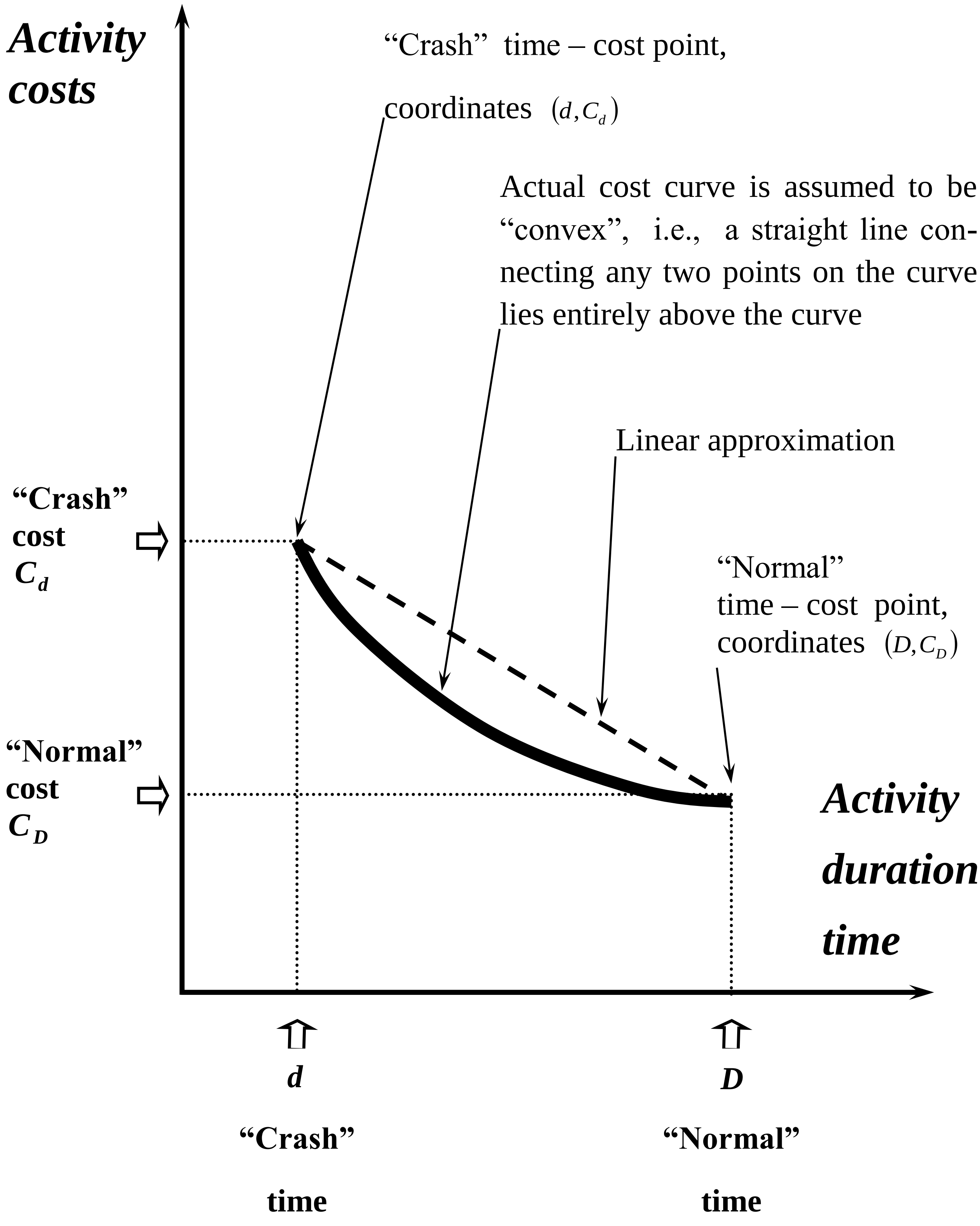

Figure 5.1. *Activity time-cost trade-off input for the CPM procedure*

Activity direct costs include the costs of the material, equipment, and direct labor required to perform the activity under consideration. If the activity is carried out in its entirety by a subcontractor, then the activity direct cost is equal to the price of the subcontract, plus any fee that may be added.

Project indirect costs may include, in addition to supervision and other customary overhead costs, the interest charges on the cumulative project investment, penalty costs for accomplishing the project after the specified deadline, and bonuses for early project completion.

Normal activity time – cost point. The normal activity cost is equal to the minimum of costs required to perform the activity, and the corresponding activity duration is called the normal time. (It is this normal time that is used in the basic critical path planning and scheduling, and the normal cost is the one usually supplied if the activity is being sub-contracted). The normal time is actually the longest time required to carry out the activity under the minimum cost constraint, which rules out the use of overtime labor or special time saving (but more costly) of materials or equipment.

Crash activity time – cost point. The crash time is the fully expedited or minimum activity duration time that is technically possible, and the crash cost is assumed to be the maximum cost required to achieve the crash performance time.

The normal and crash time – cost points are denoted by the coordinates (D, C_D) and (d, C_d), respectively, in Fig. 5.1. For the present, it will be assumed that the resources are infinitely divisible, so that all times between d and D are feasible, and the time – cost relationship is represented by the solid line. It will also be assumed that this curve is convex, and can be adequately approximated by the dashed straight line.

The CPM computational procedure chooses the duration times for each activity so as to minimize the total project direct costs and at the same time satisfy the constraints on the total project completion time and on the individual activities, the latter being dictated by both the logic of the project network and the performance time intervals (d, D) established for each activity.

The simplified time – cost trade-off model for a CPM network is as follows:

given a CPM graph $G(N,A)$ *together with functions* $t_{ij} = f_{ij}(c_{ij})$, $(i,j) \in G(N,A)$, *and values* $c_{ij\ min}$ *and* $c_{ij\ max}$, *determine:*

- the minimal total project direct costs C,

$$Min\ C, \qquad \text{and} \qquad (5.2.1)$$

- the optimal assigned budget values c_{ij}^{opt}, subject to

$$T_{cr}\left\{ t_{ij} = f_{ij}\left(c_{ij}^{opt}\right) \right\} \le D, \qquad (5.2.2)$$

$$\sum_{\{i,j\}} c_{ij}^{opt} = C, \tag{5.2.3}$$

$$c_{ij\ min} \le c_{ij}^{opt} \le c_{ij\ max}, \tag{5.2.4}$$

where D stands for a pregiven due date.

Problem (5.2.1-5.2.4) is usually solved [46-47, 142] by means of heuristic methods based on "normal" and "crash" concepts outlined above. In cases of non-linear f_{ij} the problem becomes too difficult to be solved analytically [2].

5.2.2 Stochastic time-cost trade-off procedures

In most stochastic network projects the major resources involved in the project realization are financial resources. Thus controlling the project boils down, in essence, to introducing various control actions with regard to the budget assigned to that project.

It can be well-recognized from various studies in PERT-COST [41, 47, 61, 67, 70, 77, 109, 116, 142-143, 148, 154] that activity duration is close to being inversely proportional to the budget assigned to that activity. Since random time duration t_{ij} in PERT studies is assumed to be beta-distributed (see Chapter 2), we may consider the time – cost curve for random time activities, as indicated in Fig. 5.2. Note that for all activities $(i,j) \in G(N,A)$ both time – cost pessimistic and optimistic curves are to be predetermined externally, while the time – cost average curve can be determined on the basis of the beta-distribution with pregiven lower and upper bounds a_{ij} and b_{ij} called optimistic and pessimistic values (for each t_{ij}).

Assume for simplicity that for all activities $(i,j) \in G(N,A)$ values $t_{ij}^{*} = \dfrac{a_{ij}}{c_{ij}}$ and $t_{ij}^{**} = \dfrac{b_{ij}}{c_{ij}}$, with pregiven constants a_{ij} and b_{ij}, represent optimistic and pessimistic time – cost curves, respectively, while the beta-distribution density function of the activity duration according to [67, 70, 73] may be written as

$$p_{ij}(t) = \frac{12}{\left(t_{ij}^{**} - t_{ij}^{*}\right)^4} \left(t - t_{ij}^{*}\right)\left(t_{ij}^{**} - t\right)^2. \tag{5.2.5}$$

Note that for this simplified beta-distribution relation

$$\bar{t}_{ij} = \frac{3t_{ij}^{*} + 2t_{ij}^{**}}{5} = \frac{3a_{ij} + 2b_{ij}}{5c_{ij}} \tag{5.2.6}$$

represents the time average cost curve indicated on Fig. 5.2.

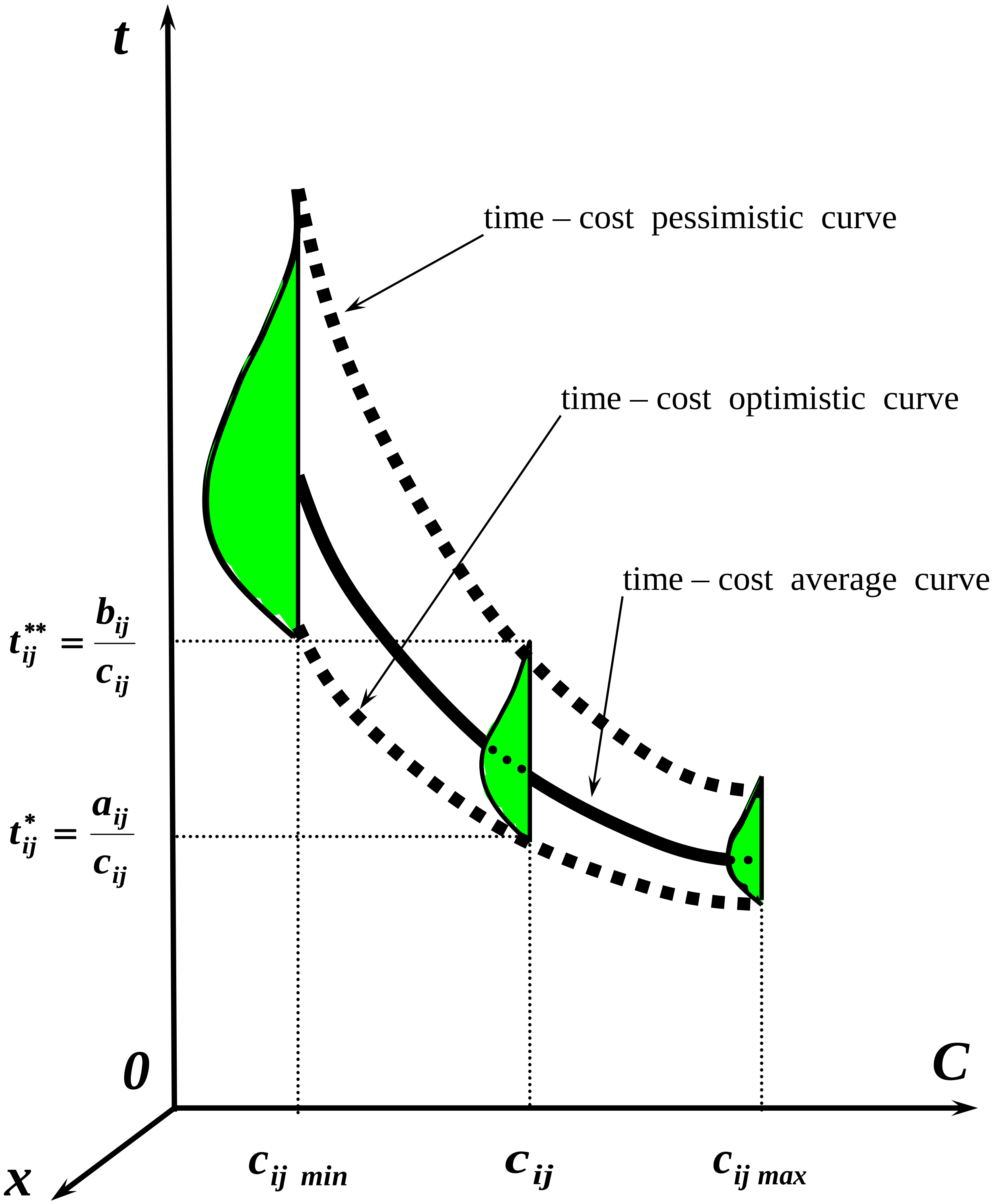

Figure 5.2. *Time-cost curves for activities with random durations*

This simplified relation has been used successfully in project management [70, 75, 109]. However, for certain practical cases generalized relations can be recommended, namely

$$t_{ij}^{*} = \frac{a_{ij}}{\left(c_{ij}\right)^{\rho}} \quad \text{and} \quad t_{ij}^{**} = \frac{b_{ij}}{\left(c_{ij}\right)^{\rho}} , \tag{5.2.7}$$

where $0.5 < \rho \le 1$. It has to be pointed out that from the *principal point of view* all PERT-COST techniques remain unchanged even when implementing assumptions (5.2.7). However, the management can adopt any suitable distribution as long as its density function presents a *linkage between time and costs*. The corresponding algorithms and control models are capable of handling and adopting different types of distribution functions.

In [76-77, 109] the trade-off model minimizes the allocated budget under given time chance constraint. The extension of problem (5.2.1-5.2.4) for a random activity duration t_{ij} is as follows:

given the PERT-COST project $G(N,A)$ *with random activity durations* t_{ij}, $(i,j) \in G(N,A)$, *where for each activity* (i,j) *its probability density function (p.d.f.)* $p_{ij}(t)$ *depends parametrically on the budget* c_{ij} *assigned to that activity: the problem is to minimize the project's budget* C

$$Min\ C, \tag{5.2.8}$$

as well as to determine the optimal budget volumes c_{ij}^{opt} assigned to each activity $(i,j) \in G(N,A)$ subject to

$$Pr\left\{ T\left[t_{ij} / c_{ij}^{opt}\right] \le D \right\} \ge p, \tag{5.2.9}$$

$$\sum c_{ij}^{opt} \le C, \tag{5.2.10}$$

$$c_{ij\ min} \le c_{ij}^{opt} \le c_{ij\ max}. \tag{5.2.11}$$

Here:

- $T\left[t_{ij} / c_{ij}^{opt}\right]$ stands for the project's random duration on condition that all the activity's durations are random values with p.d.f. $p_{ij}\left(t/c_{ij}\right)$. Value $T\left[t_{ij} / c_{ij}^{opt}\right]$ can be determined either via simulation, or by means of approximate analytical methods;
- D designates the pregiven due date;

- p is the minimal value of the chance constraint (pregiven by the project management as well).

Problem (5.2.8-5.2.11) is a very complicated problem which even for medium-scale projects cannot be solved analytically. It requires therefore heuristic solutions that are widely used nowadays in various design offices [69-70, 99, 109, 138].

As outlined below, in §5.3, a modification of problem (5.2.8-5.2.11) enables the solution of the partial harmonization problem for project management systems. This, in turn, enables solving harmonization problems for PERT-COST projects with trade-offs between three basic parameters: cost, time and reliability. In our opinion, this is an essential advancement in the area of multi-parametric optimization.

§5.3 Harmonization models in organization systems

5.3.1 Introduction

As mentioned above, in §5.1, there are very few publications (see, e.g., [100]) dealing with estimating the quality of the system itself, e.g., the system's public utility. We suggest using the term "utility" henceforth as a generalized quantitative value to estimate the quality of the system's functioning. To develop the corresponding techniques we suggest to take into account the basic parameters, which actually form the utility of the system - validity, reliability, flexibility, cost, sensitivity, forecasting (timeliness), etc. Most of those criteria are difficult to be formalized and require human judgment and rating schemes in order to turn qualitative information into quantitative estimates.

The backbone of this Chapter is to formalize the multi-parametric harmonization model in order to maximize the system's utility as a generalized quality measure of the system's functioning.

Another main result of the Chapter is the development of the principal idea of the harmonization problem's solution. As outlined above, in §1.2, we suggest to sub-divide the basic parameters into two sub-sets:

- independent parameters, where for each parameter its value may be preset and may vary independently on other parameters' values, and
- dependent parameters whose values may not depend uniquely on the values of independent parameters.

We suggest a multi-stage solution of harmonization problems. At the first stage a look-over algorithm to examine all feasible combinations of independent basic values, is implemented. The independent parameters' values obtained at that stage are used as input values at the second stage where for each dependent parameter a local subsidiary optimization problem is solved in order to raise the system's utility as much as possible. Solving such a problem enables the solely dependence of the optimized value on any combination of independent input parameters. At the next stage the system's utility value is calculated by means of basic parameters' values obtained at the previous stages, while Stage *IV* carries out the search for the extremum in order to determine the optimal combination of all basic parameters' values delivering the maximum to the system's utility.

To facilitate further discussion in this chapter, we will require additional notations.

5.3.2 Terminology

Let us introduce additional terms:

S - organization system;
M_s - the system's model;
R_k - the k-th basic system's parameter, $1 \le k \le n$ (to be optimized);
n - number of basic parameters;
$R_i^{(ind)}$ - the i-th independent basic parameter, $1 \le i \le n_1$;
n_1 - number of independent basic parameters;
$R_j^{(dep)}$ - the j-th dependent basic parameter, $1 \le j \le n_2$;
$n_2 = n - n_1$ - number of dependent basic parameters;
R_{k0} - restriction for the k-th basic parameter, $1 \le k \le n$, i.e., the worst permissible value which can be accepted;
R_{k00} - the best value of the k-th basic parameter, $1 \le k \le n$, which by no means can be refined (pregiven);
U - the system's utility (to be optimized);
α_k - local parametrical utility, $1 \le k \le n$ (pregiven);
U_0 - basic utility obtained for $R_k = R_{k0}$, $1 \le k \le n$ (pregiven);
Δ_k - search step for the k-th basic parameter (pregiven);
ε - pre-specified search tolerance for optimizing the system's utility (pregiven);

$PHM_j \left\{ R_i^{(ind)} \right\} = Max\, R_j^{(dep)}$ - partial harmonization model to maximize the j-th dependent parameter $R_j^{(dep)}$ on the basis of the n_1 input values $R_i^{(ind)}$, $1 \le i \le n_1$;

$\Delta U^* \Big/ R_{i,1 \le i \le n_1}^{(ind)} = \sum_{j=1}^{n_2} \left\{ \alpha_j^{(dep)} \cdot \left| R_j^{(dep)} - R_{j0}^{(dep)} \right| \right\} \Big/ \vec{R}_i^{(ind)}$ - additional utility on the account of depending parameters by solving the partial optimization problem on the basis of preset values $R_j^{(ind)}$, $1 \le i \le n_1$, $1 \le j \le n_2$.

$U^* = \Delta U \Big/ R_{i,\, 1 \le i \le n_1}^{(ind)} + \sum_{i=1}^{n_1} \left\{ \alpha_i^{(ind)} \cdot \left| R_i^{(ind)} - R_{i0}^{(ind)} \right| \right\}$ - the objective function for a partial harmonization problem with pregiven n_1 independent parameter values.

5.3.3 General concepts

Consider a complicated organization system which functions under random disturbances. Such a system usually comprises a variety of qualitative and quantitative attributes, characteristics and parameters, which enable the system's functioning. The problem arises to determine a generalized (usually quantitative) value which covers all essential system's parameters and can be regarded to as a system's qualitative estimate. We will henceforth call such a generalized value the system's utility.

Later on we will require some new definitions.

Definitions

I. Call the system's model M_s a formalized description of the system's structure as well as the system's functioning. M_s usually comprises the logical links between the system's elements, decision-making rules, various random parameters, etc. For project management systems various M_s may be used, e.g., network PERT-COST models (see §5.2), GANTT chart models [168], CPM models [142, 151, 154, 168], GERT models [2, 55, 67, 70, 144, 168], etc. PERT-COST network models which are widely used in project management [67, 70, 76, 109, 168], are used as M_s in Chapters 8, 16-17. Such a network model is actually a graph type simulation model comprising activities with random durations. The p.d.f. of each activity duration depends parametrically on the budget value assigned to that activity. M_s usually comprises all the basic parameters (see below) which have an influence on the system's utility.

II. Call a quantitative parameter entering the system a basic parameter on condition that changes in the parameter result in changing the system's utility. Note that the restriction value for any basic parameter is, actually, the worst permissible value that may be implemented into the system. The set of basic parameters, together with the corresponding restriction values, are externally pregiven.

III. Call the system's utility which corresponds to the pregiven restriction values for all basic system's parameters, the basic utility. Denote henceforth the basic utility by U_0 . Value U_0 is externally pregiven as well.

IV. Call the direction of changing a basic parameter's value which results in increasing the system's utility, a positive direction, and vice versa. Call the change of the system's utility caused by altering a parameter by its unit value in the positive direction, a local parametric utility. Denote henceforth the additional local parametric utility for the k-th basic parameter by $\alpha_k > 0$. Parametric utility values are also pregiven externally.

Denote henceforth the pregiven restriction values for each basic k-th parameter, $1 \le k \le n$, by R_{k0}, correspondingly. If restrictions are given in the form

$$R_k \ \ge \ (\le) \ R_{k0} \ , \quad 1 \le k \le n \ , \tag{5.3.1}$$

it can be well-recognized from (1.2.1) that the system's utility U satisfies

$$U = U_0 + \sum_{k=1}^{n} \left[\alpha_k \cdot \varepsilon \left(R_k - R_{k0} \right) \right] \tag{5.3.2}$$

subject to (5.3.1). Here

$$\varepsilon \left(R_k - R_{k0} \right) = \begin{cases} \left| R_k - R_{k0} \right| & \textit{if value } R_k \textit{ is obtained in the positive direction} \\ & \textit{from } R_{k0} \\ \Re & \textit{otherwise} \ , \end{cases} \tag{5.3.3}$$

where $\Re$ is an extremely large negative number which practically eliminates value R_k. It can be well-recognized that relation (5.3.3) ensures restrictions (5.3.1).

Note that to solve the harmonization problem, we need to define for each k-th basic parameter its <u>best</u> values which by no means can be refined. Denote those values which are externally pregiven, by R_{k00}, correspondingly.

To proceed with, we require additional definitions.

V. Call the basic system's parameters which can be pregiven <u>independently</u> from each other, <u>independent basic parameters</u>. It goes without saying that setting values of independent basic parameters honors restrictions (5.3.1).

VI. Call other basic system's parameters <u>dependent basic parameters</u>. Thus, the basic parameters can be subdivided into two groups: independent and dependent parameters. The latter do not depend uniquely on the preset values of independent parameters. Moreover, a combination of independent parameters may correspond to numerous different values (sometimes to an infinite number) of a certain dependent parameter. If, for example, a PERT-COST network project is carried out under random disturbances, setting the cost value (assigned for the project) and the time value (in the form of the project's due date) does not define solely the value of the project's reliability, i.e., its probability to meet the deadline on time. This is because the budget value C assigned to the project has to be reallocated beforehand among the project activities in order to start processing the latter. Each budget reallocation results in a certain project's reliability and, thus, different feasible (but non-optimal!) reallocations correspond to different non-optimal reliability values. However, <u>for the same preset independent basic parameters - cost and time values - it is possible to maximize the project's reliability by means of optimal budget reallocation among the</u>

project's activities. The corresponding problem together with its solution is outlined below, in *Chapter 8*.

Thus, we suggest to implement a solely dependency of each dependent basic parameter on the combination of independent input values by means of a subsidiary optimization procedure (heuristic, simulative, approximate) in order to maximize the system's utility for the fixed combination of independent parameters and the optimized dependent parameter. We will henceforth call the optimized objective (5.3.2) where at least one basic parameter value is pregiven beforehand and remains unchanged in the course of optimization, the conditional system's utility.

VII. Call a partial harmonization problem PHM_j an optimization problem (analytic, simulative, heuristic) which on the basis of preset independent basic parameters delivers an optimum value to a dependent basic parameter R_j in order to maximize the conditional system's utility. Thus, a PHM enables the solely dependence of a dependent parameter from independent ones.

Practically speaking, the partial harmonization model is mostly optimized by means of optimal budget reallocation among the system's elements. In the case of a project management system with M_s representing a PERT-COST type network model, optimal budget reallocation among the project's activities enables maximization of the project's reliability value R. In the case of a hierarchical production system with M_s based on a multi-level fault tree model together with a pregiven list of possible technical improvements for the bottom level elements, the partial harmonization model centers on an optimal budget reallocation among a chosen sub-set from the list of improvements. However, such an optimization problem is essentially more complicated than in the case of a network project, since we have to cope both with choosing an optimal sub-set from the set of improvements as well as with an optimal budget reallocation among the chosen elements to be improved.

Note, in addition, that in the case of project management systems the budget value C is always an independent basic parameter, while the project's reliability value R is a dependent one. In the case of hierarchical production systems under consideration some PHM use the budget value C as an independent parameter with reliability value R as a dependent one, while other PHM act vice versa, i.e., the reliability value us externally pregiven and is an independent parameter, while the budget value C (to be minimized) serves as a dependent one. Thus, those two basic parameters are, as a matter of fact, interchangeable.

It can be well-recognized that the efficiency of a harmonization problem depends mostly on the efficiency of partial harmonization models, since it is easy to implement a search algorithm for several independent basic parameters. Thus, the main difficulty to solve a practical harmonization problem (especially in cases of numerous basic parameters) is to develop a combination of a high-speed partial harmonization model and a relatively simple search procedure for independent basic parameters.

Thus, using *Notation* in 5.3.1, we obtain for the system's utility

$$U = \sum_{i=1}^{n_1} \alpha_i^{(ind)} \cdot R_i^{(ind)} + \sum_{j=1}^{n_2} \beta_j^{(dep)} \cdot R_j^{(dep)}, \quad 1 \le i \le n_1, \quad 1 \le j \le n_2 = n - n_1, \tag{5.3.4}$$

where

- $R_1^{(ind)},..., R_{n_1}^{(ind)}$ - independent basic parameters;
- $R_1^{(dep)},..., R_{n_2}^{(dep)}$ - dependent basic parameters.

Denoting by $PHM_j\left\{\vec{R}_i^{(ind)}\right\} = R_j^{(dep)}$, $1 \le j \le n_2$, a partial harmonization model, we finally obtain

$$U = \sum_{i=1}^{n_1} \alpha_i^{(ind)} \cdot R_i^{(ind)} + \sum_{j=1}^{n_2} \beta_j^{(dep)} \cdot PHM_j\left\{\vec{R}_i^{(ind)}\right\}. \tag{5.3.5}$$

Value U may comprise both analytic PHM_j as well as PHM_j based on simulative modeling. In some cases PHM_j can be based on subjective decision-making.

5.3.4 *Optimal harmonization problem and the general idea of the problem's solution*

Referring to *5.3.3* and using *Notation* in *5.3.1*, the harmonization problem is as follows: determine optimal values R_k, $1 \le k \le n$, to maximize the system's utility

$$\underset{\{R_k\}}{Max}\ U = U_0 + \sum_{k=1}^{n} \alpha_k \cdot |R_k - R_{k0}| \tag{5.3.6}$$

subject to

$$Min\ \{R_{k0}, R_{k00}\} \le R_k \le Max\ \{R_{k0}, R_{k00}\}. \tag{5.3.7}$$

Since U_0 remains constant, the objective can be simplified as follows

$$\underset{\{R_k\}}{Max} \sum_{k=1}^{n} \{\alpha_k \cdot |R_k - R_{k0}|\} \tag{5.3.8}$$

subject to (5.3.7).

Problem (5.3.7-5.3.8) is a very complicated optimization problem which usually does not provide analytical estimates.

Let us analyze the general harmonization problem in greater detail. Since independent basic parameters $R_i^{(ind)}$ serve as input values which can be optimized by means of a search algorithm, the harmonization problem's solution suggests itself as a combination of two sequential problems:

- to determine an optimal combination of independent basic values $\left\{R_i^{(ind)(opt)}\right\}$ by means of a lookover algorithm that checks the feasibility of each possible combination (*Problem I*),
- to solve all the partial harmonization problems by means of $PHM_j\left\{R_i^{(ind)}\right\}$ (*Problem II*), and
- to facilitate a search for the extremum in order to maximize utility value (5.3.5).

Theorem

Optimal values $R_k^{(opt)}$, $1 \le k \le n$, in problem (5.3.7-5.3.8) satisfy

$$\left\{R_k^{(opt)}\right\} \equiv \left\{R_i^{(ind)(opt)}\right\} \cup PHM_j\left\{R_i^{(ind)(opt)}\right\}. \tag{5.3.9}$$

Proof

Assume that $\left\{R_k^{(opt)}\right\}$ does not satisfy (5.3.9), i.e., there exists a combination

$$\left\{R_k^{'}\right\} \equiv \left\{R_i^{(ind)\,'}\right\} \cup \left\{R_j^{(dep)\,'}\right\} \tag{5.3.10}$$

satisfying (5.3.8) and not coinciding with (5.3.9). Note, first, that relation

$$\left\{R_j^{(dep)\,'}\right\} \equiv PHM_j\left\{R_i^{(ind)\,'}\right\} \tag{5.3.11}$$

holds, otherwise the combination $\left\{R_k^{'}\right\}$ may be improved by substituting $R_j^{(dep)\,'}$ for $PHM_j\left\{R_i^{(ind)\,'}\right\}$. This, in turn, contradicts relation (5.3.8). Secondly, relation

$$\left\{R_i^{(ind)\,'}\right\} \equiv \left\{R_i^{(ind)(opt)}\right\} \tag{5.3.12}$$

holds as well, since values $\left\{R_i^{(ind)(opt)}\right\}$ have been obtained by means of an optimal look-over algorithm which checks all possible combinations $\left\{R_i^{(ind)}\right\}$, including $\left\{R_i^{(ind)\,'}\right\}$. Thus, our assumption proves to be false and combinations (5.3.9) and (5.3.10) fully coincide. ■

5.3.5 Optimization techniques

The proved theorem enables solution of problem (5.3.7-5.3.8) by means of a sequential solution of Problem I and II. However, if, due to the high number of possible combinations $\left\{R_i^{(ind)}\right\}$, solving both problems on a look-over basis requires a lot of computational time, we suggest a simplified heuristic algorithm as follows.

Since practically most partial harmonization models PHM_j (see, e.g. [124]) for OS are complicated non-linear functions (5.3.5) of independent parameters $\left\{R_i^{(ind)}\right\}$, determining the optimal system's utility results in implementing the theory of unconstrained optimization for non-linear problems. As outlined in [133], the most effective and widely known methods for maximizing a non-linear function of several variables, e.g., the gradient method, the Newton's method, the conjugate direction method, etc., cannot be carried out without determining the gradient vector at each search step. However, solving the gradient equation for partial harmonization problems based on simulation models comprising stochastic programming constraints leads usually to futile computational efforts.

Thus, a conclusion can be drawn that more attractive and at the same time more realistic approximated algorithms have to be implemented. According to the general recommendations outlined in [133] we have replaced the precise look-over algorithm (Problem I) by the cyclic coordinate search algorithm (CCSA). The latter optimizes the non-linear function of independent parameters cyclically, with respect to coordinate variables. To implement the cyclic coordinate algorithm we suggest accepting several rules as follows:

1. An initial search point $\vec{R} = \left\{R_{10}^{(ind)}, R_{20}^{(ind)}, \ldots, R_{n_1 0}^{(ind)}\right\}$ has to be taken, i.e., the initial point corresponds to the least permissible utility U_0.

2. First, coordinate $R_1^{(ind)}$ has to be optimized, by advancing with a constant search step $\Delta R_1^{(ind)}$ in the *positive direction*, while all other $n_1 - 1$ coordinates remain unchanged. After establishing the quasi-optimal value $R_{1opt}^{(ind)}$ the latter is fixed, and the second coordinate $R_2^{(ind)}$ with other unchanged coordinates $R_{1opt}^{(ind)}, R_3^{(ind)}, \ldots, R_{n_1}^{(ind)}$ has to undergo optimization in the positive direction until obtaining value $R_{2opt}^{(ind)}$. With two coordinates $R_{1opt}^{(ind)}$ and $R_{2opt}^{(ind)}$ fixed, the third coordinate $R_3^{(ind)}$ is subject to the coordinate optimization procedure, etc. Thus, at the beginning of the search procedure all coordinates advance in their positive directions.

   In this course, objective (5.3.8) is substituted by another one, namely

$$\underset{\left\{R_i^{(ind)}\right\}}{Max}\left[\sum_{i=1}^{n_1}\left\{\alpha_i^{(ind)}\left|R_i^{(ind)} - R_{i0}^{(ind)}\right|\right\} + \underset{\left\{R_j^{(dep)}\right\}}{Max}\left[\sum_{j=1}^{n_2}\left\{\alpha_j^{(dep)}\left|R_j^{(dep)} - R_{j0}^{(dep)}\right|\right\} \Big/ \vec{R}_i^{(ind)}\right]\right] \qquad (5.3.13)$$

   subject to (5.3.7).

3. Coordinate $R_i^{(ind)}$ reaches its quasi-optimal value in three cases:

a) if in the course of the coordinate optimization value $R_{i00}^{(ind)}$ is reached. It is taken then as the optimized value $R_{i\,opt}$;

b) if in the course of the coordinate optimization at least one of the dependent parameter values $R_j^{(dep)}$ (while solving the partial harmonization problem) ceases to comply with restrictions (5.3.1) or (5.3.7). This means that we have entered a non-feasible area, and the last successful coordinate value is chosen as the quasi-optimal one;

c) if in the course of the coordinate optimization objective (5.3.13), i.e., the conditional system's utility, ceases to increase. In such a case we act similarly to case b).

4. For certain organization systems which function under random disturbances, values $R_j^{(dep)}, 1 \le j \le n_2$, may be random parameters as well. Thus, objective (5.3.13) becomes a random value too. To calculate its average value at each search point, numerous simulation runs have to be carried out to obtain representative statistics.

5. After accomplishing the first iteration, i.e., determining values $R_{1\,opt}^{(ind)}, R_{2\,opt}^{(ind)}, \dots, R_{n_1\,opt}^{(ind)}$, the corresponding search step values are usually diminished (mostly by dividing by two), and the search process proceeds anew - cyclically with respect to the coordinate variables, beginning from $R_1^{(ind)}$.

6. For all future iterations value $R_i^{(ind)}, 1 \le i \le n_1$, is calculated in two opposite points:

$$\left[R_1^{(ind)}, \dots, R_i^{(ind)} - \Delta R_i^{(ind)}, R_{i+1}^{(ind)}, \dots, R_{n_1}^{(ind)} \right], \text{ and}$$

$$\left[R_1^{(ind)}, \dots, R_i^{(ind)} + \Delta R_i^{(ind)}, R_{i+1}^{(ind)}, \dots, R_{n_1}^{(ind)} \right]$$

to determine the direction of objective's (5.3.13) increase. The search is undertaken along those directions, i.e., values

$$U^* \left(R_1^{(ind)}, \dots, R_{i-1}^{(ind)}, R_i^{(ind)} + r \cdot \Delta R_i^{(ind)}, \dots, R_{n_1}^{(ind)} \right),$$

$$r = \pm 1, \pm 2, \pm 3, \dots ,$$

are calculated.

7. The cyclic coordinate optimization algorithm terminates when the relative difference between two adjacent iterations with indices v and $v+1$

$$U^{*(v)} \left\{ \vec{R}_i^{(ind)} \right\} \quad \text{and} \quad U^{*(v+1)} \left\{ \vec{R}_i^{(ind)} \right\}$$

becomes less than the externally pregiven tolerance $\varepsilon > 0$.

In order to improve the algorithm's global convergence we have additionally implemented the highly recommended Aitken Double Sweep Method [133] which is used as follows: first *CCSA* is carried out with coordinate variables sequenced in the order $R_1^{(ind)}, R_2^{(ind)}, ... ,R_{n_1}^{(ind)}$, and afterwards - in the opposite order: $R_{n_1}^{(ind)}, R_{n_1-1}^{(ind)}, ... ,R_1^{(ind)}$. The maximal objective is taken as the algorithm's quasi-optimal solution.

It can be well-recognized that increasing the number n_1 of independent parameters results in raising the efficiency of CCSA. Note that in project management there are at least two independent parameters with continuous values - budget C assigned to the project and the due date D of accomplishing the project.

As for another class of organization multi-level technical systems with safety engineering problems, - there is usually only one independent parameter - the budget value C or the reliability value R. Using *CCSA* algorithms for such systems cannot be advised since a non-linear function of a single variable can be optimized by other methods as well. However, as outlined below, implementing other classical precise methods, e.g., the method of dynamic programming, usually leads to unavoidable computational difficulties. This is because in safety engineering optimized variables are usually various technical improvements which have to be carried out without exceeding the allocated budget. Thus, obtaining the optimal solution for a safety engineering problem results in determining an optimal subset of technical investments for the pregiven set of possible amendments. If the number of possible amendments is large enough the harmonization model boils down to an enormous amount of stages [176], each of which centers on determining the system's reliability value by means of the simulation model. However, since reliability of a technical system with possible hazardous failures has to be exceptionally close to one, evaluating reliability value by means of simulation requires a tremendous number of simulation runs to obtain sufficient statistics (in certain cases up to a million simulation runs and sometimes even more). Thus, only reasonable heuristic approaches may result in an acceptable solution.

Tables 5.1 and 5.2 present a concise description of the system's models, basic parameters and partial harmonization models for the organization systems to be discussed below: project management systems and hierarchical technical systems. Note that changing the system's model results in ultimate changes of the corresponding *PHM* techniques.

§5.4 Application areas

Besides the examples outlined above, the developed harmonization principle covers a broad spectrum of other hierarchical organization systems, especially of man-machine type. Several important examples of potential areas of implementation are presented here.

I. Consider a complicated multilevel technical system to be designed, e.g., a new commercial aircraft. Here the number of basic parameters which actually define the aircraft's utility, exceeds three by far; the basic parameters are as follows:

Table 5.1. System's model and PHM for project management systems

System's model	Parameters		Partial harmonization models
	Indep.	Dep.	
$G(N,A)$ - PERT-COST network; (i,j) - activity, $(i,j) \subset A \subset G(N,A)$; c_{ij} - budget assigned to (i,j); $c_{ij\ min}$, $c_{ij\ max}$ - lower and upper c_{ij} bounds; Total budget $C \geq \sum_{(i,j)} c_{ij\ min}$; Due date D; p.d.f. $t_{ij}(c_{ij}) = \frac{12}{(b_{ij} - a_{ij})^4}(t - a_{ij})(b_{ij} - t)^2$; $a_{ij} = \frac{A_{ij}}{c_{ij}}$, $b_{ij} = \frac{B_{ij}}{c_{ij}}$, A_{ij}, B_{ij} - const.; $T\{G \mid (c_{ij})\}$ - random project duration with assigned c_{ij}.	BUDGET C DUE DATE D	RELIABILITY R	Determine $c_{ij}^{(opt)}$ to $\underset{\{c_{ij}\}}{Max}\, R = \underset{\{c_{ij}\}}{Max}\left[Pr\{T\{G \mid (c_{ij})\} < D\}\right]$ subject to $c_{ij\ min} \leq c_{ij} \leq c_{ij\ max}$; $C = \sum_{\{i,j\}} c_{ij}^{(opt)}$; $R^{(opt)}\big|_{c_{ij}^{(opt)}} = R^{opt} = PHM(C,D)$.

- the budget assigned for constructing the new aircraft (an independent parameter);
- the number of passengers to be taken on board (an independent parameter);
- the flight distance (a partially dependent parameter);
- the average cruise speed (a dependent parameter);
- the reliability value, i.e., the probability of the aircraft within a specified exploitation period not to develop any critical failure which may result in air fleet accidents, sometimes of catastrophic nature (a dependent parameter);
- an environmental failure parameter, e.g., the level of noise (a dependent parameter);
- various technical design parameters, e.g., the aircraft's size, weight or even certain aesthetic features which nowadays may influence the aircraft's priority level

(usually dependent parameters), etc.

It goes without saying that increasing the number of basic parameters results in a dramatic increase of the level of complexity of the regarded harmonization model.

Table 5.2. System's model and PHM for hierarchical technical systems

System's model	Parameters: Indep.	Parameters: Dep.	Partial harmonization models
Fault Tree Simulation Model SM; Possible technical improvements $\{TI_k\},\ 1 \le k \le N$; Corresponding cost investments $\{C_k\},\ 1 \le k \le N$; Budget $C \ge \underset{k}{Min}\ C_k$; System's current reliability R_0;	RELIABILITY R	BUDGET C	$PHM_1(R) = C$: determine $\{TI_{\xi_q}\},\ 1 \le q \le Q \le N,\ \ \xi_q \le N$, to $\underset{\{\xi_q\}}{Min}\ C = \underset{\{\xi_q\}}{Min}\left[\sum_{q=1}^{Q} C_{\xi_q}\right]$ subject to $R_0 + \Delta R\left|\{TI_{\xi_q}\}\right. \ge R^*$.
System's desired reliability $R^* > R_0$; $\Delta R\left|\{TI_{\xi_q}\}\right.,\ 1 \le q \le Q \le N,\ \ \xi_q \le N$, is determined by means of SM.	BUDGET C	RELIABILITY R	$PHM_2(C) = R$: determine $\{TI_{\xi_q}\},\ 1 \le q \le Q \le N,\ \ \xi_q \le N$, to $\underset{\{\xi_q\}}{Max}\ R = \underset{\{\xi_q\}}{Max}\left[R_0 + \Delta R\left|\{\xi_q\}\right.\right]$ subject to $\sum_{q=1}^{Q} C_{\xi_q} \le C$.

II. In agriculture, e.g., in cotton harvesting, a multilevel decision-making control system is especially useful for cotton-growing areas with restricted resources [170]. Since all cotton harvesters are equipped with trailers, one of the independent basic parameters of the model should be the amount of trailers available to each harvester. Other basic parameters may be singled out as follows:

- the volume of the trailer (an independent parameter);

- the number of trailers used to form the so-called “cotton trains” delivering raw cotton to the cleaning factory (an independent parameter);
- the number of harvesters (an independent parameter);
- the weather forecast (a random disturbance parameter);
- the type and agricultural quality of soil (an independent parameter);
- the harvesting period for cotton (an independent parameter);
- the budget to be assigned for cotton harvesting in a cotton-growing district (a dependent parameter);
- harvesting expenses per square unit of plantation (a dependent parameter);
- the weight of cleaned cotton obtained from the above (a dependent parameter), etc.

The cotton harvesting organization system is, thus, an extremely complicated one. However, using harmonization models as suggested in this paper may result in significantly increasing the system’s utility.

III. A promising application area of the discussed theory and methodology lies in developing new approaches for designing hospitals (or providing capital investments for expanding existing medical health facilities) in rural areas [135]. The basic parameters to determine hospital’s utility may be listed as follows:

- the main costs of designing and building a new hospital (an independent parameter);
- the population to be serviced (an independent parameter);
- accessibility and the geographical distance from the hospital to most remote settlements (an independent parameter);
- the number of beds (a dependent parameter);
- various quality and quantity parameters of medical care (partially dependent parameters);
- the average number of days for a patient to stay in the hospital, i.e., the patient’s “turnover” value (a dependent parameter), etc.

Thus, a hospital is a good field for implementing harmonization trade-off problems. Note that within the last three decades numerous decision-making models on health care and health service have been described in various publications. However, attempts to define the hospital’s utility in its entirety have not been undertaken as yet.

IV. In recent years another important field for implementing the utility theory presents itself in the mobile communication business (M-Commerce). The harmonization trade-off to be optimized may be formulated as a compromise between capital investments in cellular telephones’ infrastructure (like the amount and capacity of transmitting stations), on one hand, and certain reliability parameters of providing services to local internet based enterprises (connectibility and accessibility to the web by mobile phone, throughput of information, etc.), on the other hand. Here the number of basic parameters seems to be lower than in examples outlined above, but the levels of dependency between those parameters have neither been formalized as yet, together with the system's formalized description.

§5.5 Main research stages to implement harmonization models

In order to apply harmonization models to practical OS the following research stages have to be undertaken.

Stage 1. Within the course of this stage the OS under consideration has to undergo a careful and thorough inspection in order to:

- determine all the information and the material flows which pass through the system, including income and outcome flows;
- determine all the coordination and control signals connecting various elements at different hierarchical levels;
- determine all control actions for the case when the organization system undergoes on-line control;
- determine the system's goals to be achieved in the course of the system's functioning;
- determine the main links between the system's hierarchical levels;
- determine the existing techniques for governing and monitoring subordinated hierarchical levels;
- determine the formalized description of the operations to be carried out at each hierarchical level;
- determine the main system's restrictions.

Thus, in short, implementing *Stage 1* results in undertaking preliminary inspection of the system.

Stage 2 has to be carried out in order to:

- single out all the essential, basic system's parameters which mostly affect the main quality and quantity estimates of its functioning;
- subdivide the previously obtained basic parameters into an independent and a dependent subsets;
- formalize functional dependencies between the basic parameters by means of statistical analysis, by applying theoretical approaches or by using expert information;
- determine the upper and lower bounds for all basic parameters;
- determine the system's utility on the basis of the values of essential parameters.

Note that *Stages 1* and *2* are usually carried out by system's analysts.

Stage 3 results in determining and, later on, simulating the main random factors entering the system, and the main random processes taking place within the system’s functioning. To carry out the stage one has to:

- single out all the random variables in order to determine their probability density functions;
- determine the random processes at each hierarchical level;
- develop a preliminary simulation model in order to connect all the stochastic processes entering the system;
- determine decision-making rules for essential events of the simulation model.

Stage 4 results in formalizing and developing *optimization models*, including both harmonization models and partial harmonization models. To implement the stage one has to:

- develop a simplified and high-speed version for optimizing independent parameters;
- develop a high-speed heuristic procedure (usually by means of simulation) to optimize dependent parameters on the basis of the independent ones;
- to undertake a search procedure to optimize the system’s utility.

Stages 3 and *4* have to be performed by system analysts who are qualified in harmonization modeling.

As far as we are concerned, nowadays there exist only two classes of OS being able and ready to accept the developed harmonization theory:

- semi-automated, man-machine technical systems under random disturbances which comprise several hierarchical levels and can be controlled by means of decision-making at inspection points only, and
- a broad spectrum of project management systems under random disturbances.

The reasons for the above conclusion are as follows:

- both classes of organization systems within the last five decades have already got accustomed to numerous trade-offs (cost – reliability trade-offs in technical systems, time – cost trade-offs in project management) being dealt with by means of optimization and simulation models;
- in spite of the poor techniques and the restriction to only two basic parameters, formalized descriptions of both systems have been carried out properly.

Thus, practically speaking, research Stages 1-3 as outlined above are a by-gone day for those systems, and implementing the last stage is not going to result in inflicting additional heavy cost losses. The gain to be obtained will be swift and effective.

In contrast to those systems, other OS are only starting to be formalized, i.e., are passing their baby-hood period. Since the broad scientific community is interested to enhance the development of new managing models, we suggest to take the same measures which have been taken so rapidly and so effectively in project management five decades ago: namely, we suggest to incorporate in each OS a research team or a department comprising various skilled scientists from different areas, including an expert team. The goal of such a research division would be to carry out the above four stages in order to formalize the system's functioning. One cannot hope that the success to be obtained may result from the enthusiasm of just a few individuals; but the anticipated benefit from implementing harmonization models can prove to be tremendous.

Chapter 6. Risk Management in Organization Systems

§6.1 Risk management techniques for large organization systems

6.1.1 General introduction and definitions

Risk is a major factor in organization systems (OS), especially for projects under random disturbances, e.g., various research and development (R&D) projects. In project management one usually refers to high levels of uncertainty as sources of risk [7, 39-40, 43, 45, 50, 61, 86, 115, 118, 122-123, 140, 156, 168, 180]. Principal sources of uncertainty include random variations in component and subsystem performance, inaccurate or inadequate data and the inability of proper forecasting. The following uncertainties can be taken into account in large OS [168]:

1. Uncertainty in scheduling. Changes in the environment that are impossible to forecast accurately at the outset of a project, are likely to have a critical impact on the length of certain activities.
2. Uncertainty in cost. Limited information on the duration of activities makes it difficult to predict the amount of resources required to complete them on schedule. This translates directly into an uncertainty in cost.
3. Technological uncertainty. This form of uncertainty is typically present in R&D projects where new (not well tested and approved) technologies, methods, equipment, and systems are developed or employed. Technological uncertainty may affect the schedule, the cost, and the ultimate success of the project.
4. Market regulations. New regulations may affect the market for a project, while certain changes in the policies of project management participating organizations may disrupt the project's implementation.
5. Human uncertainty stems from erroneous judgment in the course of designing the system by a human operator, especially when an emergency starts to develop.

Outlined below are some classical definitions related to risk in OS including project management [50]:

- Failure is the inability of a system, subsystem, or component to perform its required function.
- Quality assurance is the probability that a system, subsystem, or component will perform its intended function when tested.
- Reliability is the probability that a system, subsystem, or component will perform its intended function for a specified period of time or under normal conditions.
- Risk is a combination of the probability of an abnormal event or failure and the consequences of that event or failure to a project's success or system's

performance.

- Risk analysis (assessment) denotes any process and procedures of identifying, characterizing, quantifying, and evaluating risks and their significance.
- Risk management denotes any technique either used to minimize the probability of an accident or to mitigate its consequences with, for instance, good engineering design, good operating practices, or preventive maintenance.
- Uncertainty is a measure of limits of knowledge in a technical area, expressed as a distribution of probabilities around a point estimate. The four principal elements of uncertainty are statistical confidence (a measure of sampling accuracy), tolerance (a measure of the relevance of available information to the problem at hand), incompleteness and inaccuracy of the input data, and ambiguity in modeling the problem.

Once the risks are determined, managers must decide what levels are acceptable based on economic, political, and technological judgments. The decision can be controversial because it necessarily involves subjective judgments about costs and benefits of the project, the well-being of the organization, and the potential damage or liability.

Risk is tolerated at a higher level if the payoffs are high or critical to the organization. What-ever the level of risk finally judged acceptable, it should be compared with and, if necessary, used to adjust the risks calculated to be inherent in the project. The probability of failure may be reduced further by use of redundant or standby subsystems, or by parallel efforts during development. Also, managers should prepare to counter the consequences of failure or setbacks by devising contingency plans or emergency procedures.

Risks may be caused by several factors [148-149, 168]:

1. Technology. Since technology is expanding by rapid pace a new product may prove to be obsolete at the moment of the project's completion. In order to avoid this risk, the project management has a tendency to use the latest technological designs which in some cases may be unproven.
2. Complexity and Integration. Since modern complex systems are based on the integration of parts and subsystems, the interfaces between those components may be s source of risks.
3. Changes. Virtually all projects are subject to design changes through their life cycle [49, 122]. Those design changes may be risky since each change may have a different effect on the system and its components. As a result, the risk of integration may undergo an essential increase.

As a matter of fact, whenever the design process or the design itself deviates from current procedures and established techniques, technological risks are introduced. These risks can be related to the product design, to the process design, or to the design of the support system, and may vary widely in magnitude. For example, in product design a low-level risk might be one associated with a modification of an existing sub-assembly. A moderate-level risk would concern the design of a new product based on currently used technologies and parts (integration risks); a third, even higher level of risk is related to the use of new materials, such as ceramics, in a product that was previously fabricated out of conventional metal alloys.

It can be well-recognized that the risk management techniques comprise both risk assessment procedures together with a variety of techniques for evaluating cost and benefits of alternative projects or policies. The corresponding steps in risk management, thus, include determining objectives and goals for all project options, identifying constraints and developing measures of effectiveness of feasible alternatives. Thus, the main goal in project risk management is to evaluate the risks and benefits that are likely to result from project outputs. However, it is very difficult to generalize all the possible steps in project risk management. An enlarged scheme of risk assessment steps for subsystems engineering [162] is presented on Fig. 6.1.

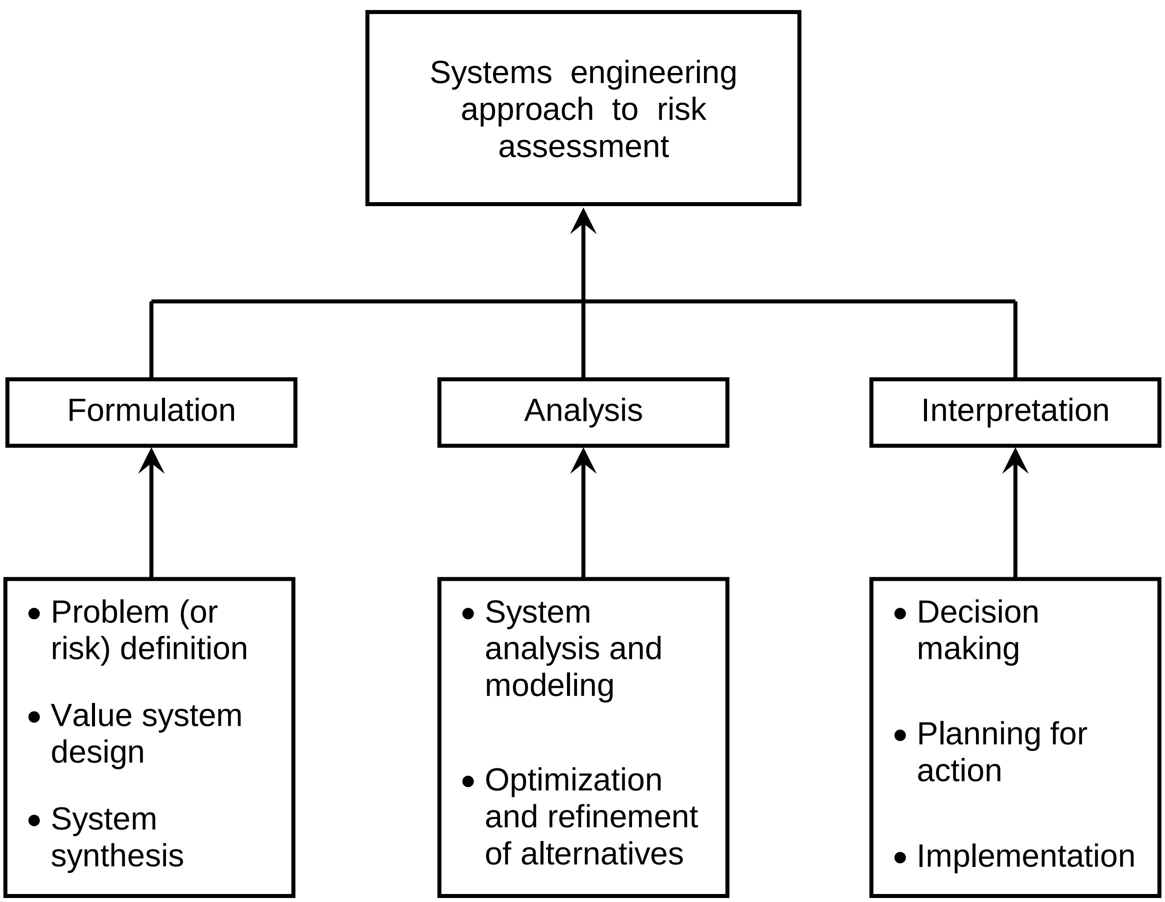

Figure 6.1. *Systems engineering approach to risk assessment*

The scheme under consideration comprises three primary steps: formulation, analysis and interpretation.

In risk formulation we determine (usually by means of experts) the types of anticipated risks. Note that risk formulation covers a very broad area of economical, organizational and technological risks.

The probability of success (or the risk of failure) should be estimated and monitored through the life cycle of any project, unique production plant and various other OS. The evaluation of alternative designs for a specific organization system, the decision to adopt or to reject proposed design changes as well as the implementation of such changes, are all parts of the risk management process. The latter deals usually with analyzing the probability of undesirable events and the consequences of those events.

In general, high risk corresponds to a strongly adverse event that has a high probability of occurrence, while low risk corresponds to a low probability of occurrence and low severity. Moderate levels of risk correspond to combinations of probabilities and consequences that fall between these extremes.

The level of risk in risk management depends both on the severity of an adverse output and its probability of occurrence. A project, e.g., may face a schedule risk related to the project's delay, a cost risk associated with the event of budget overrun. Multiple sources of risks and different aspects of organization systems that are subject to failure or delay, make risk management a demanding and important endeavor.

Besides the risk elements, all types of constraints have to be identified as well in the course of risk formulation.

In the analysis step, we forecast the failures which may occur in the course of implementing the project. All kinds of simulation modeling, mathematical programming, various trade-off models, etc., are all implemented in the analysis. It goes without saying that from the point of modeling and optimization, this step is the heart of risk assessment.

At the final step, corresponding organizational, technological, and design interpretations are assigned to any risk impact. Interpretation is usually carried out together with decision-makings. The latter are usually based on choosing the best alternative which includes the desirable combination of risk and benefit. Such a choice is also based on individual judgment by employing a group of trained experts.

As a matter of fact, nowadays there exists a large amount of different risk management and risk assessment approaches for various large-size projects. It can be well-recognized that most effective models and methods of project risk analysis are presented in [45].

6.1.2 Major risk analysis models for large projects

There exists nowadays a broad variety of risk analysis models for large projects. However, there is no single all-purpose risk analysis model, and each major project may be viewed as unique. The need for risk analysis is usually high when projects involve [45]:

- large capital outlays;
- unbalanced cash flows, requiring a large proportion of the total investment before any returns are obtained;
- significant new technology;
- unusual legal, insurance or contractual arrangements;
- important political, economic or financial parameters;
- sensitive environmental or safety issues;
- stringent regulatory or licensing requirements.

We will describe below the main risk analysis models for large projects.

I. Financial Models [40, 45, 118, 180]. Consider the main basic approaches which have been used for project risk analysis in various forms of financial models. The proposed methods use a set of measures of prediction about the project and its environment, e.g., initial capital expenditure, projected demand, projected market share, and projected cash flows. Those measures are combined in the project's financial model which can be used for decision-making. Such models comprise the following classical approaches [40]:

- discounted cash-flow models (DCF);
- cost – volume – profit analysis models (CVP).

The problem's solution may be obtained analytically by means of two-moments or four-moments approaches. In case when a large number of basic variables must be combined or where there are complex relationships between those variables, simulation modelling can be used in order to obtain approximate solutions.

II. Controlled Interval or Memory Models. More complicated methods, the so-called controlled interval and memory (CIM) models for combining independent risks, are outlined in [39, 45]. Various integration models (e.g., design plus procurement) are based on derived correction factors, interpolated correction factors, and other statistical and probability approaches. The effectiveness of CIM models is illustrated by an offshore pipeline project.

III. RAER Methods. A highly complicated risk analysis method for a project of liquefied natural gas (LNG) production and delivery system to be located in the High Arctic is a major example of risk analysis models for large projects. The authors suggest to use the critical path method (CPM) with activities of deterministic durations as the initial stage of implementing the risk analysis procedure. The CPM comprises:

- setting the list of activities;
- estimating activity durations;
- computing the project duration and activity float;
- identifying the critical path;
- undertaking time – cost trade-offs (similar to those outlined in 5.2.1-5.2.2);
- implementing the project's schedule and keeping the latter updated.

After determining the initial project planning model, a design of a reliability assessment method for LNG facilities is carried out. The so-called reliability analysis, evaluation and review (RAER) method [45] is defined in terms of four phases and a number of steps. The scope phase covers the risk formulation stage (see Fig. 6.1) and comprises the engineering review, the subsystems' and elements' identification (there are seven subsystems and twenty elements entering the LNG project), as well as other identifications (like outage source, effects and response).

The structure phase and the parameter phase cover the analysis stage (in terms of Fig. 6.1) and comprise minor risk identification, risk structure diagramming and the linkage, all probability estimations and conditional treatments. The manipulation and interpretation phase covers the final stage from Fig. 6.1 and comprises reliability computations and robustness analysis, sensitivity analysis and compatibility analysis. All the phases are carried out by analyzing possible alternative scenarios - combinations of possible risks associated with possible LNG subsystems and elements.

For example, reliability computations combined the three distributions associated with each outage source to define a "days of lost production" distribution, given one day of attempted operation. It then combined these distributions, first within elements, then across elements, to define a "days of lost production" distribution for the system as a whole, given one day of attempted operation.

Robustness analysis compared intermediate and final level outputs, given different assumptions and associated procedures with respect to the first step. For example, the adopted approach used an independent additive relationship when combining outage sources. This implies that overlapping incidents require sequential responses because of repair crew limitations.

Sensitivity analysis compared intermediate and final level outputs, given different probability distributions. For example, as the compressor in the LNG project has been recognized as the most significant source of potential outage, its estimated interval between incidents has been halved and doubled for comparative purposes.

Sparing analysis compared single and two-train compression systems and looked for other potential system plan improvements.

Compatibility analysis compared overall system reliability and availability results in a variety of forms with all available information on the overall reliability of other

LNG plants.

The research in [45] has been carried out by a large international team of risk analysts from U.S.A., Canada and the U.K. The corresponding software (the so-called SCERT) has been also used for choosing a river crossing method for a gas pipeline. Many different scenarios have been implemented in the project's realization, but the general results remain the same.

IV. Methods Based on Subjective Judgment. Various models of subjective judgment for risk analysis of large projects in the form of expert opinions are widely used [45, 150, 168]. For example, three different types of probabilistic information are usually considered on the basis of subjective experts' judgments:

(1) probabilities of particular sources of risk possible to occur;

(2) conditional probabilities of particular scenarios which may arise given the occurrence of a particular source of risk;

(3) consequence distributions, conditional on the occurrence of a particular risk.

This information has to be provided by a team of experts who are specialists in the risk area being assessed. All types of statistical adjustment and anchoring (taking into account the highly subjective level of the information) are illustrated in [45] on the basis of examples from tanks and pipes of LNG and other projects related to oil transportation systems.

V. Models of Statistical Risk Analysis. Another interesting risk analysis model for a highly complicated hydro-electronic development project covers the following areas of statistical analysis:

- statistical dependence between risks;
- determining risk – activity combinations.

The system's elements are configurations, states, risks, activities, damage scenarios, criteria and boundary conditions.

Note that such schedule risks as weather, seasons, water levels in rivers, etc., may cause essential delays. A variety of other risks (major design changes, labor problems, major floods, etc.) are considered as well.

Many other unique projects with corresponding engineering risk analysis are outlined in [45] but the general idea of risk analysis models remains unchanged (besides implementing local statistical sub-models to undertake research either for dependent and independent risks or for processes through fixed intervals of time or by the time required to complete a fixed amount of work).

VI. Decision-Trees Models. Decision trees, also known as decision flow networks and decision diagrams, are powerful means of depicting and facilitating the analysis of problems that involve sequential decisions and variable outputs over time. They have great usefulness in practice because they make it possible to look at a large complicated problem in terms of a series of smaller simple problems, while considering risk and future consequences.

A decision tree is a graphical method of expressing, in chronological order, the alternative actions that are available to the decision maker and the outputs determined by chance. In general, they are composed of the following two elements:

1) Decision nodes. At a decision node, usually designated by a square, the decision maker has to select one alternative course of action from a finite set of possibilities. Each is drawn by a branch emanating from the right side of the square. When there is a cost associated with an alternative, it is written along the branch. Each alternative branch may result in either a payoff, another decision node, or a chance node.

2) Chance nodes. A chance node, designated as a circle, indicates that a random event is expected at this point in the process. That is, one of a finite number of states of nature may occur. The states of nature are shown on the tree as branches to the right of the chance nodes. The states of nature may be followed by payoffs, decision nodes, or more chance nodes.

A tree is started on the left of the page with one or more decision nodes. From these, all possible alternatives are drawn branching out to the right. Then a chance node or second decision node, associated with either subsequent events or decisions, respectively, is added. Each time a chance node is added, the appropriate states of nature with their corresponding probabilities emanate rightward from it. The tree continues to branch from left to right until the final payoffs are reached. The tree shown in Fig. 6.2 [168] represents a single decision with two alternatives, each leading to a chance node with three possible states of nature.

Decision trees have been effectively used in various projects, e.g., constructing a major Arctic pipeline, an offshore pipeline, etc. [45].

A general conclusion can be drawn from analyzing the outlined above project risk models [45, 135]:

I. A risk analysis study requires many forms of expertise: economics, finance, medicine, environmental issues. The study is always undertaken by a qualified team management within a relatively long period (up to 10-12 weeks).

II. The general idea for solving practically any risk analysis problem for large projects involves several enlarged steps:

  1. Specify the possible system's risks in combination with the corresponding system elements.

  2. Specify the possible relationships between them.

  3. Determine alternative scenarios comprising risky situations.

  4. Calculate the desired variables (goal risk functions, objectives) for each scenario.

5. Choose the best scenario from the point of the project's goal function.

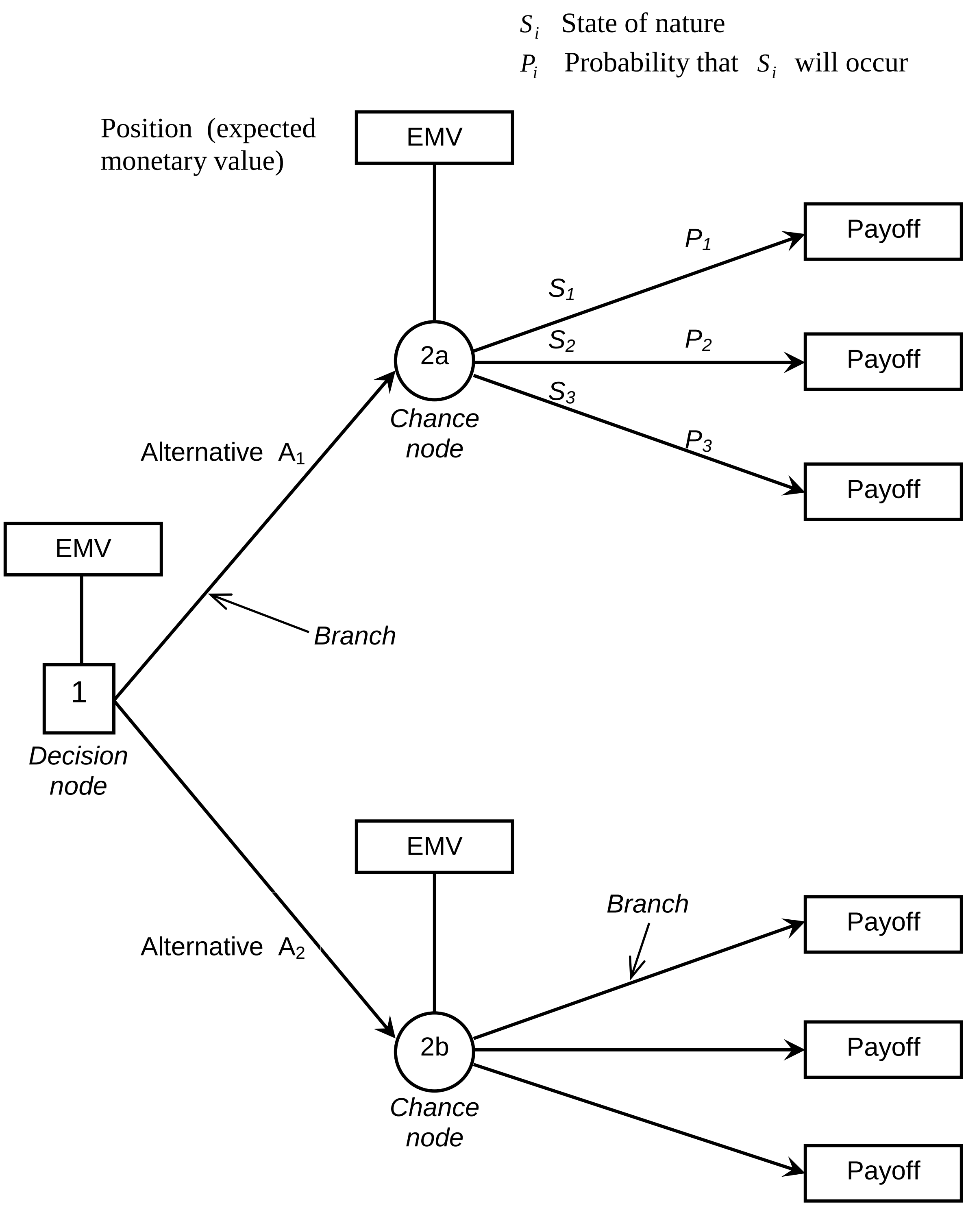

Figure 6.2. *<u>Structure of a decision tree</u>*

However, individual features of each project result in different models to be implemented on each step. Thus, while being similar from the point of general concepts, risk analysis models comprise entirely different sub-models even for relatively similar projects: a couple of additional risks may change the risk structure together with the relationships and, later on, possible alternative scenarios.

Thus, similar projects may result in different risk analysis models.

III. All projects comprising:

- new technology,
- large capital investments before any returns are obtained,
- complexity and integration,
- design changes,
- uncertainty in marketing, i.e., in the course of re-distributing large quantities of finished project's products,
- political, economical and financial considerations,
- construction or operation in new or hazardous geographical and geological areas,

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- have to be serviced by a team of risk analysts.

IV. For risks to be ascertained at all, project managers have to agree on the value of assessing them in engineering design. A conception exists [168] that a project manager can realize a poor quality project with no essential design, but he cannot enhance a poor design by a high quality project. We cannot fully agree with such an opinion, since any project (even without technological risks and future marketing uncertainties) requires good quality managing and control. This refers to a large spectrum of projects including all public service projects, e.g., construction of hospitals, bridges, theatres, stadiums, new populated areas, etc.

In addition, we have undertaken a comparative study of another approach - the so-called information gap uncertainty models [9, 149], in order to examine possible points of similarity in application areas and theoretical backgrounds. The information gap models deal with uncertainty being typical for unique engineering constructions under drastic nature's disturbances, e.g., seismic ground motions, various shape defects, etc. The type of uncertainty one often faces in technological design and analysis can be described as a gap between what is known and what is unknown. The quantification of this disparity leads to the convex info-gap model of uncertainty. However, those models have solely engineering applications and do not deal with organization systems. They are based on Banach spaces, Bayesian decision method, on the theory of statistical hypothesis to generate decision rules with severely deficient information, and are not contiguous with any points of the harmonization theory both in application areas and modeling techniques.

§6.2 Hazardous failures in risk management

It can be well-recognized that there exists a very important area where risk management, safety engineering and reliability modeling meet together. This is the area of hazardous failures on multi-level industrial plants.

In order to compare later on the risk management for hazardous failures and the harmonization modeling techniques, we will describe the existing hazard evaluation techniques for performing detailed analysis of a wide range of hazards during the detailed design of the process and after the process, in operation. These approaches are also used to identify hazardous situations.

The following hazard evaluation (HE) techniques can be typically applied in various industrial plants [6-9, 61-64, 86, 113, 115, 155-156, 162-163, 179, 185]:

- Safety Review
- Checklist Analysis
- Relative Ranking
- Preliminary Hazard Analysis
- What-If Analysis
- What-If / Checklist Analysis
- Hazard and Operability Analysis
- Failure Modes and Effects Analysis
- Fault Tree Analysis
- Event Tree Analysis
- Cause-Consequence Analysis
- Human Reliability Analysis

The Safety Review technique boils down to a detailed inspection to identify hazardous process design characteristics, plant conditions, operating practices, or maintenance activities. Using the Safety Review technique to conduct periodic inspections of an operating plant helps ensure that implemented risk management programs meet original expectations and standards. These inspections keep operating personnel alert to process hazards, since they must respond to questions from a knowledgeable inspection team. The Safety Review seeks to identify operating procedures that need to be revised, equipment or process changes that may have introduced new hazards, and inadequate maintenance or replacement of equipment. A Safety Review may also give the analyst opportunities to apply new technology to eliminate an existing hazard or reduce process risk.

A Safety Review consists of three steps:

1) preparing for the review,
2) performing the review, and
3) documenting the results.

In a traditional Checklist Analysis the hazard analyst uses a list of specific items to identify known types of hazards, design deficiencies, and potential accident situations associated with common process equipment and operations. The Checklist Analysis technique can be used to evaluate materials, equipment, or procedures. Checklists are most often used to evaluate a specific design with which a company or industry has a significant amount of experience, but they can also be used at earlier stages of development for entirely new processes to identify and eliminate hazards that have been recognized through years of operation of similar systems.

Proper use of a checklist will generally ensure that a piece of equipment conforms to accepted standards, and it may also identify areas that require further evaluation.

Once the scope of the analysis has been defined, a Checklist Analysis consists of three main steps:

1) selecting or developing an appropriate checklist,
2) performing the review, and
3) documenting the results.

Relative Ranking techniques for hazard evaluation rank process areas or plant operations by comparing the hazardous attributes of chemicals, process conditions, and operating parameters. Sometimes, Relative Ranking is used to compare process siting or design alternatives. Generally, a Relative Ranking technique attempts to distinguish between several process areas based on the magnitude of hazards, likelihood of accidents, and/or severity of potential accidents. The methods to do this vary widely in form and complexity, and can be both qualitative and quantitative. For example, analysts may create a simple ranking of process areas based on qualitative rating of expected magnitudes of hazards, likelihoods of accidents, and/or severity of accidents. Or, a more complex numerical scheme, which assigns numerical values to process characteristics, may be used to calculate numerical ranking factors.

The process of identifying and potentially quantifying parameters for a specific Relative Ranking technique may be more prescriptive than most other hazard evaluation approaches. Thus, applying some Relative Ranking techniques may require less subjective judgments, allowing either a novice hazard analyst or a knowledgeable process engineer to successfully perform the evaluation. On the other hand, because of its unique characteristics, a specific Relative Ranking technique developed by an organization may require the analysts to have a lot of experience with that particular technique before they can make the judgments necessary to correctly rank the hazards.

The Preliminary Hazard Analysis (PHA) technique was developed by the U.S. army [48]. It is customarily performed during the process plant's conceptual design or siting phases or during early development to determine any hazards that exist. A PHA does not preclude the need for further hazard assessment; in fact, it is usually a precursor to subsequent hazard evaluation studies. There are two principal advantages to using the PHA technique early in the life cycle of a process:

1) it can identify potential hazards at a time when they can be corrected at minimal cost and disruption, and
2) it can help the development team identify and/or develop operating guidelines that can be used throughout the life cycle of the process.

Thus, the principal hazards can be eliminated, minimized, or controlled from the start. A PHA can also be carried out on an existing facility when a broad-brush analysis of hazards and potential accident situations is required.

In a PHA, the team lists the basic elements of the system and the hazards of interest, which have been defined in the conceptual design stage.

The What-If Analysis technique is a creative, brainstorming examination of a process or operation. Hazard analysts review the subject process or activity in meetings that revolve around potential safety issues identified by the analysts. Each member of the HE team is encouraged to vocalize What-If questions or specific issues that concern them. The What-If Analysis technique can be used to examine virtually any aspect of facility design and operation (e.g., buildings, power systems, raw materials, products, storage, material handling, in-plant environments, operating procedures, work practices, management practices, plant security, and so forth). It is a powerful HE technique if the analysis staff is experienced; otherwise, the results are likely to be incomplete. What-If Analysis of simple systems can easily be conducted by one or two people; a more complex process would require a larger team and longer or more meetings.

The What-If / Checklist Analysis technique is a combination of the two previously discussed HE methods: What-If Analysis and Checklist Analysis. The method is usually performed by a team of personnel experienced with the subject process. The team uses the What-If Analysis technique to brainstorm the various types of accidents that can occur within the process. Then the team uses one or more checklists to help fill in any gaps that they have missed. The checklists used in this portion of the analysis differ somewhat from traditional checklists of desired design, procedural, and operating attributes. Rather than focusing on a specific list of design or operating features, checklists used in What-If / Checklist Analysis are more general and focus on sources of hazards and accidents. These checklists are intended to inspire creative thought about the types and sources of hazards associated with the process.

The combined use of these two methods emphasizes their main positive features (i.e., the creativity of What-If Analysis and the experience-based thoroughness of a checklist) while at the same time compensating for their shortcomings when used separately.

The What-If / Checklist Analysis technique can be used for any type of process or activity at virtually any stage in the life cycle of the process. Normally, the method is used to examine the potential effects of accident situations at a more general level than some of the more detailed approaches.

The Hazard and Operability Analysis technique (HAZOP) [8-9, 110, 179] is based on the principle that several experts with different backgrounds can interact in a creative, systematic fashion and identify more problems when working together than when working separately and combining their results. Although the HAZOP Analysis technique was originally developed for evaluation of a new design or technology, it is applicable to almost all phases of a process's lifetime.

The essence of the HAZOP Analysis approach is to review process drawings and/or procedures in a series of meetings, during which a multi-disciplinary team uses a prescribed protocol to methodologically evaluate the significance of deviations from the normal design intention. Imperial Chemical Industries (ICI) originally defined the HAZOP Analysis technique to require that HAZOP studies be performed by an inter-disciplinary team. Thus, while it is possible for one person to implement the HAZOP Analysis thought process, such a study cannot be actually called a HAZOP Analysis. Therefore, the HAZOP Analysis technique is distinctively different from other HE methods because, while other approaches can be carried out by single analysts (although in most cases, it is better to use an inter-disciplinary team), HAZOP Analysis, by definition, must be performed by a team of individuals with the specific, necessary skills.

Note that all techniques described above do not deal with models or even simplified algorithms, but rather with reviews implemented by qualitative analysts.

Failure Modes and Effects Analysis (FMEA) [110, 115, 177] evaluates the ways equipment can fail (or be improperly operated) and the effects these failures may have on the process. These failure descriptions provide analysts with a basis for determining where changes can be made to improve a system design. During FMEA, hazard analysts describe potential consequences and relate them only to equipment failures; they rarely investigate damage or injury that could arise if the system operated successfully.

Each individual failure is considered as an independent occurrence, with no relation to other failures in the system, except for the subsequent effects that it might produce. However, under special circumstances, common cause failures of more than one system component may be considered. The results of an FMEA are usually listed in tabular format, equipment item by equipment item. Generally, hazard analysts use FMEA as a qualitative technique, although it can be extended to give a priority ranking based on failure severity.

A typical FMEA procedure contains three steps:

1) defining the study problem,
2) performing the review, and
3) documenting the results.

As described in greater details below, in §6.3, a Fault Tree [8-9, 62-64, 86, 114-115, 119, 155, 163, 179, 181, 185] is a graphical model that illustrates combinations of failures that might cause one specific major failure of interest, called the top event. Fault Tree Analysis (FTA) is a deductive technique that uses Boolean logic symbols (i.e., AND gates, OR gates) to break down the causes of a top event into basic equipment failures and human errors (called basic events). The analyst begins with an accident or undesirable event that is to be avoided, and identifies the immediate causes of that event. Each of the immediate causes (called fault events) is further examined in the same manner until the analyst has identified the basic causes of each fault event, or reaches the boundary established for the analysis. The resulting fault tree model displays the logical relationships between basic events and the selected top event.

Top events are specific hazardous situations that are typically identified through the use of a more broad-brush HE technique (e.g., What-If Analysis, HAZOP Analysis). A fault tree model can be used to generate a list of the failure combinations (failure modes) that may cause the top event of interest. These failure modes are known as cut sets. A minimal cut set (MCS) is the smallest combination of component failures which, if they all occur or exist simultaneously, might cause the top event.

Thus, the fault tree is a graphical representation of the relationships between failures and a specific accident.

Event Tree Analysis [8-9, 110, 179] evaluates the potential for an accident that is the result of a general type of equipment failure or process upset (known as an initiating event). Unlike Fault Tree Analysis (a deductive reasoning process), Event Tree Analysis is an inductive reasoning process where the analyst begins with an initiating event and develops the possible consequences of events that lead to potential accidents, accounting for both the successes and the failures of any associated safety functions as the accident progresses. Event trees provide a systematic way of recording the accident sequences and defining the relationships between the initiating events and subsequent events that result in accidents.

Event trees are well suited for analyzing initiating events that could result in a variety of outputs. An event tree emphasizes the initial cause of potential accidents and works from the initiating event to the event's final effects. Each branch of the event tree represents a separate accident sequence that is a clearly defined set of functional relationships between the safety functions for an initiating event.

Cause-Consequence Analysis (CCA) [8-9, 110, 177] combines the inductive reasoning features of Event Tree Analysis with the deductive reasoning features of Fault Tree Analysis. The result boils down to a technique that relates specific accident consequences to their many possible causes. The advantage of this technique is that it uses a graphical method that can proceed in both directions: forward, towards the consequences of an event, and backward, towards the basic causes of an event. The main disadvantage of CCA is that only simple models can be easily displayed, since the combined fault tree and event tree diagram are somewhat cumbersome. The result of CCA is a cause-consequence diagram that displays the relationships between accident consequences and their basic causes. The solution of the cause-consequence diagram for a particular accident sequence is a list of accident sequence minimal cut sets. These sets are analogous to fault tree minimal cut sets because they represent all of the combinations of basic causes that can result in the accident sequence.

Human Reliability Analysis [7-9, 50, 110, 115] might be necessary for the success of human-machine systems and is influenced by many factors. These performance shaping factors (PSFs) may be internal attributes such as stress, emotional state, training, and experience, or external factors such as work hours, environment, actions by supervisors, procedures, and hardware interfaces. The number of PSFs that affect human performance is almost infinite. While some PSFs cannot be controlled, many can be, thus significantly influencing the success or failure of a process or operation.

Thus, practically all hazard evaluation techniques do not deal with any trade-off optimization model. However, on several stages of implementing HE procedures the appropriate techniques may collect input information for trade-off modeling and risk management (usually the Fault Tree Analysis and the Event Tree Analysis).

In the next *Paragraph* we will outline the basic structure of a complicated simulation model based on both Fault Tree and Event Tree Analysis.

§6.3 Risk management estimates for man-machine organization systems via fault tree simulation

6.3.1 Introduction

Assume that the OS under consideration comprises n hierarchical levels with one element at the top level and numerous elements at the n-th, bottom, level. Each element E_{im} entering the system's branch tree is indicated by two indices:

m - the ordinal number of the system's hierarchical level, and

i - the ordinal number of the element at the m-th level.

Note that m satisfies $1 \le m \le n$, while elements E_{in} are called basic or primary elements. A basic element cannot be subdivided into several parts, each of which functions normally, i.e., a basic element is technically indivisible.

On the basis of the system tree, a fault tree is constructed [6, 8-9, 64, 86, 113]. The general idea is to put each element E_{im}, $1 \le m \le n$, in correspondence with a set of failures which may occur within the element's work. Thus, an element of the fault tree F_{ijm} is formalized by three indices, namely, i, j, and m. Value j denotes the j-th type possible failure which may occur within the element's E_{im} work. Here, j varies from 1 to d_{im}, where value d_{im} is the number of possible failures. Thus, m is the ordinal number of the level, i is the ordinal number of the system's element at that level, and j is the ordinal number of the failure of the element. Denote by k_m the total number of elements at the m-th level, $1 \le m \le n$. Thus, the fault tree elements F_{ijm} are sorted as follows (for a fixed m–th hierarchical level):

1. Value i increases from 1 to k_m.
2. Within each value i, value j increases from 1 to d_{im}.

Thus, at the m-th level, the fault tree elements vary from F_{11m} to $F_{1d_{1m}m}$, further on from F_{21m} to $F_{2d_{2m}m}$, etc., up to $F_{k_m d_{k_m m} m}$.

Assume that at the bottom level, each fault tree element F_{ijn} has its probability to occur, namely P_{ijn}. Thus, P_{ijn} is the probability of the j-th failure of element E_{in}. These probabilities are pregiven and obtained either by expert estimation, or by using statistical analysis within the system's functioning.

If a certain failure at the bottom level occurs, it may affect some elements at higher hierarchical levels and, thus, result in secondary failures of those elements. In [9, 86] the concept of a failure signal is introduced which, with a certain probability, leaves a fault tree element F_{ijn} at the bottom level and enters another element $F_{i^* j^* m}$, $m < n$, at a higher level. Such a signal is fully denoted by a vector $\vec{S} = S\left(i, j, n, i^*, j^*, m, p\right)$. Note that several failure signals with different probabilities may leave one and the same fault tree element; these signals are income signals for different elements (at one and the same level or at different levels). Those probabilities of failure signals are called output probabilities

$$p = P\left\{i, j, n, i^*, j^*, m\right\}, \quad m < n. \tag{6.3.1}$$

The probabilities of output signals leaving one and the same fault tree element, are not obliged to represent a full group of events. Each of those output probabilities has only to be more than 0 and not more than 1.

For certain fault tree elements F_{ijm}, $m < n$, at intermediate levels, there exists a group of not less than one income signal entering those elements. By determining pregiven logical rules (via special lists of logical relations for combinations of income signals entering each element F_{ijm}, $m < n$, of the fault tree) one may undertake decision-making as follows: does a combination of income signals $\{S\}$ entering a routine element F_{ijm} cause a failure or not? Note that in cases of complicated logical relations, it is sometimes difficult to use an algorithm based on implementing only the classical logical relations "OR" and "AND" at the element's receiver. But the corresponding algorithms can be easily realized by means of a simple look-over of such "logical lists". Thus, a failure or, on the contrary, its absence, may be easily justified. If a failure occurs, it may cause new output signals to other elements, etc. Thus, the fault tree may be simulated, according to all implemented logical links, from bottom to top, i.e., until the top fault tree elements. The simulation model enables calculating the probabilities of all failures at the top level and, thus, determining the system's reliability parameters.

It can be well-recognized via simulation that the probabilities of a critical failure at the top level depend mainly on certain primary failure probabilities at the bottom level. Thus, increasing the reliability of the corresponding elements at the bottom level results in increasing the overall system's safety level. Note that most primary failure probabilities can be decreased by introducing corresponding technical alterations which require the layout of expenditure. The latter may be calculated in advance, either by using experts, or on the basis of statistical analysis of similar technical systems.

Note that in the last three decades most large-scale hierarchical industrial plants of chemical or nuclear types and with possibility of hazardous failures at the top level, are using a wide-range library of fault trees and corresponding simulation models [6, 8-9, 64, 86, 110, 113, 155-156, 185, etc.]. This enables solving problems of calculating possible decrease of the top hazardous failures' probabilities by implementing preset technical improvements at the system's bottom level. However, any results in trade-off optimization, e.g., determining an optimal sub-set of improvements in order to minimize the probability of a top hazardous failure with restricted cost expenditures, have not been published as yet.

As a matter of fact, that fault tree models may not adequately describe other hierarchical production systems, e.g., energy production systems with generators, turbines, steam generator valves, pumps, etc. For such systems the number of possible states exceeds essentially the possibilities of a branching system.

In order to simulate certain operations, one has to validate and to suggest the most reasonable probabilistic laws of the operations' durations. The reader may use the corresponding justifications presented above, in Chapter 2.

6.3.2 Notation

Let us introduce the following terms [86]:

n - number of hierarchical levels of the system's technological branch tree;

E_{im} - i-th element of the m-th level of the branch tree, $1 \le i \le k_m$;

k_m - number of elements of the branch tree at the m-th level;

d_{im} - number of failures of element E_{im};

$F_{i,j,m}$ - j-th failure of element E_{im}, $1 \le j \le d_{im}$ (in case $m = n$, the failure has pregiven probabilities, in case $m < n$ it depends on the income failure signals);

$P_{i,j,n}$ - probability of failure $F_{i,j,m}$ at the bottom level (pregiven for all $P_{i,j,n}$, $1 \le i \le k_m$,

$1 \le j \le d_{in}$);

$S\{i_1, j_1, m_1, i_2, j_2, m_2, p\}$ - output failure signal leaving $F_{i_1 j_1 m_1}$ and entering $F_{i_2 j_2 m_2}$ ($m_2 < m_1$, $m_1 > 1$) on condition that:

a) failure $F_{i_1 j_1 m_1}$ has actually occurred, and

b) later on, the signal is realized with probability p.

$OFSL(i, j, m)$ - the list of all possible output failure signals $S(i, j, m, i^*, j^*, m^*, p^*)$ leaving $F_{i,j,m}$ (pregiven for all $F_{i,j,m}$, $m > 1$);

$LIR(i, j, m)$ - logical input rules for all input failure signals entering $F_{i,j,m}$, $m < n$, given as follows:

$$\begin{bmatrix} \left(i_1^{(1)}, & j_1^{(1)}, & m_1^{(1)}, & i, j, m\right) \\ \left(i_2^{(1)}, & j_2^{(1)}, & m_2^{(1)}, & i, j, m\right) \\ \dots & \dots & \dots & \dots \\ \left(i_{q^1}^{(1)}, & j_{q^1}^{(1)}, & m_{q^1}^{(1)}, & i, j, m\right) \end{bmatrix} OR \begin{bmatrix} \left(i_1^{(2)}, & j_1^{(2)}, & m_1^{(2)}, & i, j, m\right) \\ \left(i_2^{(2)}, & j_2^{(2)}, & m_2^{(2)}, & i, j, m\right) \\ \dots & \dots & \dots & \dots \\ \left(i_{q^2}^{(2)}, & j_{q^2}^{(2)}, & m_{q^2}^{(2)}, & i, j, m\right) \end{bmatrix} OR \dots OR \begin{bmatrix} \left(i_1^{(s)}, & j_1^{(s)}, & m_1^{(s)}, & i, j, m\right) \\ \left(i_2^{(s)}, & j_2^{(s)}, & m_2^{(s)}, & i, j, m\right) \\ \dots & \dots & \dots & \dots \\ \left(i_{q^s}^{(s)}, & j_{q^s}^{(s)}, & m_{q^s}^{(s)}, & i, j, m\right) \end{bmatrix} \quad (6.3.2)$$

result in realizing failure $F_{i,j,m}$. Square brackets [ ] mean, that all signals in those packages have to be realized. Values $q^1, q^2, ..., q^s$ defining the amounts of signal packages and the number of those packages, are individual for failure $F_{i,j,m}$. All other groups of signals which do not satisfy (6.3.2), do not result in failure of $F_{i,j,m}$.

Note that such logical input rules are very easy to use. One has only to compare the signal packages in the square brackets with the actual failure signals entering $F_{i,j,m}$ (call it henceforth AFS). If a signal package in at least one of the square brackets is a part of AFS, failure $F_{i,j,m}$ occurs.

$P\{F_{i,j,1}\}$ - probability of top fault tree events (to be calculated);

$N(i, j, 1)$ - the number of times failure $F_{i,j,1}$ has occurred within N fault tree simulation runs.

6.3.3 The simulation algorithm

The enlarged step-by-step procedure of the algorithm [86] is as follows:

Step 0. Initial data accumulation; this includes values n, $k_1, k_2, ..., k_n$; values $d_{1n}, d_{2n}, ..., d_{k_n n}, d_{1,n-1}, d_{2,n-1}, ..., d_{k_{n-1} n-1}, ..., d_{11}, d_{21}, ..., d_{k_1 1}$ (note that certain values d_{in} may be equal to zero); values $P_{i,j,n}$, $1 \le i \le k_m$, $1 \le j \le d_{in}$; values $P\{\overleftrightarrow{S}\}$; lists of OFSL and LIR for all $F_{i,j,m}$ (see *6.3.2*).

Step 1. Set counter $m = n$.

Step 2. Set counter $i = 1$.

Step 3. Set counter $j = 1$.

Step 4. Does relation $j > d_{im}$ hold? If not, go to the next step. Otherwise, apply Step 10.

Step 5. Check, does relation $m = n$ hold? If yes, apply the next step. Otherwise go to Step 15 (in case $m > 1$) or to Step 16 (in case $m = 1$).

Step 6. Simulate the occurrence of failure with probability $P_{i,j,m}$. If such an event occurs, apply the next step. Otherwise go to Step 9.

Step 7. Simulate the realization of all possible output signals from $OFSL(i, j, m)$ leaving $F_{i,j,m}$. The simulation is carried out by comparing the random value α, uniformly distributed in $[0,1]$, with the output probability p. If $\alpha < p$, then the corresponding output signal is realized. This procedure is implemented for all output probabilities belonging to $OFSL(i, j, m)$.

Step 8. For all realized output signals from element $F_{i,j,m}$, check their input elements (i^*, j^*, m^*) and send the output address (i, jm) to the above outlined AFS area for all receivers F_{i^*,j^*,m^*}.

Step 9. Counter $j + 1 \Rightarrow j$ works. Go to Step 5.

Step 10. Counter $i + 1 \Rightarrow i$ works.

Step 11. Does relation $i > k_m$ hold? If not, apply the next step. Otherwise, go to Step 13.

Step 12. Set $j = 1$; go to Step 4.

Step 13. Counter $m - 1 \Rightarrow m$ works.

Step 14. Examine value m. In case $m > 1$ go to Step 2. If $m = 1$ apply Step 16. If $m = 0$ go to Step 17.

Step 15. Applying this step means that we are simulating secondary failures $F_{i,j,m}$. Examine the logical input list $LIR(i, j, m)$ (see *6.3.2*) and implement decision-making: does failure $F_{i,j,m}$ occur or not? If not, go to Step 9. Otherwise, apply Step 7.

Step 16. Applying this step means that we are simulating the top level failure $F_{i,j,1}$. Similarly to Step 15, examine $LIR(i, j,1)$ and take the decision regarding $F_{i,j,1}$'s realization. Go to Step 9.

Step 17. Clear all counters m, i, and j and simulate Steps 1÷16 N times in order to obtain representative statistics. Determine the number $N(i, j,1)$ for all failures $F_{i,j,1}$ at the top level.

Step 18. Calculate empirical frequencies $\overline{p}(i, j,1) = N(i, j,1)/N$ for all failures $F_{i,j,1}$. Assume $P\{F_{i,j,1}\}$ equal to $\overline{p}(i, j,1)$.

Step 19. End of the algorithm.

6.3.4 Reliability analysis by use of fault tree simulation

In [110, 115, 177] intensive research has been undertaken to evaluate the influence of primary failures $F_{i,j,n}$ on critical failures at the top fault tree level. Such a problem can be solved by using the simulation algorithm outlined above. The suggested methodological approach is as follows:

Step 1. For the OS under consideration, implement on the basis of the corresponding fault tree $F_{i,j,m}$, N simulation runs, in order to obtain a representative sample, to estimate values $P_{i,j,1}$. Step 1 has to be implemented in accordance with the above outlined simulation algorithm.

Step 2. "Close" failure $F_{1,1,n}$, i.e., set the probability value $P_{1,1,n}$ equal to zero. Undertake N simulation runs for such a modified fault tree, and determine new estimated values $P^*\{i, j,1/1,1,n\}$.

Step 3. Compare the relative difference

$$\frac{1}{P\{i, j,1\}}\left[P\{i, j,1\} - P^*\{i, j,1/1,1,n\}\right] \tag{6.3.3}$$

for "critical" failures at the top level.

If the difference is non-essential (e.g., does not exceed the pregiven tolerance $\varepsilon > 0$), a conclusion can be drawn that the primary failure $F_{1,1,n}$ has a small influence on top critical failures. Otherwise, the corresponding basic element E_{1n} requires refinement.

Step 4. The procedure of Steps 1-3 has to be implemented for all primary failures $F_{i,j,n}$, $1 \le i \le k_n$, $1 \le j \le d_{in}$, at the bottom level. Thus, all primary failures with an essentially "straightforward" influence on top critical failures, can be singled out.

Step 5. If, for some top critical failures $F_{i,j,1}$, their corresponding probabilities $P\{i, j,1\}$ are essential, but the calculated "straightforward" influence from primary failures (6.3.3) can be regarded as non-essential, a more detailed statistical analysis has to be undertaken. We suggest analyzing the influence of primary failures unified in couples, triples, squads, etc. Perhaps an even more complicated and sophisticated multifactor analysis has to be implemented. All this has to be carried out by means of simulation modeling.

6.3.5 Fault tree example

An example of a three-level fault tree is presented in Fig. 6.3.

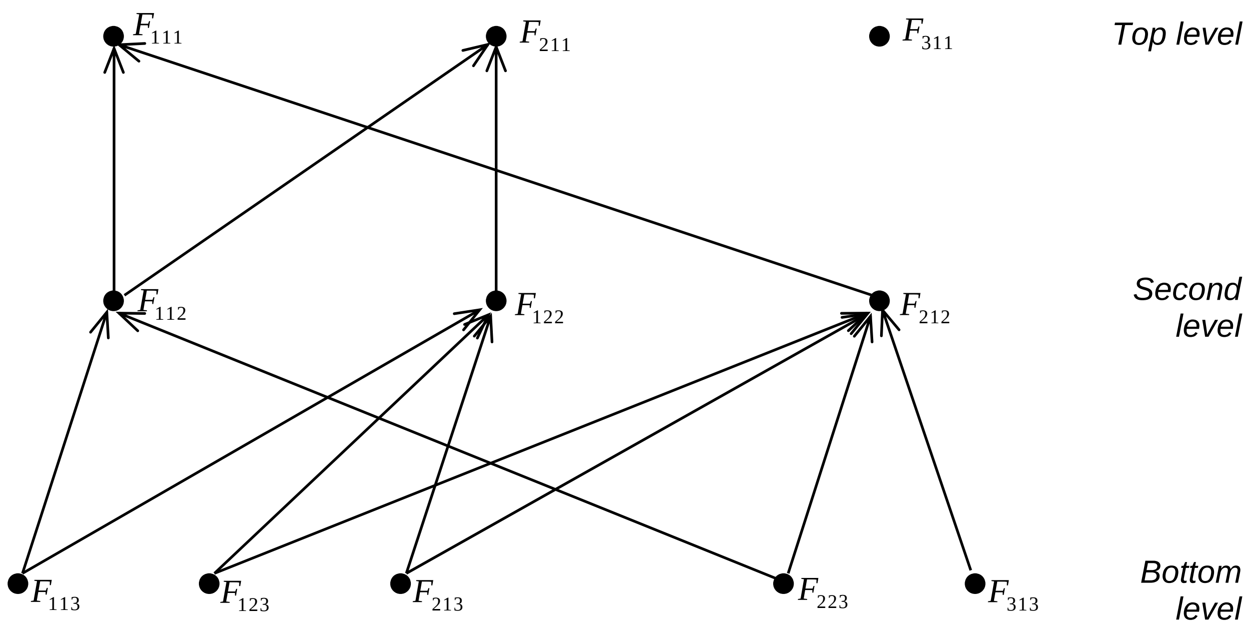

Figure 6.3. A fault tree

The top level failures represented on Fig. 6.3 may be classified as follows:

- F_{111} is a "critical" failure;
- F_{211} is a non-essential failure;
- F_{311} designates a failure that has not been realized (absence of failure).

Secondary failure F_{112} occurs when both input failure signals are realized. Failure F_{122} occurs on condition that at least one failure signal entering F_{122} is realized. Failure F_{212} occurs when:

- all three input signals $S(1,2,3,2,1,2)$, $S(2,1,3,2,1,2)$ and $S(1,2,3,2,1,2)$ are realized together, or
- at least one signal $S(3,1,3,2,1,2)$ is realized.

Top failures F_{111} and F_{211} occur if at least one input failure signal is realized.

The fault tree initial data is as follows:

$1 \le m \le 3$; $k_1 = 3$, $k_2 = 2$, $k_3 = 3$; $d_{11} = 1$, $d_{21} = 1$, $d_{31} = 1$, $d_{12} = 2$, $d_{22} = 1$, $d_{13} = 2$, $d_{23} = 2$, $d_{33} = 1$;

$P_{113} = 0.25$, $P_{123} = 0.08$, $P_{213} = 0.10$, $P_{223} = 0.05$, $P_{313} = 0.04$;

$$OFSL(1,1,3) = \begin{Bmatrix} (1,1,3,1,1,2,0.50) \\ (1,1,3,1,2,2,0.60) \end{Bmatrix}, \qquad OFSL(1,2,3) = \begin{Bmatrix} (1,2,3,1,2,2,0.40) \\ (1,2,3,2,1,2,0.80) \end{Bmatrix},$$

$$OFSL(2,1,3) = \begin{Bmatrix} (2,1,3,1,2,2,0.60) \\ (2,1,3,2,1,2,0.50) \end{Bmatrix}, \qquad OFSL(2,2,3) = \begin{Bmatrix} (2,2,3,1,1,2,0.90) \\ (2,2,3,2,1,2,0.50) \end{Bmatrix},$$

$$OFSL(3,1,3) = (3,1,3,2,1,2,0.70), \qquad OFSL(1,1,2) = \begin{Bmatrix} (1,1,2,1,1,1,0.80) \\ (1,1,2,1,2,1,0.10) \end{Bmatrix},$$

$$OFSL(1,2,2) = (1,2,2,1,2,1,0.80), \qquad OFSL(2,1,2) = (2,1,21,1,1,0.40),$$

$$LIR(1,1,2) = \begin{bmatrix} (1,1,3,1,1,2) \\ (2,2,3,11,2) \end{bmatrix}; \qquad LIR(1,2,2) = [(1,1,3,1,2,2)]OR[(1,2,3,1,2,2)]OR[(2,2,3,1,2,2)];$$

$$LIR(2,1,2) = \begin{bmatrix} (1,2,3,2,1,2) \\ (2,1,3,2,1,2) \\ (2,2,3,2,1,2) \end{bmatrix} OR[(3,1,3,2,1,2)]; \qquad LIR(1,1,1) = [(1,1,2,1,1,1)]OR[(2,1,2,1,1,1)];$$

$$LIR(1,2,1) = [(1,1,2,1,2,1)]OR[(1,2,2,1,2,1)]; \qquad LIR(3,1,1) = \varnothing .$$

To analyze the considered fault tree, we will not apply the simulation algorithm, since all calculations can be carried out analytically. First, calculate output probabilities of failure signals from the bottom level. It can be well-recognized that:

$P\{S(1,1,3,1,1,2)\} = 0.125$; $P\{S(1,1,3,1,2,2)\} = 0.150$; $P\{S(1,2,3,1,2,2)\} = 0.032$;

$P\{S(1,2,3,2,1,2)\} = 0.064$; $P\{S(2,1,3,1,2,2)\} = 0.060$; $P\{S(2,1,3,2,1,2)\} = 0.050$;

$P\{S(2,2,3,1,1,2)\} = 0.045$; $P\{S(2,2,3,2,1,2)\} = 0.025$; $P\{S(2,1,3,2,1,2)\} = 0.028$.

According to LIR, we determine the probabilities of realizing secondary failures:

- $P\{F_{1,1,2}\} = 0.50 \cdot 0.25 \cdot 0.05 \cdot 0.90 = 0.000056$;
- $P\{F_{1,2,2}\} = 1 - (1 - 0.15)(1 - 0.06)(1 - 0.032) \approx 0.224$;
- $P\{F_{2,1,2}\} = 0.064 \cdot 0.05 \cdot 0.025 + 0.028 \approx 0.028$.

The calculated output probabilities for failure signals at the second level are therefore as follows:

$P\{S(1,1,2,1,1,1)\} \approx 0.00004$; $P\{S(1,2,2,1,2,1)\} \approx 0.20$;

$P\{S(1,1,2,1,2,1)\} \approx 0.000006$; $P\{S(2,1,2,1,1,1)\} \approx 0.11$;

The calculated top level failures attain therefore the following probabilities (see LIR):

- $P\{F_{1,1,1}\} \approx 0.00004 + 0.028 \cdot 0.40 \approx 0.011$;
- $P\{F_{2,1,1}\} \approx 0.000006 + 0.20 \approx 0.20$;
- $P\{F_{3,1,1}\} \approx 0.79$.

Note that for a critical failure $P\{F_{1,1,1}\} \approx 0.011$ is high enough. It can be well-recognized that the main reason for such a high estimate is the secondary failure F_{212} which, in turn, depends mostly on the primary failure F_{313}. Setting $P_{313} = 0$ and recalculating the probabilities of the top level failures results in obtaining $P\{F_{1,1,1}\} \approx 0.00006$ which is a satisfactory value. Thus, the basic element E_{33} needs refinement, with the main target to decrease the probability of the failure's output signal.

Chapter 7. Controlling Man-Machine Organization Systems with Different Speeds

§7.1 Cost-optimization model for a single organization system

7.1.1 Introduction

In recent years extensive research has been undertaken in the area of various organization systems (OS), e.g., production control models under random disturbances, including on-line control models [56-57, 78-79, 130, 141, 170-171]. These models usually refer to man-machine production systems with random parameters, e.g., construction, metallurgy and mining, research and development projects, developments in the area of computer software and information systems, etc. For these not fully automatic systems, the actual output can be measured only at preset inspection (control) points. Such systems may normally use several possible speeds of advancement to the system's target, which can be detected and influenced by the decision-maker at control points. Given the target amount required, the due date and the amount produced up to a routine inspection point, the problem is to determine in that point the new production speed as well as the next inspection point. Two objectives have been usually embedded in the stochastic optimization model:

- to minimize the number of inspection points, and
- to maximize the probability of accomplishing the production program to the due date.

However, the number of publications on *cost-optimization on-line control models under random disturbances* remains very scanty (see, e.g., [57]), especially for control models under a chance constraint which have not been discussed elsewhere. Note that our previous publications (see, e.g., [78-82, 170-171]) considered on-line control models based on the risk averse principle, but not on the outlined below chance constraint principle applied to solving cost-optimization problems.

To fill up the gap, we suggest a newly developed production control model [87] which incorporates cost parameters. Two basic concepts are embedded in the model:

A. The objective is to minimize the manufacturing expenses of accomplishing the production program on the due date; and

B. A chance constraint, i.e., a confidence probability to meet the due date on time, has to be implemented in the model. In our opinion, such an additional restriction is important, since it guarantees the company's good name.

Thus, given:

- the target amount to be accomplished on time,

- the due date,
- several production speeds defined by their probability density functions,
- the average processing cost of realizing the manufacturing process per time unit for each production speed separately,
- the average cost of carrying out an inspection at the control point, and
- to minimize the number of inspection points, and

- the problem is to determine both control points and production speeds to be introduced at each control point, to minimize the average manufacturing expenses within the planning horizon subject to the adopted chance constraint.

This is a complicated stochastic optimization problem with a random number of decision variables. Since an optimal algorithm to solve the problem cannot be found, a heuristic one is suggested and developed. The algorithm is based on simulation and compares, one by one, sorted couples of production speeds in order to find an optimal couple which results in minimizing the average expenses. The algorithm has to be implemented at any control point to choose both, the speed to be introduced and the next control point.

Note that our previous publications [78-79, 170-171, etc.] are based on the *risk averse principle*, which is very efficient for non-cost objectives, but cannot be applied to the newly formulated cost-optimization model. It is therefore substituted for another one, namely the *chance constraint principle*, which is embedded in the heuristic algorithm and fits the cost objective [87, 173].

7.1.2 <u>Description of the organization system</u>

The OS under consideration produces a single product or a production program that can be measured by a single value, just like the system described in [170], e.g., in percentages of the planned total volume. Such an approach is often used for R&D projects, in mining, etc. the system is subject to a chance constraint, i.e., the least permissible probability of meeting the due date on time is pre-set. The system utilizes non-consumable resources that remain unchanged throughout the planning horizon. There are several alternative processing speeds to realize the program, corresponding to the same given levels of resources and depending only on the degree of intensity of the production process. However, for different speeds, the average processing costs per time unit vary. The evaluation of advancing to the goal, i.e., observing the product's actual output, can be carried out only via timely inspections at pre-set control points. At every inspection (control) point, the decision-maker observes the amount produced and has to determine both, the proper advancement speed and the next control point. Assume that it is prohibited to use unnecessarily high speeds (especially at the beginning of manufacturing the products), unless there is an *emergency situation*, i.e., a tendency to deviate from the target which may cause delay of the completion time. This is because lengthy work at higher speeds when utilizing restricted resources (e.g., manpower employed in two or three shifts, etc.) can prematurely wear out the regarded OS. Assume, further, that the inspection and the speed-reset times equal zero. The costs of all processing speeds per time unit, as well as cost of performing a single inspection at the control point, are pregiven.

7.1.3 The problem

Let us introduce the following terms:

V - the system's plan (target amount);

D - the due date (planning horizon);

$V^f(t)$ - the actual output observed at moment t, $0 < t \le D$; $V^f(0) = 0$;

$C^f(t)$ - the actual accumulated processing and control costs calculated at moment t, $0 < t \le D$; $V^f(0) = 0$;

t_i - the i-th inspection moment (control point), $i = 0,1,\ldots,N$;

N - the number of control points (a random value);

v_j - the j-th speed, $1 \le j \le m$ (a random value with pregiven probability density function $f_j(v)$);

$\bar{v}_j$ - the average speed v_j; it is assumed that speeds v_j are sorted in ascending order of the average values and are independent of t;

m - the number of possible speeds;

s_i - index of the speed chosen by the decision-maker at control point t_i;

c_j - the average processing cost per time unit of speed v_j, $1 \le j \le m$ (pregiven); note that $j_1 < j_2$ results in $c_{j1} < c_{j2}$;

c_{ins} - the average cost of carrying out a single inspection (pregiven);

Δ - the minimal value of the closeness of the inspection moment to the due date (pregiven);

d - the minimal given time span between two consecutive control points (in order to force convergence);

p - the least permissible probability of meeting the due date on time (pregiven);

a_j - lower bound of random speed v_j;

b_j - upper bound of random speed v_j;

$W_p(t,j)$ - the p-quantile of the moment when production program V will be accomplished on condition that speed v_j is introduced at moment t and will be used throughout, and the actual observed output at that moment is $V^f(t)$ (time moment to be met with pre-set probability p); in other words, $W_p(t,j)$ is the p-quantile of random value $\left[t + \left(V - V^f(t)\right)/v_j\right]$.

Assume that values $V^f(t)$, as well as the parameters of the probability density functions $f_j(v)$, $1 \le j \le m$, are given in percentages of the planned target V. We will, henceforth, implement three widely used distributions [67-70]:

1. A β-distribution with density function

$$p_j(v) = \frac{12}{(b_j - a_j)^4}(v - a_j)(b_j - v)^2; \tag{7.1.1}$$

2. A uniform distribution in the same interval;

3. A normal distribution with mean $\bar{v}_j = 0.5(a_j + b_j)$ and variance $V_j = [(b_j - a_j)/6]^2$.

Let us consider the cost-optimization control problem. The problem is to determine both, control points $\{t_i\}$ and production speeds $\{s_i\}$ to minimize the manufacturing expenses

$$J = \min_{\{t_i, s_i\}} \sum_{i=0}^{N-1} \left[c_{s_i}(t_{i+1} - t_i)\right] + N \cdot c_{ins} \tag{7.1.2}$$

such that:

$$\Pr\left\{V^f(D) \ge V\right\} \ge p, \tag{7.1.3}$$

$$t_0 = 0, \tag{7.1.4}$$

$$t_N = D, \tag{7.1.5}$$

$$t_{i+1} - t_i \ge d, \quad 0 \le i \le N-1, \tag{7.1.6}$$

$$D - t_i \ge \Delta, \quad 0 \le i \le N-1, \tag{7.1.7}$$

$$s_i \le k = \min_{1 \le q \le m}\left[q : W_p(t_i, q) \le D\right]. \tag{7.1.8}$$

Objective (7.1.2) enables minimization of all manufacturing expenses, while objective (7.1.3) reflects the chance constraint. (7.1.4) implies that the first control point to undertake decision-making is zero, namely, the starting moment to process the production program. (7.1.5) implies that the last inspection point is the due date D. (7.1.6) ensures the time span between each two consecutive control points, while (7.1.7) provides the means of ensuring the closeness of the inspection moment to the due date. (7.1.8) means that the production speed to be chosen at any routine control point must not exceed *the minimal speed which guarantees meeting the deadline on time, subject to the chance constraint*. Thus, as outlined above, *unnecessary surplus speeds are not implemented*.

The problem defined in (7.1.2-7.1.8) is a very complicated stochastic optimization problem which cannot be solved in the general case; it allows only a heuristic solution. The algorithm outlined below, in *7.1.5*, determines at each control point t_i both, the next control point t_{i+1} and the speed v_{s_i} at which to proceed until that control point.

7.1.4 The chance constraint principle

The chance constraint principle is the basic approach for determining the next control point t_{i+1} on the basis of the routine control point t_i and the actual output $V^f(t_i)$ observed at that moment. Note that such an approach has been successfully implemented in [173] for controlling stochastic network projects.

Consider a routine control point t_i, together with the actual output observed at that point, $V^f(t_i)$. For each production speed v_j, $1 \le j \le m$, calculate by means of simulation a representative statistical sample $\{T_j^{(s)}\}$, where $T_j^{(s)}$ is the simulated value of the completion time of the production program obtained by using speed v_j throughout. It can be well-recognized that the value of $T_j^{(s)}$ can be determined from

$$T_j^{(s)} = \frac{V - V^f(t_i)}{v_j^{(s)}} + t_i, \qquad (7.1.9)$$

where $v_j^{(s)}$ stands for the simulated production speed v_j at control point t_i.

After obtaining samples $\{T_j^{(s)}\}$, $1 \le j \le m$, calculate the corresponding p-quantiles and single out the subset of speeds for which:

$$W_p(t, j) < D \qquad (7.1.10)$$

holds. Note that if, for a certain speed j, (7.1.10) holds, then all speeds with higher indices also satisfy (7.1.10). Consider one of the speeds entering the subset, e.g., speed v_q. It can be well-recognized (see Fig. 7.1) that, being introduced from point $A(t_i, V^f(t_i))$ throughout, speed v_q enables the deadline to be met on time, subject to the chance constraint. *Moreover, even if no processing at all takes place within the period of length $\Delta t = D - W_p(t_i, q)$ (see the straight line AF) and afterwards speed v_q is introduced at point F, this speed v_q still enables the deadline to be met on time, under the chance constraint* (*7.1.3*). This can be well-recognized by examining two parallel straight lines: line AE, which enables accomplishing the production program with a probability exceeding p (henceforth, call this line $AE^{(q)}$) and line BF which enables the deadline to be met on time with confidence probability equal to p (call this line $BF^{(q)}$). Note that, if the production process proceeds with speed v_q from *any point* on line $BF^{(q)}$, the target will be met on time subject to the chance constraint. This basic principle will be implemented in the heuristic algorithm.

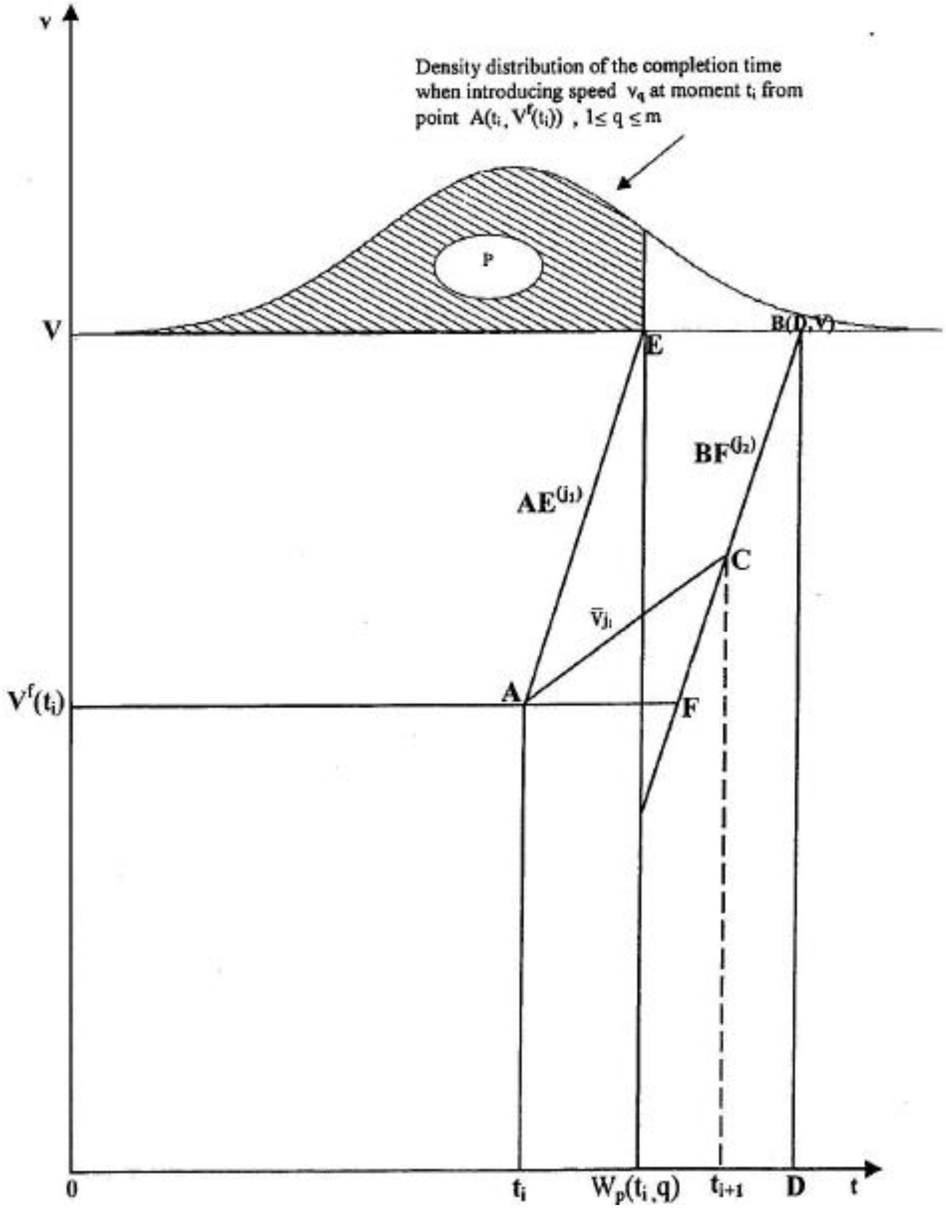

Figure 7.1. *The general idea of the chance constraint principle*

7.1.5 The heuristic algorithm

Referring to [78-79, 87, 124, 173], the heuristic control algorithm at each routine control point t_i, enables minimization of the manufacturing expenses (7.1.2) during the *remaining time* $(D-t_i)$. Thus, the objective function for optimizing decision-making at point t_i includes only *future expenses*, while past expenses, as well as past decision-makings, are considered to be irrelevant for the on-line control procedure. At each control point t_i, decision-making centers around the assumption (see [79, 87, 98]) that there is no more than one additional control point before the due date.

It can be well-recognized that the backbone of the heuristic control algorithm is *Subalgorithm I* which, at each routine control point t_i determines both, index s_i of the speed to be introduced and the next control point t_{i+1}. Following the assumption outlined above, two speeds have to be chosen at point t_i:

1. Speed v_{j_1}, $j_1 = s_i$, which *has to be actually introduced at point* t_i *up to the next control point* t_{i+1};
2. Speed v_{j_2}, $j_2 = s_{i+1}$, which *is forecast to be introduced at control point* t_{i+1} *within the remaining period* $[t_{i+1}, D]$.

Note that, if speed v_{j_2} is forecast to be the *last processing speed* before the due date D, control point t_{i+1} has to be necessarily on straight line $BF^{(j_2)}$ (see Fig, 7.1), otherwise chance constraint (7.1.3) might not be met. We suggest singling out, at each routine control point t_i, all possible couples (j_1, j_2) satisfying restriction (7.1.8), with subsequent choosing the one delivering the minimum of forecasted manufacturing and control expenses, namely

$$\min_{\{j_1, j_2\}} \left\{ c_{j_1}(t_{i+1} - t_i) + c_{j_2}(D - t_{i+1}) + c_{ins} \right\} \tag{7.1.11}$$

such that:

$$j_1 \le k = \min_{1 \le q \le m} \left[q : W_p(t_i, q) \le D \right], \tag{7.1.12}$$

$$j_2 \ge k \quad if \quad j_1 < k, \tag{7.1.13}$$

$$j_2 \le k \quad if \quad j_1 = k. \tag{7.1.14}$$

Restriction (7.1.12) is embedded in the algorithm to satisfy restriction (7.1.8). Restriction (7.1.13) holds, since case $j_1 < k$, $j_2 < k$ contradicts chance constraint (7.1.3). Case $j_1 = k$, $j_2 > k$ is a pointless one since, for both couples (k, k) and $(k, j_2 > k)$, chance constraint (7.1.3) will be met, but the second possibility proves to be more costly.

As to value t_{i+1}, we suggest calculating the latter on the assumption that, being introduced at t_i, the actual processing speed is $\bar{v}_{j_1}$. Thus, t_{i+1} may be determined as the abscissa of the intersection point C (see Fig. 7.1) of two straight lines:

$$AC: \; v = V^f(t_i) + \bar{v}_{j_1}(t - t_i); \tag{7.1.15}$$

$$BV^{(j_2)}: \; v = \frac{V - V^f(t_i)}{W_p(t_i, j_2) - t_i} t + V - D \frac{V - V^f(t_i)}{W_p(t_i, j_2) - t_i}. \tag{7.1.16}$$

Note that case $j_1 = j_2 = k$ is possible if using speed v_{j_1} throughout, until the due date D, results in the cheapest realization. In such a case, value c_{ins} has to be excluded from (7.1.11).

The enlarged step-by-step procedure of the heuristic algorithm is as follows:

Step 1. Start with $i = 0$, $t_0 = 0$, $V^f(t_0) = 0$.

Step 2. For each speed v_j, $1 \le j \le m$, determine by means of simulation values $W_p(t_i, j)$ (see *7.1.4*).

Step 3. Determine $k = \min\limits_{1 \le q \le m}\left[q : W_p(t_i, q) \le D\right]$. If k cannot be established, the problem defined in (7.1.2-7.1.8) has no solution. Otherwise apply the next step.

Step 4. Consider couples as follows:

$$\begin{cases} (1,k);(2,k);\ldots;(k-1,k); \\ (1,k+1);(2,k+1);\ldots;(k-1,k+1); \\ (1,k+2);(2,k+2);\ldots;(k-1,k+2); \\ \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \\ (1,m);(2,m);\ldots;(k-1,m); \\ (k,k);(k,k-1);\ldots;(k,2);(k,1) \end{cases} \tag{7.1.17}$$

in accordance with restrictions (7.1.12-7.1.14).

Step 5. For each combination (j_1, j_2) of couples (7.1.17) calculate value t_{i+1} as abscissa of the intersection point of lines (7.1.15) and (7.1.16).

Step 6. For each combination of couples (7.1.17), check if $t_{i+1} - t_i \ge d$ holds. If not, calculate $t_{i+1} = t_i + d$.

Step 7. For each combination of couples (7.1.17), check if $t_{i+1} > D$ or $D - t_{i+1} < \Delta$. If one of these relations holds, set $t_{i+1} = D$.

Step 8. For each combination of couples (7.1.17), besides $j_1 = k$ and $j_2 = k$, simulate the random, forecasted output product:

$$V(j_1, j_2) = v_{j_1} \cdot (t_{i+1} - t_i) + v_{j_2} \cdot (D - t_{i+1}) + V^f(t_i) \tag{7.1.18}$$

many times in order to obtain representative statistics. In case $j_1 = j_2 = k$, the required simulation has already been undertaken at Steps 2 and 3. Assume [78-79, 98, 124, 170] that any speed v_j is a *random variable* which, being simulated at the routine control point, remains unchanged until the next control point.

Step 9. Calculate for each pairwise combination (j_1, j_2) by means of statistical analysis, the approximate probability $\Pr\{V(j_1, j_2) \geq V\}$.

Step 10. Exclude from the list of combinations (7.1.17) all couples satisfying $\Pr\{V(j_1, j_2) \geq V\} < p$.

Step 11. Add to the remaining list of combinations, the combination (k,k) and calculate for all those combinations the forecasted average costs $C(j_1, j_2)$ as follows:

$$C(j_1, j_2) = c_{j_1} \cdot (t_{i+1} - t_i) + c_{j_2} \cdot (D - t_{i+1}) + c_{ins} \quad \text{if} \quad j_1 \neq j_2 \quad \text{and} \quad t_{i+1} \neq D, \tag{7.1.19}$$

$$C(j_1, j_2) = c_{j_1} \cdot (D - t_i) \quad \text{if} \quad j_1 \neq j_2 \quad \text{and} \quad t_{i+1} = D, \tag{7.1.20}$$

$$C(j_1, j_2) = (D - t_i) \quad \text{if} \quad j_1 = j_2 = k. \tag{7.1.21}$$

Step 12. Determine the optimal couple (j_1, j_2) which delivers the minimum to $C(j_1, j_2)$. If $j_1 \neq j_2$ and $t_{i+1} \neq D$ (e.g., (7.1.19) holds), go to the next step. In case (7.1.20) holds, apply Step 16. From (7.1.21) proceed to Step 15.

Step 13. Administer speed v_{j_1} up to the next control point t_{i+1}.

Step 14. Observe $V^f(t_{i+1})$ at moment t_{i+1}, and calculate value $C^f(t_{i+1}) = C^f(t_i) + (t_{i+1} - t_i)c_{j_1} + c_{ins}$; then set $i = i+1$ and go to Step 2. Thus, at the next control point t_{i+1} Subalgorithm I has to be implemented anew.

Step 15. Introduce speed $v_j = v_k$ until $t_{i+1} = D$. Go to Step 17.

Step 16. Introduce speed v_{j_1} up to the due date D.

Step 17. Observe the output product at due date D. Calculate $C^f(D) = C^f(t_i) + (D - t_i)c_k + c_{ins}$ in case (7.1.20).
End of the algorithm.

The algorithm is implemented in real time, although it is based on statistical trials and simulation modeling. Each interaction of the algorithm can be carried out only after value $V^f(t_i)$ is actually observed. However, the efficiency of the algorithm can be examined by simulating the actual speed v_{s_i} in each interval $\left[t_i, t_{i+1}\right]$. An illustrative numerical example is being outlined below, in *7.1.6*.

7.1.6 Numerical example

Let us consider the following example: a production OS with five possible speeds uniformly distributed:

$v_1 = U(1.8,\ 2.3)$; $c_1 = 10$;
$v_2 = U(2.0,\ 2.5)$; $c_2 = 20$;
$v_3 = U(2.5,\ 3.1)$; $c_3 = 40$;
$v_4 = U(3.0,\ 3.4)$; $c_4 = 50$;
$v_5 = U(3.5,\ 4.0)$; $c_5 = 60$;

Other parameters are:

$V = 77$, $D = 30$, $p = 0.75$, $c_{ins} = 40$, $d = 3$, $\Delta = 3$.

The solution according to the heuristic algorithm outlined above (for one simulation run) is as follows:

Stage I

Set $i = 0$, $t_0 = 0$, $V^f(0) = 0$.

Values $W_p(0, j)$ obtained via simulation (Step 2) are:

$W_p(0,1) = 40.0$, $W_p(0,2) = 36.3$, $W_p(0,3) = 29.1$, $W_p(0,4) = 24.9$, $W_p(0,5) = 21.2$.

Thus, $k = 3$ (see Step 3 of the heuristic algorithm), and the possible couples (j_1, j_2) to be examined (see Step 4) are as follows:

$(1,3)$, $(1,4)$, $(1,5)$, $(2,3)$, $(2,4)$, $(2,5)$, $(3,3)$, $(3,2)$, $(3,1)$.

Values t_{1+d} and $\Pr\{V(j_1, j_2) \geq V\}$ for each couple (j_1, j_2) are calculated by implementing (7.1.15) and (7.1.16) (see Step 5) and (7.1.18) (see Steps 8-9), respectively:

Couple (j_1, j_2)	Value t_1	Value $\Pr\{V(j_1, j_2) \geq V\}$
$(1,3)$	3.871	0.759
$(1,4)$	15.077	0.721
$(1,5)$	20.224	0.582
$(2,3)$	5.840	0.772
$(2,4)$	18.672	0.643
$(2,5)$	23.143	0.563
$(3,3)$	30.000	>0.75 (not calculated, since $W_p(0,3) < 30$)
$(3,2)$	19.664	0.622
$(3,1)$	21.994	0.556

Thus, only couples $(1,3)$, $(2,3)$ and $(3,3)$ are left after Step 10 for further examination.

The corresponding average processing costs $C(j_1, j_2)$ obtained in Step 11 by (7.1.19-7.1.21) are as follows:

$C(1,3) = 1,123.87$;
$C(2,3) = 1,123.20$;
$C(3,3) = 1,200.00$.

Thus, couple $(2,3)$ has to be chosen at Step 12.

The simulated speed of v_2 (see Step 13) from $t_0 = 0$ to $t_1 = 5.84$ equals $v_2 = 2.3311$. The simulated output at the next control point $V^f(t_1) = 13.61$, while the calculated processing and control costs are $C^f(t_1) = 5.84 \cdot 20 + 40 = 156.80$.

Stage II

From the previous stage we import $i = 1$, $t_1 = 5.84$, $V^f(t_1) = 13.61$.

Values $W_p(t_1, j)$ obtained via simulation (see Step 12) are:

$W_p(t_1,1) = 38.61$, $W_p(t_1,2) = 35.76$, $W_p(t_1,3) = 29.85$, $W_p(t_1,4) = 26.30$, $W_p(t_1,5) = 23.30$.

Thus $k = 3$, and couples (j_1, j_2) delineated above have to be examined anew. The corresponding values t_2 and $\Pr\{V(j_1, j_2) \geq V\}$ have been calculated at point $t_1 = 5.84$. Note that, since the use of relations (7.1.15) and (7.1.16) for couples $(1,3)$ and $(2,3)$ results in values $t_2 = 6.52$ and $t_2 = 6.87$, respectively, $t_2 - t_1 < d$ and values t_2 have been updated according to Step 6. Thus, for both couples $(1,3)$ and $(2,3)$, values $t_2 = t_1 + 3 = 8.84$. Values t_{i+1} and $\Pr\{V(j_1, j_2) \geq V\}$ for all couples (j_1, j_2) under examination are as follows:

Couple (j_1, j_2)	Value t_2	Value $\Pr\{V(j_1, j_2) \geq V\}$
$(1,3)$	8.84	0.749
$(1,4)$	16.77	0.722
$(1,5)$	21.23	0.635
$(2,3)$	8.84	0.746
$(2,4)$	19.35	0.660
$(2,5)$	23.46	0.618
$(3,3)$	30.00	>0.75 (not calculated, since $W_p(t_1,3) < 30$)
$(3,2)$	23.75	0.550
$(3,1)$	25.08	0.545

Thus, only couple $(3,3)$, which results in $\Pr\{V(3,3) \geq V\} > 0.75$, remains after Step 12. The simulated speed v_3 at Step 13 and up to the due date $D = 30$ is $v_3 = 2.7325$.

Stage III

It can be well-recognized that at this stage $t_2 = D = 30$. The actual simulated output at moment $t_2 = 30$ will be determined as $V^f(30) = 13.61 + 2.7325 \cdot (30 - 5.84) = 79.63 > 77$.

The actual simulated total expenses are $C^f(30) = 156.80 + 40 \cdot (30 - 5.84) + 40 = 1{,}163.2$. Thus, the system has met its target on time with two inspection points.

§7.2 Case of several stochastic organization systems with different speeds and resource reallocation

7.2.1 Introduction

Several OS (subsystems) entering a company (OS as well) are considered. The progress of each subsystem cannot be inspected and measured continuously, but only at preset inspection points. An on-line control model has to determine both inspection points and control actions to be introduced at those points to alter the progress of each subsystem in the desired direction. Those on-line control models are playing an increasing role in operation management. Two different cases may be examined:

A. The subsystem comprises production units, each of which, being supplied with resources of pregiven capacities, can be operated at one speed only.

B. Each unit can be operated at several possible speeds that are subject to random disturbances and correspond to one and the same resource capacity. That is, these speeds depend only on the degree of intensity of the subsystem's functioning (e.g., construction projects or design offices where different speeds may correspond to different hours a day per worker). The number of possible speeds is common to all units. All units, being employed between two adjacent control points, have to be operated with speeds of one and the same index.

In both cases cost-optimization problems can be formulated with different objectives and restrictions. We will consider case (B), where OS under random disturbances with different possible speeds have to be controlled. However, the number of publications on developing on-line control models in this area remains very scanty (see, e.g., [82, 87]).

The company under consideration comprises several simultaneously realized subsystems with random activity durations. The accomplishment of each subsystem is measured in percentage of its program. All the production units are to be operated by one of the identical *comprehensive resource units* (CRU) which may use several possible speeds subject to random disturbances. The speeds depend only on the intensity of the subsystem's functioning. They are indexed and the number of speeds is common to all CRU.

It is assumed that the progress of any subsystem can be evaluated only via periodical inspection in control points. At any moment $t > 0$ units that start to operate at that moment for one and the same subsystem, have to use speeds with similar indices (ordinal numbers). Speeds can be changed only at a control point. Within the subsystems' functioning a CRU can be transferred from one subsystem to another only at a so-called emergency moment common to all subsystems.

The subsystems' due dates and their chance constraints, i.e., their minimal permissible probabilities of accomplishing the target on time, are pregiven. All CRU have to be delivered to the company store at the subsystems starting time and are released when the last subsystem reaches its target. The cost of hiring and maintaining a CRU, together with the average processing costs per time unit for operating each unit under each speed, the average cost of performing a single inspection at a control point (common to all subsystems) and the average cost of reallocating CRU among still-operating subsystems at each emergency moment, are pregiven.

In §7.1 we have formulated and solved a cost-simulation problem for a *single* subsystem as follows: given the *fixed* number of CRU, at each routine control point t_i determine the next control point t_{i+1} and the new index of the speeds for all units to be operated at that point. The objective is to minimize the subsystem's total expenses. This basic problem (we will henceforth call it *Problem AI* [87]) will be used in order to develop a much more complicated realistic cost-optimization model as follows: determine the optimal number of CRU to minimize the total value of all subsystems' expenses subject to their chance constraints.

The problem's solution is as follows:

- at the company level a combination of a search procedure to determine the number of CRU together with a resource reallocation model among the subsystems is considered,
- at the subsystem level a basic cost-optimization on-line control *Model A1* is applied for each subsystem independently.

Both resource reallocation model and *Model A1* are implemented into a simulation model in order to obtain representative statistics to check the fitness of the problem's solution.

It is assumed that all non-accomplished subsystems have to be operated at any moment $t > 0$ with a speed exceeding zero. Thus, at least one CRU has to be assigned to each subsystem. At any moment each CRU can operate only one unit.

7.2.2 Notation

Let us introduce the following terms:

S - the organization system (company);
S_e - the e-th subsystem subordinated to S, $1 \le e \le f$;

f - number of subsystems with variable speeds;

f_t - number of subsystems which at moment t, $t \geq 0$, haven't reached their target yet;

U_{qe} - the q-th unit entering the e-th subsystem;

S_{et} - subsystem S_e observed at moment $t \geq 0$; $S_{e0} = S_e$;

$v_{qe}^{(k)}$ - the k-th speed of unit U_{qe}, $1 \leq e \leq f$, $1 \leq k \leq m$;

m - number of possible speeds common to all subsystems and units (pregiven);

n_{et} - number of identical generalized resource units CRU assigned to S_e at emergency moment $t \geq 0$, $n_{e0} = n_e$;

n - total number of CRU to be hired and maintained throughout the planning horizon by the company (optimized variable, to be determined beforehand);

ρ_{qe} - percentage of unit U_{qe} in subsystem S_e, $1 \leq e \leq f$ (pregiven);

D_e - due date of subsystem S_e (pregiven);

p_e - chance constraint to meet the deadline D_e on time (pregiven);

$V_e^f(t)$ - actual subsystem's S_e output in percentages of the total subsystem's S_e target (observed at moment t, $t \geq 0$);

$C_e^f(t)$ - the actual accumulated processing and control costs of subsystem S_e calculated at moment t, $t \geq 0$;

$W_p\left[t,k,V_e^f(t)\right]$ - the p-quantile of the moment subsystem S_e will reach its target on condition that the k-th speed for all units will be introduced at control point t and will be used throughout, and the actual observed output at that moment is $V_e^f(t)$;

t_{ge} - the g-th control point of the e-th subsystem, $g = 0,1,\ldots,N_e$, $t_{0e} = 0$, $t_{N_e e} = D_e$;

t_r^* - the company's emergency moment, $t_0^* = 0$, $r = 0,1,\ldots,N^*$;

N_e - number of control points of the e-th subsystem (a random value);

N^* - number of company's emergency moments (a random value);

Δ_{1e} - the minimal value of the closeness of the inspection moment to the due date D_e (pregiven);

Δ_{2e} - the minimal time span between two adjacent control points of the e-th subsystem (pregiven);

$t_{qe}^{(k)}$ - random duration of processing unit U_{qe} using speed $v_{qe}^{(k)}$ throughout;

$c_{qe}^{(k)}$ - the average processing cost per time unit for unit U_{qe} to be operated with speed $v_{qe}^{(k)}$ (pregiven);

c_{ins} - the average cost of undertaking a routine subsystem's inspection (common to all subsystems and pregiven);

c^* - the average cost of the CRU reallocation among the subsystems at a routine moment t_r^*;

V_e - the planned volume assigned to subsystem S_e (pregiven);

V_{et} - the actual processed volume of subsystem S_e at moment t (a random value);

S_{qe} - the actual moment unit U_{qe} starts operating (a random value);

F_{qe} - the actual moment unit U_{qe} accomplishes its production program (a random value); $F_{qe} = S_{qe} + t_{qe}^{(k)}$;

c_{cru} - the average cost of hiring and maintaining a CRU per time unit (pregiven);

F_e - the actual moment subsystem S_e accomplishes its production program (a random value); $F_e = \max_{\{U_{qe} \in S_{et}\}} F_{qe}$;

s_{ge} - the index of the speed to be introduced for all units U_{qe} starting in the interval $\left[t_{ge}, t_{g+1,e}\right]$, $1 \le s_{ge} \le m$.

It can be well-recognized that two kinds of control points are imbedded in the model:

1. *Regular* control (inspection) points t_{ge} to introduce proper speeds in order to alter the subsystem's speed in the desired direction.
2. *Emergency* control points t_r^* to reallocate all CRU at the company level among the non-accomplished subsystems, beginning from $t = 0$. Emergency moments t_r^* are as follows:

- $t = 0$;
- t is the moment of one of the subsystem's completion;
- t is the control moment for one of the subsystems when it is anticipated that with the previously assigned for that subsystem CRU the subsystem cannot meet its deadline on time.

7.2.3 The problem's formulation

The cost-optimization on-line control problem for several stochastic network projects [98] is as follows: determine the optimal value $n^{(opt)}$ of CRU (a deterministic value to be determined beforehand, i.e., before the subsystems start to be operated) together with values n_{et} assigned to all subsystems, all control points t_{ge}, the speeds to be introduced at that points for all subsystems' units $v_{qe}^{(k_e)}$, $k_e = s_{ge}$, and the actual moments S_{qe} production units U_{qe} start being operated (random values conditioned on decision-making of the control model), in order to minimize all operational, control, resource reallocation, hiring and maintenance expenses subject to the subsystems' chance constraints

$$J = \underset{\{n, n_{et}, t_{ge}, S_{qe}, s_{ge}, v_{qe}^{(k)}\}}{\mathrm{Min}} E\left\{ \sum_{e=1}^{f} \sum_{U_{qe} \in S_{et}} \left(c_{q1}^{(k_e)} \cdot t_{qe}^{(k_e)} \right) + \sum_{e=1}^{f} \left(N_e \cdot c_{ins} \right) + n \cdot c_{cru} \cdot \underset{e}{Max} F_e + N^* \cdot c^* \right\} \qquad (7.2.1)$$

subject to

$$\mathrm{k_e} = \mathrm{s_{ge}} \ \forall U_{qe} : S_{qe} = \mathrm{t_{eg}}, \ 0 \le g < N, \ 1 \le e \le f, \tag{7.2.2}$$

$$\Pr\left\{\mathrm{F_e} \le \mathrm{D_e}\right\} \ge \mathrm{p_e}, \ 1 \le e \le f, \tag{7.2.3}$$

$$t_{0e} = 0, \ 1 \le e \le f, \tag{7.2.4}$$

$$t_{N_e} = D_e, \ 1 \le e \le f, \tag{7.2.5}$$

$$D_e - t_{ge} \ge \Delta_{1e}, \ 0 \le g \le N_e, \ 1 \le e \le f, \tag{7.2.6}$$

$$t_{g+1,e} - t_{ge} \ge \Delta_{2e}, \ 0 \le g \le N_e, \ 1 \le e \le f, \tag{7.2.7}$$

$$\mathrm{s_{ge}} \le \mathrm{s^*_{ge}} = \underset{1 \le q \le m}{\mathrm{Min}} \left\{ \mathrm{q} : \mathrm{W_p}\left[\mathrm{t_{ge}}, \mathrm{q}, \mathrm{V^f_e}\left(\mathrm{t_{ge}}\right)\right]\right\}, \tag{7.2.8}$$

$$\sum_{e=1}^{f_t} n_{et} = n \text{ for any emergency moment } t \ge 0, \ n_{et} \ge 1. \tag{7.2.9}$$

Note that the on-line control model undertakes decision-making either at regular routine control point t_{ge} (determining S_{qe}, $v^{(k)}_{qe}$, $k = s_{ge}$), or at emergency points t^*_r (determining n_{et}, $t = t^*_r$), on the basis of future expenses only, i.e., during the remaining time $D_e - t_{ge}$ (for a single subsystem), or by taking into account values D_e and p_e, $1 \le e \le f$. Past expenses and past decision-makings, are not relevant for the on-line control model. Relation (7.2.3) honors the chance constraints. As to relation (7.2.8), it refers to the on-line cost-optimization algorithm for a single project (see [87]). (7.2.8) means that the speed to be chosen at any routine control point t_{ge} must not exceed the minimal speed s^*_{ge} that enables meeting deadline D_e on time, subject to be chance constraint p_e. It can be well- recognized that operating a unit at a higher speed always results in higher costs to accomplish the target than by using a lower speed. Thus, (7.2.8) prohibits using unnecessary high speeds. (7.2.9) ensures reallocation of n CRU at the company's disposal among the non-accomplished subsystems at any emergency moment $t \ge 0$. Relations (7.2.4-7.2.7) are obvious while (7.2.2) ensures assignment of one and the same speed index k_e to all units which start processing at a routine control point t_{ge}. Note that a unit cannot start at the moment between two adjacent control points t_{ge} and $t_{g+1,e}$.

7.2.4 Subsidiary models

Consider several important subsidiary models which will be used henceforth.

I. Subsidiary Model A1

As outlined above, the *basic subsidiary model A1* (see §7.1) centers on controlling a single subsystem, without taking into account any resource hiring and maintaining costs. The number of CRU is taken as a fixed and pregiven value. *Model A1* is an on-line cost-optimization model and is based on the chance constraint principle [87].

Given the average processing costs per time unit for each unit to be operated under each speed, together with the average cost of performing a single inspection at the control point, the problem at a routine control point t_g is to introduce the proper speed $v^{(k)}$ and the next control point t_{g+1}, in order to minimize the total processing costs within the planning horizon, subject to a chance constraint. At each control point, decision-making centers around the assumption that there is no more than one additional control point before the due date. Following that assumption, two speeds $v^{(k_1)}$ and $\mathrm{v}^{(\mathrm{k}_2)}$ have to be chosen at a routine control point t_g:

- Speed $v^{(k_1)}$ which has to be actually introduced at point t_g up to the control point t_{g+1};
- Speed $v^{(k_2)}$ which is forecast to be introduced at control point t_{g+1} up to the due date.

Couple $\left(v^{(k_1)}, v^{(k_2)}\right)$ providing the minimal cost expenses, has to be accepted.

The model is mostly effective when each processed unit can be measured as a partial accomplishment of the whole planned program. The problem is to determine both control points $\{t_g\}$ and activity speeds $\{v_q^{(k)}\}$ to minimize the average subsystem's expenses

$$\mathrm{J} = \underset{\{\mathrm{t_g}, \mathrm{v_q^{(k)}}, \mathrm{s_g}\}}{\mathrm{Min}} \mathrm{E}\left\{\sum_{\mathrm{U_q} \in S}\left(\mathrm{c_q^{(k)}} \cdot \mathrm{t_q^{(k)}}\right) + \mathrm{N} \cdot \mathrm{c_{ins}}\right\} \tag{7.2.10}$$

subject to

$$\mathrm{k} = \mathrm{s_g} \;\; \forall \mathrm{U}_q : \mathrm{t_g} = \mathrm{S_q}, \;\; 0 \le g < N, \tag{7.2.11}$$

$$\Pr\left\{\underset{\mathrm{U_q} \in S_{t_g}}{\mathrm{Max}} \mathrm{F_q} \le \mathrm{D}\right\} \ge \mathrm{p}, \tag{7.2.12}$$

$$t_0 = 0, \tag{7.2.13}$$

$$t_N = D, \tag{7.2.14}$$

$$D - t_g \ge \Delta_1, \;\; 0 \le g < N, \tag{7.2.15}$$

$$t_{g+1} - t_g \ge \Delta_2, \;\; 0 \le g < N, \tag{7.2.16}$$

$$\mathrm{s_g} \le \mathrm{s_g^*} = \underset{1 \le \mathrm{q} \le \mathrm{m}}{\mathrm{Min}}\left\{\mathrm{q} : \mathrm{W_p}\left[\mathrm{t_g}, \mathrm{q}, \mathrm{V^f}\left(\mathrm{t_g}\right)\right] \le \mathrm{D}\right\}. \tag{7.2.17}$$

II. Subsidiary Model A2

The model differs from *Model A1* by implementing the cost of hiring and maintaining CRU resources within the planning horizon. Thus, objective (7.2.10) is substituted by

$$J = \underset{\{t_g, v_q^{(k)}, s_g\}}{Min} E\left\{ \sum_{U_q \in S} \left(c_q^{(k)} \cdot t_q^{(k)}\right) + \left(\underset{U_q \in S}{Max} F_q\right) \cdot n\, c_{cru} + N \cdot c_{ins} \right\} \quad (7.2.18)$$

subject to (7.2.11-7.2.17), while the on-line heuristic algorithm remains unchanged.

III. Subsidiary Model A3

Determine the minimal number of CRU $n^{(opt)}$ for a single subsystem in order to meet the given chance constraint, i.e.,

$$Min\ n, \quad (7.2.19)$$

subject to (7.2.11-7.2.17).

The Solution

Start ascending value n, beginning from 1. For each n solve *Problem A1* taking into account for each unit U_{qe} its highest speed $v_q^{(m)}$, i.e., t_q refers to one speed only. Value n, for which relation

$$Pr\left\{ \underset{(i,j)}{Max}\ F_{ij} \le D \right\} < p \quad (7.2.20)$$

ceases to hold, is taken as the solution. Cost parameters are, thus, not taken into account. Denote the optimal number $n^{(opt)}$ by $n(A3)$.

IV. Subsidiary Model A4

Determine the minimal number of CRU in order to minimize objective (7.2.18) for *Model A2* subject to the chance constraint. Thus, two objectives are imbedded in the model

$$Min\ n, \quad (7.2.21)$$

$$J = \underset{\{n, t_g, v_q^{(k)}, s_g\}}{Min} E\left\{ \sum_{U_q \in S} \left(c_q^{(k)} \cdot t_q^{(k)}\right) + \left(\underset{U_q \in S}{Max} F_q\right) \cdot n\, c_{cru} + N \cdot c_{ins} \right\} \quad (7.2.22)$$

subject to (7.2.11-7.2.17).

The Solution

Solve *Problem A3* in order to determine value $n(A3)$. Afterwards proceed ascending value n, beginning from $n(A3)$, and for each value $n \ge n(A3)$ solve *Problem A2*. Value $n(A4)$ which delivers the minimum to (7.2.22) is taken as the solution of *Problem A4*.

7.2.5 The general idea of the problem's solution

The problem (7.2.1-7.2.9) to be considered (see Sections *7.2.2* and *7.2.3*) is a very complicated problem and allows only a heuristic solution. Denote the optimal solution of problem (7.2.1-7.2.9) by $n(A)$. A basic assertion can be formulated as follows:

Assertion

Let $n_e(A4)$ be the solution of *Problem A4* for each subsystem S_e, $1 \le e \le f$, independently. Relation

$$n(A) \le \sum_{e=1}^{f} n_e(A4) = n_{max} \tag{7.2.23}$$

holds.

Proof

Any additional CRU which results in exceeding value $\sum_{e=1}^{f} n_e(A4)$, has to be assigned to one of the subsystems S_e. For that subsystem, as it turns from *Model A4*, the unit becomes redundant. ■

Thus, the general idea of determining $n(A)$ is based on the following concepts:

Concept 1

At the company level the search for an optimal solution is based on examining all feasible solutions $\{n\}$, by decreasing n by one, at each search step, beginning from $n_{\max}$.

Concept 2

Examining a feasible solution centers on simulating the system. Multiple simulation runs have to be undertaken in order to obtain a representative statistics to check the fitness of the model.

Concept 3

A simulation model comprises two-levels. At the higher level – the company level – Subalgorithm I (outlined in *7.2.6*) reallocates n CRU among f_t non-completed subsystems at all emergency moments t, beginning from $t = 0$. At the lower level (the subsystem level) Subalgorithm II (outlined above, in §7.1; see also model (7.2.10-7.2.17)) undertakes on-line control for each subsystem independently between two adjacent emergency points t_r^* and t_{r+1}^*, by the use of a single-subsystem algorithm of *Problem A2*.

Concept 4

Each value n is examined via M simulation runs to provide a representative statistics to calculate values $\Pr\{F_e \le D_e\}$, $1 \le e \le f$, and objective (7.2.22).

Concept 5

The search process proceeds by decreasing n by one, i.e., substituting n by $n-1$, if all relations $\Pr\{F_e \le D_e\} \ge p_e$, $1 \le e \le f$, hold and value (7.2.22) decreases monotonously.

Concept 6

If even for one subsystem S_e relation $\Pr\{F_e \le D_e\} \ge p_e$ ceases to hold, or value (7.2.22) ceases to decrease, the last successful feasible solution n has to be taken as the optimal solution $n(A)$.

7.2.6 *<u>The enlarged procedure of resource reallocation (Subalgorithm I)</u>*

At each emergency point $t \ge 0$ (each emergency point is a control point for all subsystems S_e as well) reassign n CRU among f_t subsystems with non-accomplished production target, as follows:

<u>Step 1</u>. At moment t inspect values V_{et}, $1 \le e \le f$. Note that for subsystems with already accomplished production target, their corresponding values $V_{et} = 0$.

<u>Step 2</u>. By any means reassign n CRU among f_t subsystems subject to:

- $\sum_e n_{et} = n$;
- n_{et} must be whole numbers;
- n_{et} must be not less than 1;
- relations $n_{et} \ge \left[n \cdot \frac{V_{et}}{\sum_e V_{et}} \right]$, $V_{et} > 0$, $1 \le e \le f$, hold, where $[x]$ denotes the maximum whole number being less than x. Thus, Step 2 obtains a *non-optimal*, feasible solution.

<u>Step 3</u>. Take value $Z = 10^{17}$, i.e., an extremely large positive value.

<u>Step 4</u>. For all subsystems S_e with non-accomplished production target solve *Problem A2*, independently for each subsystem, with due dates $D_e - t$, chance constraints p_e, resource units n_e and non-accomplished volumes V_{et}. Denote the *actual* probability of meeting the due date on time by $\bar{p}_e$. Values $\bar{p}_e$, $1 \le e \le f$, are obtained via M simulation runs.

Step 5. Calculate values

$$\gamma_e = \frac{\overline{p}_e - p_e}{p_e}, \quad 1 \le e \le f .$$

Step 6. Calculate values

$$\gamma_{\xi_1} = \underset{e}{Max}\,\gamma_e ,$$

$$\gamma_{\xi_2} = \underset{e}{Min}\,\gamma_e .$$

Step 7. Calculate $\Delta = \gamma_{\xi_1} - \gamma_{\xi_2}$.

Step 8. If $\Delta < Z$, go to the next step. Otherwise apply Step 12.

Step 9. Set $Z = \Delta$.

Step 10. Transfer one CRU from subsystem $S_{\xi_1 t}$ to $S_{\xi_2 t}$, i.e., $n_{\xi_1 t}$ is diminished by one, and $n_{\xi_2 t}$ is increased by one.

Step 11 is similar to Step 4, with the exception of solving *Problem A2* for subsystems $S_{\xi_1 t}$ and $S_{\xi_2 t}$ only. Go to Step 5.

Step 12. Values n_{et}, $1 \le e \le f$, which refer to the last successful iteration, are taken as the optimal solution of Subalgorithm I.

7.2.7 The enlarged two-level heuristic algorithm of simulating the system

The enlarged step-by-step procedure of the problem's algorithm is based on simulating the system. The input of the simulation model is as follows:

- value $n \ge f$ of CRU (to be examined by simulation);
- pregiven values D_e, p_e, $1 \le e \le f$;
- speeds' parameters $v_{qe}^{(k)}$, $U_{qe} \in S_e$, $1 \le k \le m$;
- cost parameters $c_{qe}^{(k)}$, c_{ins}, c_{cru}, c^*;
- target parameters V_e, $1 \le e \le f$.

A simulation run comprises the following steps:

Step 1. Set $r = 1$, $t_r^* = 0$.

Step 2. Reallocate at $t = t_r^*$ n CRU units among subsystems S_e, $1 \le e \le f$, according to Subalgorithm I.

Step 3. Reassign values n_{et} obtained at Step 2, to subsystems S_e.

Step 4. Each subsystem S_e is operated independently according to *Problem A2* (see

Section 7.2.4). In the course of operating each subsystem any routine control point t_{ge} is examined as follows:

- is moment t_{ge} the moment subsystem S_e has reached its production target? If yes, go to Step 9. Otherwise proceed examining inspection point t_{ge}.
- is moment t_{ge} the moment when it is anticipated that subsystem S_e cannot meet its deadline on time even by introducing the highest speed with index m? If yes, go to the next step. Otherwise proceed realizing the project until the next routine control point $t_{g,e+1}$.

Step 5. Counter $r+1 \Rightarrow r$ works.

Step 6. Set $t_r^* = t_{ge}$.

Step 7. Inspect all subsystems S_e with non-accomplished production target at the routine emergency point t_r^*. Calculate values $V_e^f(t)$, $1 \le e \le f$, $t = t_r^*$.

Step 8. Update all remaining targets $V_e - V_e^f(t) \Rightarrow V_e$, $1 \le e \le f$. Go to Step 2 to undertake resource reallocation among subsystems with non-accomplished production target.

Step 9. Are there at moment $t = t_{ge}$ other subsystems with non-accomplished production target? If yes, go to Step 5. Otherwise apply the next step.

Step 10. The simulation run terminates.

In the course of carrying out Steps 2 and 4 the cost-accumulated value J of objective (7.2.1) has to be calculated.

The problem's solution is, thus, based on realizing procedures described in *Sections 7.2.5-7.2.7*.

§7.3 Experimentation

The experimental design considers an OS company comprising there different aggregated systems ($f = 3$) in the form of consecutive chain operations with five possible speeds ($m = 5$), which are subject to disturbances with given density probability functions. The speeds' parameters are as follows:

System 1

$v_1^{(1)} = U, N, B(0.5,1.2)$; $c_1^{(1)} = 10$.
$v_1^{(2)} = U, N, B(2.0,2.8)$; $c_1^{(2)} = 20$.
$v_1^{(3)} = U, N, B(2.9,3.8)$; $c_1^{(3)} = 40$.
$v_1^{(4)} = U, N, B(3.5,4.5)$; $c_1^{(4)} = 60$.
$v_1^{(5)} = U, N, B(4.0,5.0)$; $c_1^{(5)} = 70$.

System 2

$v_2^{(1)} = U, N, B(0.1,1.2)$; $c_2^{(1)} = 15$.
$v_2^{(2)} = U, N, B(1.0,2.5)$; $c_2^{(2)} = 25$.
$v_2^{(3)} = U, N, B(2.4,3.5)$; $c_2^{(3)} = 40$.
$v_2^{(4)} = U, N, B(2.5,3.9)$; $c_2^{(4)} = 50$.
$v_2^{(5)} = U, N, B(3.0,5.0)$; $c_2^{(5)} = 60$.

System 3

$v_3^{(1)} = U, N, B(0.5,1.2)$; $c_3^{(1)} = 10$.

$v_3^{(2)} = U, N, B(1.0,2.0)$; $c_3^{(2)} = 20$.

$v_3^{(3)} = U, N, B(1.8,2.8)$; $c_3^{(3)} = 30$.

$v_3^{(4)} = U, N, B(2.0,4.0)$; $c_3^{(4)} = 45$.

$v_3^{(5)} = U, N, B(3.5,5.5)$; $c_3^{(5)} = 60$.

Here terms U, N and B denote the uniform, the normal and beta-distributions, correspondingly.

The system's time parameters are as follows: the due date $D = 30$, $\Delta_{1e} = 3$, $\Delta_{2e} = 3$.

The cost parameters are as follows: $c^* = c_{cru} = 20$, $c_{ins} = 10$.

In addition, three other cost parameters have been imbedded in the example:

$C_e^* = 1{,}000$ - the single paid penalty cost of system S_e in case of delay;

$C_e^{**} = 100$ - the penalty cost for each time unit of delay, i.e., in case $D_e < F_e$;

$C_e^{***} = 10$ - storage costs charged for the system's achieving its production target ahead of time (before the due date) - per time unit;

In this experimental design, we assume the total number of the company's CRU being equal 40. Other parameters of the design are presented in Table 7.1:

Table 7.1. The experimental design

Variable	Values implemented in the experiment	Number of values
The least permissibility p in the chance constraint	0.75; 0.85	2
Distribution of $v_e^{(k)}$	Uniform, normal, beta	3
The target amount V	350; 365	2
The minimal time span values Δ	3; 5	2

Four parameters are varied, namely, p, Δ, V and the distribution of $v_e^{(k)}$.

Thus, the regarded company comprises three aggregated systems with five possible speeds. Three distributions of $v_e^{(k)}$ are considered:

- $v_e^{(k)}$ is a random variable uniformly distributed in the interval $\left[a_e^{(k)}, b_e^{(k)}\right]$.

- $v_e^{(k)}$ is a random variable normally distributed with mean value $\mu_e = 0.5\left(a_e^{(k)} + b_e^{(k)}\right)$ and variance $\left[\sigma_e^k\right]^2 = \frac{1}{36}\left(b_e^{(k)} - a_e^{(k)}\right)^2$ (a truncated distribution).
- $v_e^{(k)}$ has a beta distribution density function $P_e^{(k)}(v) = \frac{12}{\left(b_e^{(k)} - a_e^{(k)}\right)^4}\left(v - a_e^{(k)}\right)\left(b_e^{(k)} - v\right)^2$.

It can be well-recognized that implementing beta-distribution results in lower production speeds than in the case of normal or uniform distribution. This is because the average value of the asymmetric beta distribution is by $0.1\left(b_e^{(k)} - a_e^{(k)}\right)$ lower than the corresponding average values $0.5\left(a_e^{(k)} + b_e^{(k)}\right)$ for symmetric normal and uniform density functions.

Note that all pregiven average processing costs $c_e^{(k)}$, $1 \le e \le f$, $1 \le k \le m$, depend only on the couple of values $a_e^{(k)}, b_e^{(k)}$, but not on the type of distribution within the interval $\left[a_e^{(k)}, b_e^{(k)}\right]$.

A total of 24 combinations $(2 \times 3 \times 2 \times 2)$ were considered. For each combination 1,000 simulation runs were carried out. Several output parameters were considered as follows:

$\overline{C}$ - the optimal average value of total expenses within one simulation run;
$\overline{p}$ - the average actual probability of meeting the due date on time;
$\overline{N}_{ins}$ - the average number of inspection points for all systems within one simulation run;
$\overline{N}_{em}$ - the average number of emergency points (excluding $t = 0$);
$\overline{s}_{ge}$ - the average index of the speed to be introduced by the decision-maker at a routine control point t_{ge} (within one simulation run);
$\overline{p}_e$ - the average quasi-optimal chance constraint values for each system determined at a routine emergency moment t, $0 \le t \le T$ (within one simulation run);

Note that value $\overline{s}_{ge}$ for one simulation run is determined as follows:

$$\overline{s}_{ge} = \frac{1}{nD}\sum_{e=1}^{f}\sum_{g=0}^{N_{e-1}}\left[\left(t_{g+1,e} - t_{ge}\right)\cdot s_{g1}\right]. \qquad (7.3.1)$$

The summary of the experimentation is demonstrated in Table 7.2. The following five conclusions may be drawn from the summary:

Table 7.2. The summary of the experimentation

Distribution	p	$\Delta_{1,2}$	V	$\bar{p}_1$	$\bar{p}_2$	$\bar{p}_3$	$\bar{p}$	$\bar{C}$	$\bar{s}_{ge}$	$\bar{N}_{ins}$	$\bar{N}_{em}$
Uniform	0.75	3	350	0.61	0.62	0.60	0.82	5,660	4.32	14.57	2.87
			365	0.62	0.62	0.60	0.76	5,991	4.59	14.89	3.32
		5	350	0.57	0.56	0.55	0.80	5,688	4.35	8.50	1.14
			365	0.62	0.61	0.62	0.83	5,943	4.68	9.47	1.83
	0.85	3	350	0.68	0.66	0.69	0.88	5,682	4.43	15.08	3.06
			365	0.67	0.68	0.67	0.92	5,959	4.69	21.47	5.35
		5	350	0.63	0.63	0.62	0.90	5,626	4.46	8.89	1.31
			365	0.65	0.66	0.66	0.85	5,991	4.75	12.50	2.79
Normal	0.75	3	350	0.48	0.49	0.49	0.81	5,451	4.11	9.77	1.34
			365	0.63	0.63	0.63	0.81	5,842	4.49	8.99	1.45
		5	350	0.47	0.47	0.46	0.77	5,570	4.21	8.23	1.00
			365	0.47	0.47	0.47	0.78	5,849	4.48	7.29	0.79
	0.85	3	350	0.64	0.63	0.64	0.85	5,564	4.18	10.02	1.70
			365	0.66	0.67	0.65	0.90	5,777	4.49	8.16	5.35
		5	350	0.56	0.55	0.57	0.86	5,538	4.31	7.57	0.84
			365	0.61	0.60	0.62	0.90	5,781	4.57	6.57	0.73
Beta	0.75	3	350	0.58	0.58	0.56	0.78	5,859	4.47	10.75	1.93
			365	0.64	0.65	0.64	0.81	6,283	4.93	20.81	5.47
		5	350	0.51	0.50	0.50	0.77	5,888	4.50	7.46	0.92
			365	0.64	0.62	0.63	0.77	6,299	4.96	13.97	3.41
	0.85	3	350	0.67	0.68	0.66	0.89	5,857	4.59	12.79	2.73
			365	0.73	0.73	0.71	0.87	6,376	4.94	28.11	7.65
		5	350	0.60	0.61	0.63	0.88	5,814	4.60	7.49	1.12
			365	0.91	0.92	0.92	0.87	6,297	4.98	18.11	4.95

1. Increasing value $\bar{p}$ results in increasing the optimal average value $\bar{C}$, together with values $\bar{s}_{ge}$, $\bar{N}_{ins}$ and $\bar{N}_{em}$.
2. The average actual probability $\bar{p}$ of meeting the due date on time for all types of distributions exceeds its corresponding pregiven chance constraint value p. Thus, the control algorithm minimizes objective $\bar{C}$ with respect to (7.2.1).
3. Using the normal distribution yields in lower total cost expenses $\bar{C}$ than by using uniform or beta distributions. Using the beta distribution results in the highest total cost expenses. Thus, practically speaking, normal distribution enables the cheapest management while beta distribution proves to be the least effective one.

4. Increasing values Δ_1 and Δ_2 results usually in decreasing the efficiency of the control model, e.g., in increasing values $\overline{C}$, $\overline{N}_{em}$, $\overline{N}_{ins}$, $\bar{s}_{ge}$.

5. With the exception of one combination (beta distribution, $p = 0.85$, $V = 365$, $\Delta_1 = \Delta_2 = 5$) average values $\overline{p}_e$ of local chance constraints are usually smaller than the pregiven probability value p. This phenomenon occurs since decision-making [87, 98] enables implementing non-exhausting speeds at non-emergency inspection moments in combination with introducing maximal speeds for all systems in case of critical emergency situations.

§7.4 Conclusions

1. It can be well-recognized that the developed cost-optimization simulation algorithm for solving problem (7.2.1-7.2.9) can be applied to a wide range of organization systems. The outlined model (7.2.1-7.2.9) enables managing complicated building and construction systems, various R&D systems with different speeds and inspection points, etc.

2. The newly developed on-line control model is a generalized model: it satisfies a variety of chance constraints and develops cost-minimization for a broad spectrum of expenses in the course of the system's functioning.

3. The structure of the algorithm is as follows: at the system's level (the higher level) a search of the optimal number of CRU is undertaken. At the lower level a basic cost-optimization model for a single OS (see §7.1) is implemented in the simulation model.

4. The main connection between those two levels is carried out via a newly developed resource reallocation subalgorithm. The latter is carried out by undertaking probability control to be as close as possible to the systems' chance constraints.

5. In the considered Chapter the previously used risk averse principle [78-82] has been substituted by a more effective chance constraint principle [87, 173]. Extensive experimentation [173] has justified the fitness of the newly developed theoretical concepts.

PART II

ESTIMATING ORGANIZATION SYSTEMS' QUALITY BY MEANS OF HARMONIZATION MODELS

Chapter 8. Quality Models via Harmonization in Stochastic Project Management

§8.1 Harmonization model for a single PERT-COST project

8.1.1 *Introduction*

In this Chapter we will outline a new three-parametrical trade-off model for a PERT-COST type network project with random activity durations. As outlined in *5.2.2*, a PERT-COST project is characterized by the following parameters:

- the budget C assigned to the project which has to be redistributed among the project's activities;
- the due date D for the project to be accomplished;
- the project's reliability R, i.e., the probability of meeting its due date on time subject to the pre-assigned budget C.

The trade-off optimization problem is developed and formalized within the framework of the outlined in Chapter 5 theory of harmonization models for multi-parametrical organization systems [9, 11-13, 18]. To maximize the project's utility, a three-parametrical harmonization model is developed. The model's algorithm is a unification of a cyclic coordinate search algorithm in the two-dimensional space (budget and time values) and the partial harmonization model (see §5.3) to maximize the project's reliability subject to the preset budget and due date values. The model comprises a heuristic budget reallocation procedure together with a simulation model of the project's realization.

In order to outline the developed theory we will need the following terms:

$G(N,A)$ - finite, connected, oriented activity – on – arc network of PERT-COST type;
$(i,j) \subset G(N,A)$- activity leaving node i and entering node j;
t_{ij} - random time duration of activity (i,j);
c_{ij} - budget assigned to activity (i,j);

$c_{ij\ min}$ - minimal budget capable of operating activity (i,j) (pregiven);

$c_{ij\ max}$ - maximal budget required to operate activity (i,j) (pregiven); in case $c_{ij} > c_{ij\ max}$ additional value $c_{ij} - c_{ij\ max}$ is redundant;

C - budget assigned to carry out project $G(N,A)$;

D - the due date for the project $G(N,A)$;

R - the project's reliability value, i.e., its probability of meeting the deadline D on time;

M - number of simulation runs in course of determining R on the basis of preset values C and D;

$P_{h\,f}$ - the probability of a hazardous failure in the course of carrying out the project;

$T(G)_{c_{ij}}$ - the project's random duration on condition that budget values c_{ij} are assigned to activities (i,j);

$R(G)_{c_{ij}}$ - the project's local reliability, i.e., the probability of meeting its deadline on time on condition that values c_{ij} are assigned to $(i,j) \subset G(N,A)$, $R(G)_{c_{ij}}$ = $\Pr\left\{ T(G)_{c_{ij}} \le D \right\}$;

$R(G)_{C,D} = \underset{\{c_{ij}\},\sum_{(i,j)} c_{ij}=C}{Max} R(G)_{c_{ij}}$ - the project's conditional reliability (on condition that values C and D are preset beforehand; to be calculated);

$PHM/_{C,D}$ - the partial harmonization model to optimize reliability R with independent input values C and D;

$P_{h\,f}(C,D)$ - the formalized deterministic dependency from values C and D (obtained by means of expert information);

C_0 - the maximal possible budget to be assigned to project $G(N,A)$ (pregiven);

D_0 - the maximal permissible due date for the project $G(N,A)$ to be accomplished (pregiven);

R_0 - the minimal permissible reliability value for project $G(N,A)$ (pregiven);

P_0 - the maximal permissible probability value for the hazardous failure to appear (pregiven);

ΔC - budget search step (pregiven);

ΔD - due date's search step (pregiven);

C_{00} - the best possible value of parameter C (pregiven);

D_{00} - the best possible value of parameter D (pregiven);

R_{00} - the best possible value of parameter R (pregiven);

P_{00} - the best possible value of parameter P (pregiven);

δ_C - budget unit value (pregiven);

δ_D - time unit value (pregiven);

δ_R - reliability unit value (pregiven);

δ_P - hazardous failure probability unit value (pregiven);

α_C - partial utility value for parameter C (pregiven);

α_D - partial utility value for parameter D (pregiven);

α_R - partial utility value for parameter R (pregiven);

α_P - partial utility value for parameter P (pregiven);

$U(G) = U(C,D,R)$ - the project's utility;

$U_0 = U(C_0, D_0, R_0)$ - the project's basic utility;

$U = U(C,D,R,P)$ - the project's utility with P_{hf};

$U_0 = U(C_0, D_0, R_0, P_0)$ - the project's basic utility with P_{hf};

$q \geq 1$ - number of a current iteration for value $R(G)_{c_{ij}}$ in problem $PHM/_{C,D}$;

$R^{(q)}(G)_{c_{ij}}$ - value of the project's local reliability, i.e., $R^{(q)}(G)_{c_{ij}} = Pr^{(q)}\left\{ T(G)_{c_{ij}} \leq D \right\}$;

$CCSA\{G\}$- the cyclic coordinate search algorithm which undertakes a search in the E^2 space of budget values C and due dates D;

$v \geq 1$ - ordinal number of a current iteration in $CCSA\{G\}$;

$CCSA^{(v)}$ - the results of the v-th current iteration in the course of carrying out $CCSA\{G\}$;

$\varepsilon > 0$ - pregiven search tolerance (accuracy) in the course of optimizing the project's utility.

We suggest evaluating the project's utility by

$$U = \alpha_C \cdot [C_0 - C] + \alpha_D \cdot [D_0 - D] + \alpha_R \cdot [R - R_0],$$

where C_0, D_0 and R_0 are the least permissible budget, due date and reliability values which can be implemented in a PERT-COST project, while values C, D and R are the corresponding current values for a project under consideration. Linear coefficients α_C, α_D and α_R define additional partial utilities which the project obtains by refining its corresponding parameter by a unit's value. Note that parameters C and D are independent parameters since they can be preset beforehand independently on each other, while parameter R is practically defined by values D and C and, thus, is a dependent parameter. For the case $C = C_0$, $D = D_0$ and $R = R_0$, the project's utility is called the basic utility and is usually pregiven beforehand. Note that quantitative relations between parameters C, D and R are complicated, since setting a couple of values C and D

results in a variety of possible values R depending on the budget reassignment among the project's activities. Thus, an additional optimization problem to maximize the reliability value on the basis of preset values C and D has to be imbedded in the project's harmonization model.

8.1.2 The system's description

Consider a PERT-COST network project with random activity durations. It can be assumed (see §5.2) that for each activity time duration its density function depends parametrically on the budget which is assigned to that activity.

It can be well-recognized from various studies in PERT-COST [2, 41, 70, 77, 109, 116] that for most activities (i, j) entering the network model, their random time duration t_{ij} is close to be inversely proportional to the budget c_{ij} which is assigned to that activity. Thus, three different distributions may be considered:

- random activity durations are assumed to have a beta-distribution, with the probability density functions (p.d.f.) as follows:

$$p_{ij}(t) = \frac{12}{\left(b_{ij} - a_{ij}\right)^4}\left(t - a_{ij}\right)\left(b_{ij} - t\right)^2, \tag{8.1.1}$$

  where $b_{ij} = \dfrac{B_{ij}}{c_{ij}}$ and $a_{ij} = \dfrac{A_{ij}}{c_{ij}}$, A_{ij} and B_{ij} being pregiven constants for each activity (i, j) entering the PERT-COST network model.

- random activity durations are assumed to be normally distributed with the p.d.f. $N\left(a, \sigma^2\right)$

$$p_{ij}(t) = \frac{1}{\sigma\sqrt{2\pi}}\, e^{-\frac{2(x-a)}{2\sigma^2}}\, dx, \tag{8.1.2}$$

  where the mean value a and the variance σ^2 are calculated by

$$a = 0.5 \cdot \frac{A_{ij} + B_{ij}}{c_{ij}}, \qquad \sigma = \frac{B_{ij} - A_{ij}}{6c_{ij}}. \tag{8.1.3}$$

- random activity durations are assumed to be distributed uniformly in the interval $\left(\dfrac{A_{ij}}{c_{ij}}, \dfrac{B_{ij}}{c_{ij}}\right)$, with the p.d.f.

$$\frac{c_{ij}}{B_{ij} - A_{ij}} = \frac{1}{b_{ij} - a_{ij}}. \tag{8.1.4}$$

In the problem under consideration all those cases will be examined.

As outlined above, the following restrictions will be implemented in the model:

- $C \le C_0$, where C_0 is the maximal permissible budget to be assigned to the project;
- $D \le D_0$, where D_0 is the maximal permissible due date to be accepted by the project management;
- $R \ge R_0$, where R_0 is the least permissible reliability of meeting the project's deadline on time, i.e., the minimal probability of accomplishing the project before its due date.

Besides those *worst* permissible pregiven values C_0, D_0 and R_0, one can define the *best* pregiven possible correspondent values - the minimal budget C_{00} to be assigned to the project, the earliest due date D_{00} (there is no need in accomplishing the project before D_{00}), and the maximal reliability value R_{00} (usually $R_{00} = 1$). It can be well-recognized that any project values C, D and R satisfy

$$\begin{cases} C_{00} \le C \le C_0 \\ D_{00} \le D \le D_0 \\ R_0 \le R \le R_{00} . \end{cases} \tag{8.1.5}$$

Directions which refine the corresponding parametric characteristics (i.e., from C_0 to C_{00}, from D_0 to D_{00}, from R_0 to R_{00}) are called (see §5.3) *positive directions*.

8.1.3 Harmonization model

The harmonization model (see §§1.2 and 5.3) is as follows: determine optimal non-contradictive project parameters $C^{(opt)}$, $D^{(opt)}$ and $R^{(opt)}$ resulting in the maximal project's utility

$$\underset{\{C,D,R\}}{Max}\ U(G) = \underset{\{C,D,R\}}{Max} \left\{ U_0 + \alpha_C (C_0 - C) + \alpha_D (D_0 - D) + \alpha_R (R - R_0) \right\} \tag{8.1.6}$$

subject to

$$C_{00} \le C^{(opt)} \le C_0 , \tag{8.1.7}$$

$$D_{00} \le D^{(opt)} \le D_0 , \tag{8.1.8}$$

$$R_{00} \ge R^{(opt)} \ge R_0 . \tag{8.1.9}$$

Note that since the basic utility U_0 is a constant value which remains unchanged, it may be canceled and, thus, the model satisfies

$$\underset{\{C,D,R\}}{Max}\ U(G) = \underset{\{C,D,R\}}{Max} \left\{ \alpha_C (C_0 - C) + \alpha_D (D_0 - D) + \alpha_R (R - R_0) \right\} \tag{8.1.10}$$

subject to (8.1.7-8.1.9). Values C, D and R are called *non-contradictive* if budget C can be reassigned among the project activities to satisfy

$$\Pr\left\{T(G)_{c_{ij}} \le D\right\} = R \tag{8.1.11}$$

subject to

$$\sum_{(i,j)} c_{ij} = C . \tag{8.1.12}$$

Theorem

Optimal value $R^{(opt)}$ in problem (8.1.7-8.1.10) satisfies

$$R^{(opt)} = PHM\Big/C^{(opt)}, D^{(opt)} = \underset{\{c_{ij}\}, \sum_{(i,j)} c_{ij} = C^{(opt)}}{Max} \left[\Pr\left\{ T(G)_{c_{ij}} \le D^{(opt)} \right\} \right]. \tag{8.1.13}$$

The ***Proof*** of the Theorem stems from the proof of the general Theorem outlined in §5.3.

It can be well-recognized from the theorem that solving problem (8.1.7-8.1.10) can be carried out by solving two sequential problems: to determine an optimal budget value C and an optimal due date D (*Problem 1*) and to carry out the PHM (*Problem 2*).

Problem 1 centers on determining an optimal couple $\left(C^{(opt)}, D^{(opt)}\right)$ by means of a look-over algorithm that checks the feasibility of each possible combination (C, D). If the number of combinations is high enough and taking into account that:

- each combination requires a PHM solution, and
- *Problem 1* is a NP-complete one [66, 176],

- solving both problems on a look-over basis requires a lot of computational time. To avoid this obstacle, we suggest a two-level high-speed approximate heuristic algorithm. At the upper level a heuristic simplified search procedure, e.g. a cyclic coordinate sub-algorithm [133], has to be carried out in the two-dimensional space in order to determine an optimal combination (C, D). At the bottom level, a heuristic high-speed procedure to optimize the partial harmonization model $PHM\Big/C, D$ with independent input values C and D, has to be implemented. Thus, we substitute objective (8.1.10) by

$$\underset{C,D}{Max} \left\{ CCSA\{C, D\} \cup PHM\Big/C, D \Rightarrow U(C, D, R) \right\}, \tag{8.1.14}$$

where $\cup$ stands for a unification sign.

Both *Problems 1* and *2* together with their solutions will be outlined below.

8.1.4 Partial harmonization model PHM(C,D)=R

As outlined above, parameters C and D are input values of the model as well as values $c_{ij\ \min}$, $c_{ij\ \max}$, A_{ij} and B_{ij}, $(i,j)\subset G(N,A)$. The problem is as follows: determine optimal reassigned budget values c_{ij} for each activity $(i,j)\subset G(N,A)$, to maximize the project's conditional reliability, i.e.,

$$\underset{\{c_{ij}\},\sum_{(i,j)} c_{ij}=C}{Max}\left[\ Pr\left\{T(G)_{c_{ij}}\le D\right\}\right] \tag{8.1.15}$$

subject to

$$C_{ij\ min} \le C_{ij} \le C_{ij\ max}, \tag{8.1.16}$$

$$\sum_{(i,j)\subset G(N,A)} c_{ij} = C. \tag{8.1.17}$$

We will henceforth use a refined version of the procedure outlined in [9, 76, 109]. The step-by-step procedure is as follows:

Step 1. By any means reassign budget C among the project's activities $(i,j)\subset G(N,A)$ subject to (8.1.16) and (8.1.17) to obtain a feasible solution of the problem. It is suggested to implement the step by using the bisection method [176] as follows:

1.1 Start with $\alpha=0,\ \beta=1$.

1.2 Determine two values:

$$\Sigma_1 = \sum_{(i,j)\subset G(N,A)} c_{ij\ \min} = \sum_{(i,j)}\left[(1-\alpha)\cdot c_{ij\ \min} + \alpha\cdot c_{ij\ \max}\right],$$

$$\Sigma_2 = \sum_{(i,j)\subset G(N,A)} c_{ij\ \max} = \sum_{(i,j)}\left[(1-\beta)\cdot c_{ij\ \min} + \beta\cdot c_{ij\ \max}\right].$$

1.3 Calculate value

$$\Sigma_3 = \frac{1}{2}\cdot(\Sigma_1+\Sigma_2) = \sum_{(i,j)}\left[\left(1-\frac{\alpha+\beta}{2}\right)\cdot c_{ij\ min} + \frac{\alpha+\beta}{2}\cdot c_{ij\ max}\right].$$

1.4 Compare values Σ_1 and Σ_2. If $\Sigma_2-\Sigma_1<\delta C$ go to *1.8*; otherwise go to *1.5*. Note that δC is a pregiven budget unit value.

1.5 Examine relation $\Sigma_1\le C\le\Sigma_3$. If it holds go to *1.6*; otherwise go to *1.7*.

1.6 Set

$\Sigma_3 \Rightarrow \Sigma_2, \quad 1-\frac{\alpha+\beta}{2} \Rightarrow 1-\beta, \quad \frac{\alpha+\beta}{2} \Rightarrow \beta.$

Go to *1.3*.

1.7 Applying *1.7* means that $\Sigma_3 < C < \Sigma_2$ holds. Set

$\Sigma_3 \Rightarrow \Sigma_1, \quad 1-\frac{\alpha+\beta}{2} \Rightarrow 1-\alpha, \quad \frac{\alpha+\beta}{2} \Rightarrow \alpha,$

and go to *1.3*.

1.8 Value $\Sigma_3 \cong C$ with $c_{ij} = 0.5\cdot(2-\alpha-\beta)\cdot c_{ij\ min} + 0.5\cdot(\alpha+\beta)\cdot c_{ij\ max}$ is the feasible solution.

Step 2. Calculate $a_{ij} = \frac{A_{ij}}{c_{ij}}$ and $b_{ij} = \frac{B_{ij}}{c_{ij}}$ for all activities $(i,j) \subset G(N,A)$.

Step 3. Simulate values t_{ij} with p.d.f. (8.1.1), (8.1.2) or (8.1.4).

Step 4. Calculate the critical path length $L_{cr}\left[t_{ij}\right]$ and determine all activities $(i,j) \subset G(N,A)$ belonging to the critical path.

Step 5. Compare values D and $L_{cr}\left[t_{ij}\right]$. If $D \geq L_{cr}\left[t_{ij}\right]$ counter $W \Rightarrow W+1$ works; then go to Step 6. In case $D < L_{cr}\left[t_{ij}\right]$ go to Step 6 directly.

Step 6. If a routine activity (i,j) belongs to the critical path, counter $W_{ij} \Rightarrow W_{ij}+1$ works. The step is carried out for all $(i,j) \subset G(N,A)$.

Step 7. Repeat Steps 2-6 M times in order to obtain representative statistics.

Step 8. Calculate the average value

$$R^{(q)}(C) = R^{(q)}(G)_{c_{ij}} = \Pr^{(q)}\left\{T(G)_{c_{ij}} \leq D\right\}, \quad \sum_{(i,j)} c_{ij} = C, \tag{8.1.18}$$

where q is the number of the current iteration.

Step 9. Compare two adjacent average values $R^{(q)}(C)$ and $R^{(q-1)}(C)$. If $R^{(q)}(C) > R^{(q-1)}(C)$ holds, go to the next step. Otherwise go to Step 16.

Step 10. Calculate the frequency of each activity (i,j) belonging to the critical path (on the basis of M simulation runs carried out in Step 7). Denote those frequencies by $\overline{P}(i,j/L_{cr})$, $\overline{P}(i,j/L_{cr}) = \frac{W_{ij}}{M}$.

Step 11. Reschedule all the activities (i,j) as follows [9, 109]:
For activities (i,j) with $\overline{P}(i,j/L_{cr}) > 0$ reschedule them in *descending order of the product*

$\overline{P}\left(i,j/L_{cr}\right)\cdot\upsilon_{ij}\,,$ (8.1.19)

where:

- $\upsilon_{ij} = \dfrac{3A_{ij} + 2B_{ij}}{5c_{ij\ min}\cdot c_{ij\ max}}$ for the case of p.d.f. (8.1.1);
- $\upsilon_{ij} = \dfrac{A_{ij} + B_{ij}}{2c_{ij\ min}\cdot c_{ij\ max}}$ for the case of p.d.f. (8.1.2);
- $\upsilon_{ij} = \dfrac{A_{ij} + B_{ij}}{2c_{ij\ min}\cdot c_{ij\ max}}$ for the case of p.d.f. (8.1.4).

Decreasing the mean value of the activity duration has to be carried out by taking into account not only values υ_{ij} but their probabilities as well.

Activities (i,j) with $\overline{P}\left(i,j/L_{cr}\right) = 0$ have to be rescheduled at the end of the schedule in *descending order* of values υ_{ij} only since the product $\overline{P}\left(i,j/L_{cr}\right)\cdot\upsilon_{ij}$ is equal to zero.

Step 12. Determine activity $\left(i_\xi\,,j_\xi\right)$ with the *highest order* for which relation $Z_1 = C_{i_\xi j_\xi\ max} - C_{i_\xi j_\xi} > 0$ holds. It goes without saying that activity $\left(i_\xi\,,j_\xi\right)$ is placed at the beginning of the schedule and refers to the critical zone, $\overline{P}\left(i_\xi\,,j_\xi/L_{cr}\right) > 0$.

Step 13. Determine activity $\left(i_\eta\,,j_\eta\right)$ with the *lowest order* for which relation $Z_2 = C_{i_\eta j_\eta} - C_{i_\eta j_\eta\ min} > 0$ holds. Activity $\left(i_\eta\,,j_\eta\right)$ is at the end of the schedule and is a non-critical activity, having practically no influence on the entire project's duration.

Step 14. Reassign cost values $Z = min\left(Z_1, Z_2\right)$ from activity $\left(i_\eta\,,j_\eta\right)$ to activity $\left(i_\xi\,,j_\xi\right)$.

Step 15. Clear counter W and go to Step 2.

Step 16. Introduce changes in the heuristic procedure as follows:

(a) in Step 9: for case $R^{(q)}(C) \le R^{(q-1)}(C)$ go to Step 18 and not to Step 16.

(b) in Step 14: value Z to be transferred from activity $\left(i_\eta\,,j_\eta\right)$ to $\left(i_\xi\,,j_\xi\right)$ is to be equal now to δC. Afterwards go to Step 17.

Step 17. Take the rescheduled activities (i,j) obtained at Step 11 for the $(q-1)$-th iteration. Go to Step 12.

Step 18. End of the heuristic procedure. Further application of the procedure will not lead to any increase of the confidence probability.

Values c_{ij} obtained in the course of the $(q-1)$-th iteration are considered as the optimal ones. The optimal value of the objective, i.e., the project's conditional reliability $R(G)_{C,D}$, is value $R^{(q-1)}(C)$ calculated on Step 8.

It can be well-recognized that in cases $C < \sum_{(i,j)\subset G(N,A)} c_{ij\ \min}$ or $C > \sum_{(i,j)\subset G(N,A)} c_{ij\ \max}$ the corresponding reliability values are 0 and 1, i.e., the problem obtains trivial solutions. It is in the case of $\sum_{(i,j)\subset G(N,A)} c_{ij\ \min} < C < \sum_{(i,j)\subset G(N,A)} c_{ij\ \max}$ that the above heuristic procedure has to be implemented.

Note, in conclusion, that subdividing the above procedure into two parts is aimed to enhance its sensitiveness. In the course of carrying out initial iterations (at Step 14), $Z \geq 1$ cost unit values are transferred from the most "idle" activity to the most "tense" one. Later on, to refine the procedure's convergence, only one budget unit is transferred in the course of a routine iteration.

8.1.5 Two-dimensional cyclic coordinate search algorithm

As outlined in Chapter 5, we suggest to substitute an exact look-over algorithm that checks the feasibility of each possible combination (C,D), by a cyclic coordinate search algorithm ($CCSA$) [11, 133]. The enlarged step-by-step procedure of the algorithm is as follows:

Stage I. First iteration, i.e., $v=1$.

Step 1. Set $C = C_0$ and $D = D_0$.

Step 2. Start diminishing value C (beginning from C_0) by the search step ΔC, i.e., $C \Rightarrow C - r\Delta C$, $r = 0, 1, \ldots$, while value D remains unchanged. For each couple (C,D) under consideration calculate $R(G)_{C,D}$ by means of the PHM as outlined in *8.1.4*.

Step 3. On the basis of value R established at Step 2, calculate the project's utility parameter $U(C,D,R)$.

Step 4. The process of diminishing value C with fixed value D proceeds until either:

- utility $(\mathrm{C,D,R})$ in the course of decreasing value C ceases to increase. In this case the last "successful" value C is taken as the quasi-optimal value *for the first iteration*.
- value R determined at Step 2, proves to be less than value R_0. This means that we have reached the non-feasible zone. In this case we act similarly to the preceding case.
- Value C in the course of its decreasing reaches value C_{00}. In this case

$C = C_{00}$ is taken as the quasi-optimal value.

Step 5. Fix value C and start decreasing value D by search step ΔD, similarly to Steps 2-4.

Thus, in the course of Steps 1-5, we determine values $\left[C^{(v)},\ D^{(v)} \right]$ for the first iteration (v=1) of the $CCSA$. Note that the couple obtained satisfies restrictions (8.1.7-8.1.9) and results in the highest value $U(G)$ from all the couples already observed.

Stage II. Routine v-th iteration, $v>1$.

Step 6. Diminish search step values ΔC and ΔD by dividing them by two.

Step 7. Fix $D = D^{(v-1)}$ and check two alternative directions: $C^{(v-1)} + \Delta C$, $C^{(v-1)} - \Delta C$, where ΔC has been already decreased at Step 6. If couple $\left[C^{(v-1)} + \Delta C, D^{(v-1)} \right]$ delivers higher values of the reliability parameter $R(G)$(calculated by means of the PHM) than the alternative attempt $\left[C^{(v-1)} - \Delta C, D^{(v-1)} \right]$, and if value $R(G)_{C^{(v-1)}+\Delta C, D^{(v-1)}}$ exceeds $R(G)_{C^{(v-1)}, D^{(v-1)}}$, choose the search point $\left[C^{(v-1)} + \Delta C, D^{(v-1)} \right]$ and proceed in that direction, i.e., use $C = C^{(v-1)} + r\Delta C$, $r = 1, 2, \ldots$, with fixed $D = D^{(v-1)}$. The search process for value C terminates in cases discussed at Step 4. Thus, value C is fixed and remains constant.

Step 8. A local search procedure similar to that outlined at Step 7, has to be implemented for value $D^{(v-1)}$ (with fixed $C = C^{(v-1)}$). Thus, at the end of the step a triple $C^{(v)}$, $D^{(v)}$, $R(G)_{C^{(v)}, D^{(v)}}$ is determined, resulting in higher project's utility than the previous point $\left(C^{(v-1)}, D^{(v-1)}, R^{(v-1)} \right)$.

Step 9. Iterations with increasing index v are carried out similarly to Steps 6-8. After each routine iteration $v>1$, the corresponding project's utility $U^{(v)} = U\left(C^{(v)}, D^{(v)}, R^{(v)} \right)$ is compared with that obtained at the previous, $(v-1)$ -th, iteration. If relation

$$\frac{U^{(v)} - U^{(v-1)}}{U^{(v-1)}} < \varepsilon \tag{8.1.20}$$

holds, the search procedure terminates and values $\left[C^{(v)}, D^{(v)}, R^{(v)} = R(G)_{C^{(v)}, D^{(v)}} \right]$ are taken as the quasi-optimal ones.

§8.2 Harmonization models for several stochastic network projects

8.2.1 Introduction

This section is actually a continuation of the previous §8.1 and considers a complicated hierarchical system comprising a variety of projects of different significance. Such projects usually emerge in constructing new industrial and populated areas, where the significance of certain local projects entering the system may undergo changes within the projects' realization. The latter often happens in the course of changing management policy as well as the economic situation.

Another harmonization model covers a simplified although important case when all projects happen to be of equal significance and do not undergo drastic changes in the course of their implementation. The harmonization model becomes simpler in usage, and is based on determining optimal utility values via minimax principles.

In order to outline the harmonization model we will require the following additional terms:

$G_k(N,A)$ - the k-th PERT-COST type stochastic network project, $1 \le k \le n$;

n - number of projects entering the project management system;

D_k - due date for the k-th project (pregiven);

η_k - priority value of the k-th project (pregiven);

C_k - budget assigned to the k-th project (to be determined);

C - budget at the management's disposal, $C = \sum_{k=1}^{n} C_k$;

$R_k = R_k(C_k, D_k)$- the maximal project's reliability value corresponding to the optimal reassignment of budget C_k among the project's activities;

U_S - the system's utility for projects of different significance;

U_k - the k-th project's utility:
$U_k = \alpha_{C_k}(C_{0k} - C_k) + \alpha_{D_k}(D_{0k} - D_k) + \alpha_{R_k}[R_k - R_{0k}]$;

α_{C_k} - the budget partial utility coefficient (pregiven);

C_{0k} - the least permissible budget value to be assigned to the k-th project (pregiven);

α_{D_k} - the due date partial utility coefficient (pregiven);

D_{0k} - the least permissible due date for the k-th project (pregiven);

α_{R_k} - the reliability partial utility coefficient (pregiven);

R_{0k} - the least permissible reliability value for the k-th project to be accomplished on time (pregiven);

$(i,j)_k$ - activity (i,j) of the k-th project;

$c(i,j)_k$ - budget value assigned to $(i,j)_k$;

$T_k\left(c(i,j)_k\right)$ - random duration of the k-th project on condition that budget $c(i,j)_k$ is assigned to each activity $(i,j)_k$ entering $G_k(N,A)$;

C_{00k} - the minimal possible budget value to be assigned to the k-th project (pregiven);

D_{00k} - the minimal due date for the k-th project (pregiven);

R_{00k} - the maximal possible reliability value of the k-th project (pregiven);

$c(i,j)_{k\ min}$ - the minimal possible budget value to be assigned to the activity $(i,j)_k$ (pregiven);

$c(i,j)_{k\ max}$ - the maximal required budget value for $(i,j)_k$ (pregiven);

ρ_{kD_k} - a constant value defining the linear dependence of parameter $R_k(C_k, D_k)$ from C_k with fixed D_k (to be calculated);

$t(i,j)_k$ - random duration of activity $(i,j)_k$;

$a(i,j)_k = \dfrac{A(i,j)_k}{c(i,j)_k}$ - the lower bound of $t(i,j)_k$;

$b(i,j)_k = \dfrac{B(i,j)_k}{c(i,j)_k}$ - the upper bound of $t(i,j)_k$;

$A(i,j)_k$, $B(i,j)_k$ - pregiven values for each activity $(i,j)_k$, $1 \le k \le n$;

ΔC_k - search step length in linear programming models for the optimal value C_k, $1 \le k \le n$ (pregiven);

ΔD_k - search step length in the cyclic coordinate search algorithm for the optimal value D_k, $1 \le k \le n$ (pregiven);

N_{kC} - the number of possible search steps for variable C_k (pregiven); note that relation $C_{0k} - C_{00k} = N_{kC} \cdot \Delta C_k$ holds;

N_{kD} - the number of possible search steps for variable D_k (pregiven); note that relation $D_{0k} - D_{00k} = N_{kD} \cdot \Delta D_k$ holds;

δC - cost unit for budget values (pregiven);

δD - time unit for the due date values (pregiven).

8.2.2 The system's description

The project management (company) is faced with managing n PERT-COST type network projects $G_k(N,A)$, $1 \le k \le n$, which have to be carried out. The projects are of different importance and significance; for each k-th project the corresponding priority index η_k is externally pregiven. However, in the course of the projects' realization values η_k, $1 \le k \le n$, may undergo changes. The total budget C at the management's disposal to carry out all the projects, is limited. For each project $G_k(N,A)$, $1 \le k \le n$, its due date D_k, as well as the assigned budget C_k, have to be determined.

For each activity $(i,j)_k$ entering project $G_k(N,A)$ two budget values are externally pregiven:

- $c(i,j)_{k\ min}$ - the minimal possible budget still enabling to carry out activity $(i,j)_k$;
- $c(i,j)_{k\ max}$ - the maximal budget required to carry out activity $(i,j)_k$.

Thus, the actual budget value $c(i,j)_k$ assigned to activity $(i,j)_k$, is restricted by its upper and lower bounds. In case $c(i,j)_k > c(i,j)_{k\ max}$ additional budget is redundant.

Activity duration $t(i,j)_k$ is a random value with a beta-distribution p.d.f.

$$p_{ij}(x)_k = \frac{12}{\left[b(i,j)_k - a(i,j)_k\right]^4}\left[x - a(i,j)_k\right]\left[b(i,j)_k - x\right]^2, \qquad (8.2.1)$$

where $a(i,j)_k = \dfrac{A(i,j)_k}{c(i,j)_k}$, $b(i,j)_k = \dfrac{B(i,j)_k}{c(i,j)_k}$, values $A(i,j)_k$ and $B(i,j)_k$ being pregiven, $1 \le k \le n$.

Regarding Chapter 5 and §8.1, each project comprises three essential, basic parameters which define the project's utility:

- budget C_k assigned to each project $G_k(N,A)$, $1 \le k \le n$;
- the appropriate due date D_k;
- reliability parameter $R_k(C_k, D_k)$,

$$R_k(C_k, D_k) = \underset{\{c(i,j)_k\}}{Max} \Pr\left\{ T_k \le D_k \,\middle|\, \sum_{(i,j)_k} c(i,j)_k = C_k \right\}, \qquad (8.2.2)$$

where T_k signifies the moment project $G_k(N,A)$ is completed (a random value), on condition that budget C_k is assigned to $G_k(N,A)$ and optimally reallocated between activities $(i,j)_k$. It goes without saying that relation $C_k > \sum_{(i,j)_k} c(i,j)_k$ holds, otherwise project $G_k(N,A)$ cannot be carried out.

For each k-th project its utility U_k is calculated as follows:

$$U_k = \alpha_{C_k}\left(C_{0k} - C_k\right) + \alpha_{D_k}\left(D_{0k} - D_k\right) + \alpha_{R_k}\left[R_k\left(C_k, D_k\right) - R_{0k}\right] , \qquad (8.2.3)$$

where C_{0k}, D_{0k} and R_{0k} are the least permissible basic values that can be accepted in the course of the project's realization, $1 \le k \le n$, while α_{C_k}, α_{D_k} and α_{R_k} stand for local (partial) utilities per each parametrical unit.

For the models under consideration we will henceforth accept a reasonable assumption [9, 70, 109] stating that:

- *reliability parameter $R_k\left(C_k, D_k\right)$ depends on budget value C_k linearly, i.e., within each project $G_k\left(N, A\right)$ with fixed due date D_k relation*

$$\frac{R_k\left(C_k^{'}, D_k\right) - R_k\left(C_{00k}, D_k\right)}{C_k^{'} - C_{00k}} = \frac{R_k\left(C_k^{''}, D_k\right) - R_k\left(C_{00k}, D_k\right)}{C_k^{''} - C_{00k}} = \rho_{kD_k} \qquad (8.2.4)$$

holds, where $\sum_{(i,j)_k} c(i,j)_{k\,\max} \ge C_{0k} > C_k^{'} > C_k^{''} > C_{00k} \ge \sum_{(i,j)_k} c(i,j)_{k\,\min}$ and ρ_{kD_k} depends only on the project's index k and the due date D_k. Thus, C_{00k} presents the minimal possible budget value assigned to the k-th project. Note that ρ_{kD_k} may alter only when budget reallocation is undertaken at a fixed moment, otherwise the structure of the project $G_k\left(N, A\right)$ may undergo drastic changes. It goes without saying that values ρ_{kD_k} may differ from project to project. Assumption (8.2.4) has been verified by undertaking extensive experimentation [109].

Note, in conclusion, that budget reallocation among the projects has to be carried out:

- at the beginning of the planning horizon, i.e., at $t = 0$;
- at a certain moment t when at least for one k-th routine project values D_k and η_k undergo changes;
- at a certain moment t when one of the projects is accomplished.

Note that for a project management system with projects of different importance we suggest to solve the harmonization problem with objective

$$J_1 = \sum_{k=1}^{n} \left(\eta_k \cdot U_k\right). \qquad (8.2.5)$$

Maximizing objective (4.2.5) means that the project management takes all possible measures first to support projects with higher priorities. Only afterwards it takes care of other, less important, projects.

In case of projects with equal priorities we suggest (see Section *1.3.1*), with respect to [9, 70, 109], implementing another objective satisfying

$$J_2 = \underset{\{C_k, D_k\}}{Max} \; \underset{k}{Min} \; U_k . \tag{8.2.6}$$

Objective (8.2.6) means that for projects with equal significance the project management takes all measures to support the "weakest" projects on the account of the "stronger" and the "faster" ones. That means, in turn, implementing a policy resulting in control actions aimed on projects' leveling, in order to smooth the differing projects' utilities.

8.2.3 Harmonization model for projects with different priorities

As outlined above, the problem is as follows: for each k-th project determine optimal due date D_k and budget value C_k, $1 \le k \le n$, as well as optimal reassignment values for each activity $c(i,j)_k$, $(i,j)_k \subset G_k(N,A)$, to maximize the objective:

$$\begin{aligned} Max\ J_1 &= \underset{\{C_k, D_k\}\{c(i,j)_k\}}{Max} \sum_{k=1}^{n} \left(\eta_k \cdot U_k \right) = \\ &= \underset{\{C_k, D_k\}\{c(i,j)_k\}}{Max} \left\{ \sum_{k=1}^{n} \left[\eta_k \cdot \left[\alpha_{C_k} \left(C_{0k} - C_k \right) + \alpha_{D_k} \left(D_{0k} - D_k \right) + \alpha_{R_k} \left(R_k - R_{0k} \right) \right] \right] \right\} \end{aligned} \tag{8.2.7}$$

subject to

$$\sum_{k=1}^{n} C_k = C , \tag{8.2.8}$$

$$\sum_{\{(i,j)_k\}} c(i,j)_k = C_k , \tag{8.2.9}$$

$$c(i,j)_{k\ min} \le c(i,j)_k \le c(i,j)_{k\ max} , \tag{8.2.10}$$

$$C_{00k} \le C_k \le C_{0k} , \tag{8.2.11}$$

$$D_{00k} \le D_k \le D_{0k} , \tag{8.2.12}$$

$$R_{0k} \le R_k \le R_{00k} , \tag{8.2.13}$$

$$\left(C_k - C_{00k} \right) \rho_{kD_k} = R_k \left(C_k , D_k \right) - R_k \left(C_{00k} , D_k \right), \tag{8.2.14}$$

$$C_{00k} \ge \sum_{\{(i,j)_k\}} c(i,j)_{k\ min} , \tag{8.2.15}$$

$$C_{0k} \le \sum_{\{(i,j)_k\}} c(i,j)_{k\ max} , \quad 1 \le k \le n . \tag{8.2.16}$$

It can be well-recognized that problem (8.2.7-8.2.16) is of extremely complicated nature and excels essentially the most sophisticated models outlined in [9, 11-13, 30].

Let us analyze restrictions (8.2.8-8.2.16) in greater detail. Restrictions (8.2.8-8.2.10) are obvious, as well as parametrical restrictions (8.2.11-8.2.13). Restrictions (8.2.15-8.2.16) determine the lower and upper bounds for the budget value C_k to be assigned to the k-th project, $1 \le k \le n$. As to restriction (8.2.14), it is based on assumption (8.2.4) which has been discussed in Section *8.2.2*. Values ρ_{kD_k}, $1 \le k \le n$, are calculated beforehand by means of simulation on the basis of different combinations (C_{00k}, D_k), (C_{0k}, D_k) for several possible due date values D_k, $D_{00k} \le D_k \le D_{0k}$. The quasi-optimal procedure to be implemented for calculating ρ_{kD_k}, $1 \le k \le n$, is outlined in [9, 11-13].

8.2.4 <u>The problem's solution for projects with different priorities</u>

As outlined above, to optimize problem (8.2.7-8.2.16) we have to solve *an auxiliary problem* as follows: for each k-th project, $1 \le k \le n$, separately, determine reliability values:

$$R_k(C_{00k}, D_{0k}),\ R_k(C_{00k}, D_{0k} - \Delta D_k),\ R_k(C_{00k}, D_{0k} - 2\Delta D_k), \dots ,\ R_k(C_{00k}, D_{00k}) \tag{8.2.17}$$

and

$$R_k(C_{0k}, D_{0k}),\ R_k(C_{0k}, D_{0k} - \Delta D_k),\ R_k(C_{0k}, D_{0k} - 2\Delta D_k), \dots ,\ R_k(C_{0k}, D_{00k}). \tag{8.2.18}$$

Values R_k are determined by means of simulation, in combination with a budget reassignment procedure outlined in Section *8.1.4*.

Later on, by calculating

$$\frac{R_k(C_{0k}, D_{0k} - r \cdot \Delta D_k) - R_k(C_{00k}, D_{0k} - r \cdot \Delta D_k)}{C_{0k} - C_{00k}} = \rho_{k, D_{0k} - r \cdot \Delta D_k}, \tag{8.2.19}$$

$$0 \le r \le N_{kD_k},$$

we obtain approximated values of coefficients ρ_{kD_k} for each k-th routine project, $1 \le k \le n$.

After obtaining values ρ_{kD_k}, $D_k = D_{0k} + r \cdot \Delta D_k$, $0 \le r \le N_{kD_k}$, we suggest the problem (8.2.7-8.2.16) to be solved by implementing the two-level heuristic algorithm as follows:

<u>The upper level</u>

A look-over algorithm to examine the feasibility and efficiency of each of all possible combinations $\{D_k\}$ has to be operated. Note that the total number of combinations N is equal to $\prod_{k=1}^{n}\left(N_{kD_k}+1\right)$, and for large numbers n and N_{kD_k}, $1 \le k \le n$, value N may become extremely high. Thus, checking all combinations on a full look-over basis may require a lot of computational time. For such cases a high-speed approximated algorithm on the basis of a cyclic coordinate search algorithm (see (8.1.5)), is suggested.

The enlarged step-by-step procedure of the cyclic coordinate search algorithm $(CCSA)$ is as follows:

Step 1. Choose an initial search point $\vec{D}^{(0)} = \{D_{01}, D_{02}, \dots, D_{0n}\}$ in the n-dimensional space. In the course of carrying out the cyclic coordinate algorithm each search point $\vec{D}$ will serve as the input information for the algorithm at the lower level, in order to maximize objective (8.2.7) subject to restrictions (8.2.8-8.2.16). Note that *since due dates D_{0k}, $1 \le k \le n$, are maximal possible due dates for each project $G_k(N,A)$, the initial search point $\vec{D}^{(0)}$ has to be in a feasible area - otherwise problem (8.2.7-8.2.16) has no solution*.

Step 2. Start using the cyclic coordinate search method which maximizes the problem's objective (8.2.7) at the lower level with respect to the coordinate variables $D_1, D_2, \dots, D_n$. Decrease the first coordinate D_1 by the search step ΔD_1, while all other coordinates $D_2, D_3, \dots, D_n$ are fixed and remain unchanged. In the course of undertaking a routine search step the feasibility of every routine search point $\vec{D}$ is examined, in order to verify relations $R_k(C_k, D_k) \ge R_{0k}$, $1 \le k \le n$. The process of decreasing the first coordinate D_1 terminates in three cases:

- if for a certain value D_1 relation (8.2.13) ceases to hold;
- if value D_1 reaches its lower bound D_{001};
- if for a certain value D_1 objective (8.2.7) ceases to increase.

In cases 1) and 3) the last successful value D_1 is taken as the quasi-optimal value, while in case 2) value D_{001} is taken as the quasi-optimal one.

Step 3. After the first coordinate D_1 is optimized, its value is fixed, and the search process proceeds by decreasing the second coordinate D_2 by a constant search step, i.e., $D_2^{(0)} - \Delta D_2 = D_2^{(1)}$, while all other coordinates (including D_1) remain fixed. After examining the coordinate D_2 by a step-wise decrease, its newly obtained value is fixed, similarly to D_1, and we proceed with the third coordinate D_3, and so forth, until D_n is reached and checked by the constant step decreasing procedure.

Step 4. After all coordinates are checked by means of the cyclic coordinate search method (first iteration), the process is then repeated starting with D_1 again. The cyclic coordinate search algorithm terminates after a current iteration does not succeed in bringing any changes in the search point $\vec{D}$ and in the value of objective (8.2.7) at the lower level.

The lower level

Two problems have to be solved at the lower level. We will henceforth call them *Problems A1* and *A2*.

Problem A1

After obtaining input information from the upper level - the routine search point-vector $\vec{D}$ - the feasibility of that search point has to be checked. For each k -th coordinate D_k, $1 \le k \le n$, independently, the *minimal* budget value C_k^* satisfying

$$R_k\left(C_k^*, D_k\right) = R_{0k}, \tag{8.2.20}$$

has to be determined. Note that problem (8.2.20) is a dual problem for the direct problem (8.2.2) and can be solved by implementing heuristic approaches and the bisection method [176] in combination with the direct problem (see, e.g. [70, 72, 109]). Thus, for each project $G_k(N, A)$, the dual problem (8.2.20) can be formulated as follows:

$$\underset{\{c(i,j)_k\}}{Min}\left\{C_k = \sum_{\{(i,j)_k\}} c(i,j)_k\right\}, \tag{8.2.21}$$

subject to

$$Pr\left\{T_k \le D_k \middle| \sum_{\{(i,j)_k\}} c(i,j)_k = C_k\right\} = R_{0k}, \tag{8.2.22}$$

$$c(i,j)_{k\ min} \le c(i,j)_k \le c(i,j)_{k\ max}. \tag{8.2.23}$$

After determining values C_k^*, $1 \le k \le n$, checking the feasibility of a routine vector search point $\vec{D}$ results in examining values C and $C^* = \sum_{k=1}^{n} C_k^*$. If $C \ge C^*$, search point $\vec{D}$ does belong to the feasible area. Otherwise the search step which led to that point, should be regarded to as unsuccessful, and search point $\vec{D}$ under consideration has to be substituted by another one.

Problem A2

If the vector search point $\vec{D}$ is found to be a feasible one, optimizing model (8.2.7-8.2.16) can be accomplished by solving *Problem A2* as follows:

From the set of feasible n-dimensional vector-points $\vec{D}$ obtained in the course of carrying out *CCSA* and solving *Problem A1*, determine:

- optimal values of due dates D_k^*,
- optimal budget values C_k^*, $1 \le k \le n$,

in order to maximize objective (8.2.7) subject to restrictions (8.2.8-8.2.16).

Thus, the direct solution of problem (8.2.7-8.2.16) is substituted by a sequential approximate solution of three problems:

CCSA $\Rightarrow$ *Problem A1* $\Rightarrow$ *Problem A2*.

The suggested approximate solution of *Problem A2* is as follows:

Since $R_k(C_k, D_k)$ depends on C_k linearly, we obtain from (8.2.14)

$$R_k(C_k, D_k) = R_k(C_{00k}, D_k) + \rho_{kD_k} \cdot (C_k - C_{00k}). \qquad (8.2.24)$$

Substituting $R_k = R_k(C_k, D_k)$ in (8.2.7) for (8.2.24), we obtain

$$\underset{\{C_k, D_k\}\{c(i,j)_k\}}{Max} \left\{ \sum_{k=1}^{n} \eta_k \cdot \left[\begin{array}{l} C_k \cdot \left(\alpha_{R_k} \cdot \rho_{kD_k} - \alpha_{C_k} \right) + \alpha_{C_k} \cdot C_{0k} + \alpha_{D_k} \cdot D_{0k} - \\ - \alpha_{D_k} \cdot D_k - \alpha_{R_k} \cdot R_{0k} + \alpha_{R_k} \cdot R_k(C_{00k}, D_k) - \\ - \alpha_{R_k} \cdot \rho_{kD_k} \cdot C_{00k} \end{array} \right] \right\}. \qquad (8.2.25)$$

Denoting

$$\begin{cases} \eta_k \cdot \left[\alpha_{R_k} \cdot \rho_{kD_k} - \alpha_{C_k} \right] = \gamma_{kD_k} \\ \eta_k \cdot \left[\begin{array}{l} \alpha_{C_k} \cdot C_{0k} + \alpha_{D_k} \cdot D_{0k} - \alpha_{D_k} \cdot D_k - \alpha_{R_k} \cdot R_{0k} + \\ + \alpha_{R_k} \cdot R_k(C_{00k}, D_k) - \alpha_{R_k} \cdot \rho_{kD_k} \cdot C_{00k} \end{array} \right] = \omega_{kD_k} \end{cases} \qquad (8.2.26)$$

we substitute objective (8.2.25) for

$$\underset{\{C_k, D_k\}\{c(i,j)_k\}}{Max} \left\{ \sum_{k=1}^{n} \left[C_k \cdot \gamma_{kD_k} + \omega_{kD_k} \right] \right\} \tag{8.2.27}$$

subject to (8.2.8-8.2.13, 8.2.15-8.2.16).

Taking into account that ω_{kD_k} does not depend on C_k, objective (8.2.27) can be simplified, namely,

$$\underset{\{C_k, D_k\}\{c(i,j)_k\}}{Max} \left\{ \sum_{k=1}^{n} C_k \cdot \gamma_{kD_k} \right\} \tag{8.2.28}$$

subject to (8.2.8-8.2.13, 8.2.15-8.2.16).

Note that, due to (8.2.4), value γ_{kD_k} may be close to zero. In case $\gamma_{kD_k} > 0$ for all indices k and due dates D_k, $1 \le k \le n$, model (8.2.28) obtains a precise step-wise analytical solution [9, 76, 109] as follows:

Step 1. Consider a routine n-dimensional vector $\vec{D} = D_1, D_2, ..., D_n$.

Step 2. Assign to all projects their minimal budget values C_k^* determined by solving *Problem A1* with due dates $D_1, D_2, ..., D_n$.

Step 3. Reorder sequence γ_{kD_k}, $1 \le k \le n$, in descending order. Let their new ordinal numbers be $f_1, f_2, ..., f_n$.

Step 4. Set $a = 1$.

Step 5. Calculate $Z_a = Min\left[C_{0k} - C_{00k}, \Delta C\right]$, where $\Delta C = C - \sum_{k=1}^{n} C_k^*$.

Step 6. Determine for project G_{f_a} its final budget $C_{f_a} = C_{f_a}^* + Z_a$.

Step 7. Update the remaining budget $\Delta C - Z_a \Rightarrow \Delta C$. If $\Delta C = 0$ go to Step 10. Otherwise apply the next step.

Step 8. Set $a + 1 \Rightarrow a$.

Step 9. If $a \le n$ go to Step 5. Otherwise apply the next step.

Step 10. If all n-dimensional vectors $\vec{D}$ (obtained in the course of carrying out the *CCSA*) have been already examined, go to Step 13. Otherwise apply the next step.

Step 11. Calculate objective (8.2.7) for the routine vector $\vec{D}$ under consideration. Denote the obtained value by $J\left(\vec{D}\right)$. If $J\left(\vec{D}\right)$ exceeds the *maximal utility obtained by examining previous vectors* $\vec{D}$, apply the next step. Otherwise go to Step 1.

Step 12. Set $J\left(\vec{D}\right)$ equal to the maximal utility which has been obtained as yet in the course of solving problem (8.2.7-8.2.16). Remember vector $\vec{D}$ together with optimal values $\vec{C}_k$, $1 \le k \le n$, obtained at Steps 2-6, in a special array. Go to Step 1.

Step 13. Determine the final problem's solution, i.e., n-dimensional optimal vectors $\vec{C}_k$ and $\vec{D}_k$, together with the system's maximal utility U_S.

Step 14. The algorithm of solving *Problem A2* terminates.

Note that if even for one index k or due date value D_k multiplicand γ_{kD_k} becomes negative, a precise analytical solution by implementing Steps 2-7 cannot be obtained. In that case problem (8.2.8-8.2.13, 8.2.15-8.2.16, 8.2.28) has to be solved by using standard linear programming models. We recommend a standard computer software package called LINDO [165] which is both simple in usage and can be applied to solve *Problem A2* on common personal computers.

Note, in conclusion, that after obtaining the solution of harmonization problem (8.2.7-8.2.16), one may calculate the system's utility parameter as

$$U_S = \sum_{k=1}^{n} U_k , \tag{8.2.29}$$

without using priority indices η_k. Thus, the system's utility U_S and objective (8.2.7) may not coincide. However, choosing the proper way to calculate U_S is the sole prerogative of the system's management.

8.2.5 Harmonization model for several PERT-COST projects with different priorities

It can be well recognized that the outlined above harmonization model is essentially more complicated than in the case of a single project. First, we use practically a multi-level model. Second, a $PHM(C,D)=R$ model for single projects, together with inverse PHM problems of $PHM(D,R)=C$ type, are implemented at various stages of the harmonization process. An enlarged step-wise description of the harmonization algorithm is as follows [9]:

Stage 1. Undertake a one-dimensional search for choosing the optimal total budget value C within the interval $[C_0, C_{00}]$. For each routine search value C apply the next stage.

Stage 2. Use partial harmonization models $PHM(C_{00k}, D_{0k} - r\Delta D_k) = R$ and $PHM(C_{0k}, D_{0k} - r\Delta D_k) = R$ *for a single project* in order to determine values ρ_{kD_k}, $1 \le k \le n$.

Stage 3. Undertake a cyclic coordinate look-over search in the n-dimensional space of local due dates D_k, $1 \le k \le n$.

Stage 4. For each feasible vector search point $\vec{D}$ obtained at Stage 2, calculate the minimal possible local budgets C_k^*. The latter may be determined by solving a dual problem to the direct problem $PHM(C_k, D_k) = R_k$, i.e., $PHM(D_k, R_k) = C_k$, with preset values $R_k = R_{0k}$.

Stage 5. After obtaining values C_k^*, $1 \le k \le n$, use a precise step-wise analytical procedure to determine corrected values C_k, satisfying $\sum_{k=1}^{n} C_k = C$.

Stage 6. Calculate the system's utility U_S by (8.2.7). Return to Stage 3.

Stage 7. Stages 3-6 terminate if in the course of performing a routine *CCSA* iteration (for a fixed C) the system's utility has not increased. Thus, we obtain a local maximum. Go to Stage 1.

Stage 8. The harmonization algorithm terminates by calculating the global maximum utility value U_S after undertaking a search for the optimal value C.

Thus, practically speaking, for the problem under consideration, the total budget value C is the only independent parameter, while local budgets C_k, due dates D_k and reliabilities R_k prove to be dependent ones.

8.2.6 Harmonization model for projects of equal significance

As outlined above, in Section *8.2.2*, the problem of maximizing the system's utility can be formalized as follows: for each k-th project determine budget value C_k and due date D_k, $1 \le k \le n$, *to maximize utility of the project with the least utility value*, namely,

$$J_2 = \underset{\{C_k, D_k\}}{Max} \; \underset{k}{Min} \; U_k = \underset{\{C_k, D_k\}}{Max} \; \underset{k}{Min} \left[\alpha_{C_k} \left(C_{0k} - C_k \right) + \alpha_{D_k} \left(D_{0k} - D_k \right) + \alpha_{R_k} \left(R_k - R_{0k} \right) \right] \quad (8.2.30)$$

subject to (8.2.8-8.2.16).

Since the maximin approach has presented itself in a very good light both in production planning and control [9, 56-57, 124] and in project management [76-77, 109], objective (8.2.30) may be suggested as a priority technique for improving utility values in complicated project management systems operating under random disturbances. Note that providing the optimal value to objective J_2 in the course of solving problem (8.2.8-8.2.16, 8.2.30), does not define directly the system's utility parameter U_S. For calculating the latter value, we recommend using relation (8.2.29) with values U_k obtained while solving the problem.

As to the algorithm for solving problem (8.2.8-8.2.16, 8.2.30), its several main stages, namely,

- the auxiliary problem to calculate values ρ_{kD_k};
- the cyclic coordinate search algorithm $CCSA$ at the upper level;
- *Problem A1* to verify the feasibility of the routine n-dimensional vector $\vec{D}$ obtained by using the $CCSA$,

\- fully coincide with the corresponding stages outlined in the preceding Section. The only difference lies in the approximate solution of *Problem A2* to maximize objective (8.2.30) subject to restrictions (8.2.8-8.2.16).

The suggested approximate solution of Problem A2 for the case of projects with equal significance is as follows.

Substituting $R_k\left(C_k, D_k\right)$ in (8.2.30) for (8.2.24), we obtain

$$\underset{\{C_k, D_k\}}{Max}\left[\underset{k}{Min}\left\{\begin{array}{l}\alpha_{C_k}\cdot\left(C_{0k}-C_k\right)+\alpha_{D_k}\cdot\left(D_{0k}-D_k\right)+\\+\alpha_{R_k}\cdot\left[R_k\left(C_{00k}, D_k\right)+\rho_{kD_k}\left(C_k-C_{00k}\right)-R_{0k}\right]\end{array}\right\}\right]. \quad (8.2.31)$$

Implementing notations (8.2.26), we obtain an optimization problem as follows:

Maximize

$$J_2 = \underset{\{C_k, D_k\}}{Max}\left[\underset{k}{Min}\left\{\gamma_{kD_k}\cdot C_k+\omega_{kD_k}\right\}\right] \quad (8.2.32)$$

subject to (8.2.8-8.2.13, 8.2.15-8.2.16).

Substitution

$$\underset{k}{Min}\left\{\gamma_{kD_k}\cdot C_k+\omega_{kD_k}\right\} = Z \quad (8.2.33)$$

modifies problem (8.2.8-8.2.13, 8.2.15-8.2.16, 8.2.32) to the following one:

$$\underset{\{C_k, D_k\}}{Max}\ Z \quad (8.2.34)$$

subject to

$$Z \le \gamma_{kD_k}\cdot C_k+\omega_{kD_k} \quad (8.2.35)$$

and restrictions (8.2.8-8.2.13, 8.2.15-8.2.16).

Problem (8.2.8-4.2.13, 8.2.15-8.2.16, 8.2.34-8.2.35) can be solved by implementing linear programming methods, e.g., the standard software package LINDO [165].

The problem has to be solved for each feasible combination $D_1, D_2, \dots, D_n$ separately. The combination delivering the maximum value to objective J_2, has to be taken as the optimal one, together with the corresponding values $C_1, C_2, \dots, C_n$. The optimal system's utility has to be determined by (8.2.29).

It can be well-recognized that the corresponding harmonization algorithm outlined in *8.2.5*, does not undergo drastic changes for the case of several projects with equal priorities. The only difference results in substituting the precise solution at Stage 5 of the problem by a linear programming procedure.

§8.3 Harmonization models in project management with safety engineering concepts

8.3.1 Introduction

In previous §§8.1-8.2 we have developed algorithms for determining the utility values in project management for both a single PERT-COST type project with random activity durations as well as for the case of several simultaneously realized projects of different importance and significance. A number of basic parameters:

- the budget assigned to the project;
- the project's due date;
- the project's reliability, i.e., the probability of meeting the project's due date on time,

\- enter the harmonization model which results in a trade-off between the basic parameters. We suggest supplementing the latter by a new essential parameter defining the quality of the project as a whole, namely, the probability of a hazardous failure in the course of carrying out the project.

The backbone of §8.3 is to introduce a new basic criterion which is usually difficult to be formalized; it requires human judgment and rating schemes in order to obtain quantitative estimates by means of expert information.

Note that the harmonization theory outlined in Chapter 5 and applied to project management in §§8.1-8.2, is based on determining and formalizing functional dependencies between the basic parameters and, later on, developing optimization models including both harmonization and partial harmonization models. Expert information has not been taken into account. From the other side, the widely used multi-attribute utility theory (MAUT, outlined in §5.1) is practically based on expert information in the form of various interview questions being addressed to experts. However the MAUT, as distinct from the harmonization theory, does not deal with any system's functioning and is restricted to market competitive problems alone.

Another main achievement outlined in §8.3 is the development of a mixed type harmonization model: it deals with the quality of the system's functioning but, besides formalized techniques and optimization models, is partially based on expert information.

From expert information a conclusion can be drawn that in the case of a PERT-COST project a hazardous failure capable of jeopardizing environmental or personnel's safety, depends mostly on the following project's control actions:

- decreasing the project's due date, and
- increasing the intensity of the project's realization without undertaking proper safety engineering measures.

This discussion is a further development of our previous §§8.1-8.2 where three-parametrical harmonization models have been suggested. A formalized four-parametrical harmonization model accompanied by a heuristic solution is developed. The model, thus, is based on a cost – time – reliability – safety trade-off.

8.3.2 Basic parameters

In order to formalize the four-parametrical harmonization problem the suggested basic parameters [9, 22-23] are as follows:

- the budget C assigned to the project (an independent parameter);
- the due date D for the project to be accomplished (an independent parameter);
- the project's reliability R, i.e., the probability of meeting its due date on time subject to budget C (a dependent parameter, which depends on C and D and can be obtained through simulation by means of solving a partial harmonization problem outlined in Chapter 5);
- the project's probability P_{hf} of a hazardous failure within the project's realization (a dependent parameter, which is defined by values C and D and can be obtained through expert information).

We suggest to evaluate the project's utility by (see *Notation* in §8.1)

$$U = \alpha_C \cdot [C_0 - C] + \alpha_D \cdot [D_0 - D] + \alpha_R \cdot [R - R_0] + \alpha_P \cdot [P_0 - P], \qquad (8.3.1)$$

where C_0, D_0, R_0 and P_0 are the least permissible budget, due date, reliability and hazardous probability values which can be implemented in a PERT-COST project, while values C, D, R and P_{hf} are the corresponding current values for a project under consideration. Linear coefficients α_C, α_D, α_R and α_P, which are pregiven beforehand, define partial utilities per corresponding unit values δ_C, δ_D, δ_R and δ_P. As to the corresponding unit values $\delta_C \div \delta_P$, they are usually defined in money terms (both for R and P_{hf}), correspondingly.

8.3.3 Harmonization model

The harmonization model is as follows (see *Notation* in §8.1): determine optimal non-contradictive project parameters $C^{(opt)}$, $D^{(opt)}$, $R^{(opt)}$, $P^{(opt)}$ resulting in the maximal project's utility (8.3.1) subject to

$$C_{00} \le C^{(opt)} \le C_0, \tag{8.3.2}$$

$$D_{00} \le D^{(opt)} \le D_0, \tag{8.3.3}$$

$$R_{00} \ge R^{(opt)} \ge R_0, \tag{8.3.4}$$

$$P_{00} \le P^{(opt)} \le P_0 . \tag{8.3.5}$$

Values C, D, R and P are called *non-contradictive* if relations

$$R = R(C,D) = {}^{PHM}\!/_{C,D}, \tag{8.3.6}$$

$$P = P_{hf}(C,D) \tag{8.3.7}$$

subject to

$$\sum_{(i,j)} c_{ij} = C \tag{8.3.8}$$

hold. As outlined above, parameters C and D are input values of the model. Value (8.3.6) is optimized by means of a heuristic procedure outlined in §8.1. Value P_{hf} is calculated on the basis of dependency $P_{hf}(C,D)$ obtained by means of statistical analysis and expert information.

Solving problem (8.3.1-8.3.5) can be achieved by solving the main problem (to determine an optimal budget value C and an optimal due date D) and two subsidiary problems as following:

- solving the optimal PHM problem, i.e., determining $R(C,D)$, and
- calculating $P_{hf}(C,D)$ on the basis of expert information.

According to §5.3 and §8.1, we suggest to substitute an exact lookover algorithm, that checks the feasibility of each possible combination (C,D), by a cyclic coordinate search algorithm $(CCSA)$ outlined in *8.1.5*.

8.3.4 Decision-making rules for estimating hazardous failures' probabilities

It can be well-recognized that the hazardous failure's probability value $P_{hf}(C,D)$ fully depends on values C and D. Thus, any changes in value P_{hf} depend on alterations of those values.

In the course of applying the developed theoretical results to a variety of real projects [9], various interview questions have been addressed to experts. All the answers have been carefully analyzed in order to develop decision-making rules to forecast changes in value P_{hf} by changing values C and D. Note that in the course of undertaking $CCSA$ in the two-dimensional space, only one coordinate (either C or D) can be altered by taking a search step; both coordinates can never be changed simultaneously. The developed decision-making rules are as follows [9]:

I. In case of decreasing value C with fixed D the probability of a hazardous failure remains unchanged.

II. In case of decreasing the due date D with fixed budget value C probability P_{hf} may increase. If the project's due date value D_1 is diminished by ΔD, a ratio

$$\gamma = \frac{\Delta D}{D_{max} - D_{min}} \tag{8.3.9}$$

has to be calculated. Here D_{max} is the maximal duration to carry out the project from the practical point of view, while D_{min} represents the minimal project's duration which by no means can be further decreased. The approximated estimate of value P_{hf} can be evaluated by

$$\Delta P_{hf} \cong \gamma \cdot f(D_1), \tag{8.3.10}$$

where $f(D_1)$ depends on the closeness to value D_1 and can be estimated by the following relation:

$$f(D_1) \cong \begin{cases} A & \text{if} \quad 0.67D_{max} + 0.33D_{min} \le D_1 \le D_{max} \quad \text{holds}, \\ 1.5A & \text{if} \quad 0.33D_{max} + 0.67D_{min} \le D_1 < 0.67D_{max} + 0.33D_{min} \quad \text{holds}, \\ 2A & \text{if} \quad D_{min} < D_1 \le 0.33D_{max} + 0.67D_{min} \quad \text{holds}. \end{cases} \tag{8.3.11}$$

Here A depends on the project's peculiarities and is estimated by the expert team beforehand, i.e., before the project starts to be carried out.

III. In case of increasing the due date D with fixed budget value C from D_1 to D_2, probability P_{hf} will decrease on a scale equal to that which can be estimated by decreasing value D from D_2 to D_1 (see Case II).

IV. In case of increasing budget value C with fixed due date D, probability P_{hf} is not likely to undergo drastic changes. On the one hand, increasing budget value C results in enhancing the project's speed and, thus, may stimulate increasing the hazardous failure's probability. On the other hand, a part of the supplementary budget will certainly be assigned to strengthening various safety engineering related issues relevant to the regarded project. This usually equilibrates the raised intensity of the project's realization.

As outlined above, the developed decision-making rules have been formulated partially on the basis of interviews held with experts, and partially by analyzing similar projects carried out in recent years. Those rules have been implemented in the newly developed harmonization theory, at the stage of calculating the project's utility value.

§8.4 Using harmonization models in project risk analysis, planning and control

8.4.1 Harmonization models for managing PERT-COST network projects

We will consider in greater detail the case when a PERT-COST activity-on-arc network project serves as a system's model in order to undergo harmonization. The project $G(N,A)$ comprises activities $(i,j)\subset A\subset G(N,A)$ of random durations t_{ij} with pregiven p.d.f. depending parametrically on the budget value c_{ij} assigned to each activity (i,j). It is assumed that for each activity (i,j) its p.d.f. $p(t_{ij}/c_{ij})$ satisfies the beta-distribution

$$p(t_{ij}/c_{ij}) = \frac{12}{(b_{ij}-a_{ij})^4}(t-a_{ij})(b_{ij}-t)^2, \qquad (8.4.1)$$

where $a_{ij}=\frac{A_{ij}}{c_{ij}}$, $b_{ij}=\frac{B_{ij}}{c_{ij}}$, A_{ij}, B_{ij} - constants, and c_{ij} satisfies restrictions

$$c_{ij\ min} \le c_{ij} \le c_{ij\ max} \qquad (8.4.2)$$

with pregiven values of $c_{ij\,\min}$ and $c_{ij\,\max}$.

Given the total project's budget $C\ge\sum_{\{(i,j)\}} c_{ij\,\min}$ and the due date D, the problem is to evaluate the project's optimal utility value given in the linear form

$$U = \underset{C,D,R}{Max}\ \{\alpha_C(C_0-C)+\alpha_D(D_0-D)+\alpha_R(R-R_0)\}, \qquad (8.4.3)$$

where C_0, D_0 and R_0 are the least permissible budget, due date and reliability values which can be implemented in a PERT-COST project, while values C, D and R are current values for the project under consideration. Linear rates α_C, α_D and α_R, i.e., the partial utilities, are pregiven as well. It has been demonstrated earlier in this Chapter that reliability value R is obtained by means of optimizing the partial harmonization model $PHM\{C, D\} = R$ as follows:

determine $c_{ij}^{(opt)}$ *to maximize the objective*

$$\underset{\{c_{ij}\}}{Max}\, R = \underset{\{c_{ij}\}}{Max} \left[Pr\left\{ T\{G/c_{ij}\} < D \right\} \right] \tag{8.4.4}$$

subject to (8.4.2) and

$$C = \sum_{\{i,j\}} c_{ij}^{(opt)}, \tag{8.4.5}$$

$$C_{00} \leq C \leq C_0, \tag{8.4.6}$$

$$D_{00} \leq D \leq D_0, \tag{8.4.7}$$

$$R_0 \leq R \leq R_{00}. \tag{8.4.8}$$

Undertaking a relatively simple lookover in the two-dimensional area of C and D and determining for each couple (C, D) the corresponding $R = PHM\{C, D\}$ in order to maximize objective (8.4.3) enables establishing the project's utility U.

In case when project $G(N, A)$ is represented in a formalized shape and activities $(i, j) \subset G(N, A)$ do not bear any engineering definitions and have an abstract meaning, we suggest using harmonization modeling as the project's planning and control technique. Note that undertaking harmonization modeling for the project under consideration results in optimal budget reallocation among the project's activities. This basic assertion will be used later on, by implementing the project's on-line control.

We suggest a step-wise procedure to control the PERT-COST network project by means of harmonization as follows:

Step 0. Given the input information:

- PERT-COST project $G(N, A)$;
- pregiven values $c_{ij\ min}$, $c_{ij\ max}$, A_{ij} and B_{ij} for each activity $(i, j) \subset A \subset G(N, A)$;
- pregiven partial utilities α_C, α_D and α_R;
- pregiven admissible intervals $[C_{00}, C_0]$, $[D_{00}, D_0]$ and $[R_0, R_{00}]$.

Step 1. Undertake harmonization modeling for $G(N, A)$ beforehand, i.e., before the project actually starts to be carried out. Denote the corresponding optimized values which define the maximal project's utility, by C^*, D^* and R^*. Note that restrictions

$$\begin{cases} C_{00} \le C^* \le C_0 \\ D_{00} \le D^* \le D_0 \\ R_0 \le R^* \le R_{00} \end{cases} \quad (8.4.9)$$

hold, otherwise harmonization cannot be accomplished.

Step 2. If budget value C^* is accepted, reassign C^* among the project's activities according to values $c_{ij}^{(opt)}$ obtained in the course of undertaking harmonization at Step 1. Afterwards the project starts to be carried out.

Step 3. In [70, 109], a control model for PERT-COST projects is outlined. The model determines planned trajectories, observes at each control point the progress of the project and its deviation from the planned trajectory, and establishes the next control point. This control model has to be implemented at Step 3, in order to determine the routine control point $t > 0$.

Step 4. At each control point t the progress of the project is observed, i.e., network graph $G(N, A)$ has to be updated at point t, as well as the remaining budget C^*. Denote those values by $G_t(N, A)$ and C_t^*, correspondingly.

Step 5. At each routine control point $t > 0$ solve harmonization problem in order to reallocate later on the remaining budget C_t^* among remaining activities $(i, j) \subset A_t \subset G_t(N, A)$. Denote the corresponding optimal budget values by $c_{ij\,t}^{(opt)}$.

Step 6. Reallocate, if necessary, budget C_t^* among activities $(i, j) \subset A_t$ according to the results of Step 5. Note that *implementing numerous budget reallocations is actually the only control action in the course of performing on-line control*. Go to Step 3.

Step 7. The algorithm terminates after inspecting the project at the due date D, i.e., at the last control point.

It can be well-recognized that, besides undertaking on-line procedures, the suggested step-wise algorithm comprises both harmonization modeling and risk analysis models. Indeed, the latter are not similar to traditional risk management methods which involve technological risks, uncertainties in products' marketing, etc. However, optimal budget reallocation serves actually as a regulation model under random disturbances and can be regarded as a risk analysis element.

Note that in the course of the project's realization certain parameters entering the input information may undergo changes, e.g., restriction values C_0, C_{00}, R_0, R_{00}, D_0, D_{00}, as well as partial utility values α_C, α_D and α_R. New values have to be implemented in the harmonization model in order to facilitate optimal budget reallocation among the remaining project's activities at Step 5 of the algorithm. If problem (8.4.4-8.4.8) has no solution, the decision-making to be undertaken at the company level results either in obtaining additional budget value ΔC or in increasing the due date by ΔD. Both values can be determined by means of harmonization.

8.4.2 Harmonization models for analyzing alternatives and scenarios

We will consider the case of a large complicated project with a high level of uncertainty both in technology and at the marketing stage of the project's life cycle. To manage such projects risk analysis methods similar to RAER or SCERT [39, 45] have to be implemented. Those methods, which we will use henceforth as benchmarks, deal with analyzing various alternatives or scenarios which may be presented in the form of deterministic network sub-projects of CPM type. On the basis of those sub-projects “time – cost” trade-offs outlined in §5.2, are usually carried out. We will henceforth call those deterministic trade-offs the CPM-COST projects.

We suggest, if possible, to present those scenarios in the form of stochastic PERT-COST network projects and to substitute the former "time – cost" trade-off by a harmonization model. We will demonstrate that the newly developed trade-off optimization model is more effective than the former CPM-COST ones outlined in [45].

In order to perform a proper comparison we have to use similar input information. Since an overwhelming majority of both researchers and practitioners accept the beta-distribution as a probability law for random activities' durations [9, 16-17, 31, 42, 54, 58, 67, 70, 109, 111, 116, 134, 138, 166-167, 183-184] with the p.d.f. of the activity time t_{ij}

$$f_{ij}(t) = \frac{\Gamma(\alpha+\beta)}{\Gamma(\alpha)\,\Gamma(\beta)} \frac{\left(t-a_{ij}\right)^{\alpha-1}\left(b_{ij}-t\right)^{\beta-1}}{\left(b_{ij}-a_{ij}\right)^{\alpha+\beta-1}}, \quad a<t<b, \quad \alpha,\beta>0, \tag{8.4.10}$$

where a_{ij} stands for the optimistic time and b_{ij} is the pessimistic time.

In order to simplify the model the p.d.f. in the PERT statements can be modified [9, 67, 70, 109] to

$$f_{ij}(t) = \frac{12}{\left(b_{ij}-a_{ij}\right)^4}\left(t-a_{ij}\right)\left(b_{ij}-t\right)^2 \tag{8.4.11}$$

with the mean

$$\mu_{ij} = 0.2\left(3a_{ij}+2b_{ij}\right). \tag{8.4.12}$$

Thus, introducing the assumption about the p.d.f. (8.4.10) and taking into account that (8.4.10) depends on c_{ij} parametrically, relations (8.4.1-8.4.2) hold.

Since trade-off models in RAER [45] are based on deterministic time – cost trade-off for CPM-COST models (5.2.1-5.2.4), the similarity of input information for both models (harmonization and RAER) can be provided by setting

$$t_{ij} = \frac{0.6\,A_{ij} + 0.4\,B_{ij}}{c_{ij}} \tag{8.4.13}$$

for

$$c_{ij} = \begin{cases} c_{ij\ min} \\ 0.5\left(c_{ij\ min} + c_{ij\ max}\right) \\ c_{ij\ max} \end{cases}, \tag{8.4.14}$$

where values A_{ij}, B_{ij}, $c_{ij\,\min}$, $c_{ij\,\max}$ are similar to those outlined in (8.4.1), and (8.4.13) are obtained by substituting the p.d.f. (8.4.11) by its mean value (8.4.12).

Let us compare the "time – cost" trade-off CPM-COST model (5.2.1-5.2.4) and the harmonization model (8.4.4-8.4.8), taking into account that *actually* activity durations t_{ij} are random variables. It is widely known [see, e.g., 67, 70, 109, 142-144, 174, 184] that substituting all p.d.f. activities by their mean values results in essential statistical bias errors for optimization models' objectives (sometimes up to 40-50%). Those errors underestimate the objective, e.g., the critical path's duration. Thus, substituting deterministic trade-offs by harmonization modeling prevents this shortcoming.

Second, implementing harmonization modeling by means of PERT-COST network projects enables decision-making by using reliability parameter R which is difficult to be analyzed by means of solving CPM-COST problem (5.2.1-5.2.4). And, third, by using CPM-COST trade-offs of type (5.2.1-5.2.4) only several scenarios can be examined (since no risk analyst can take into account numerous alternatives), as distinct from harmonization models when the whole spectrum of possible couples (C,D) is looked through and later on optimized. This, in turn, enables a more reasonable and realistic decision-making. Thus, harmonization procedures when considered a *risk assessment technique*, are more effective than the former similar risk assessment by means of CPM-COST network models.

§8.5 Experimentation

8.5.1 Practical applications for the case of a single project

This section refers to considering practical achievements on the basis of implementing harmonization models for monitoring various PERT-COST network projects. We will henceforth consider a PERT-COST type project with random activity durations and p.d.f. satisfying (8.1.1), (8.1.2) or (8.1.4). The project's initial data is presented in [9]. The basic project's parameters are as follows: project's budget C, due date D and reliability

R. Partial utility coefficients are $\alpha_C = 1.0$, $\alpha_D = 0.5$ and $\alpha_R = 1.0$, while the initial search steps (first iteration) for $CCSA$ are $\Delta C = 4$ and $\Delta D = 2$. The number M of simulation runs for the PHM is taken $M = 2{,}000$. Other project's parameters are as follows: $R_0 = 0.7$, $R_{00} = 0.95$, $C_0 = 250$, $C_{00} = 230$, $D_0 = 95$, $D_{00} = 85$, $\delta C = 10$, $\delta D = 2.0$, $\delta R = 0.1$ and $\varepsilon = 0.001$.

The second iteration for the $CCSA$ is carried out with $\Delta C = 2.0$ and $\Delta D = 1.0$, while all further iterations, $v \geq 2$, are realized with $\Delta C = 1.0$ and $\Delta D = 1.0$.

The performance of the harmonization model's algorithm is illustrated on Tables 8.1-8.3 (for the case of p.d.f. satisfying (8.1.1), (8.1.2) and (8.1.4), correspondingly). It can be well-recognized that:

1. The cyclic coordinate search algorithm for determining the optimal utility of a medium-size project requires only four iterations to carry out the optimization process. The increase of the project's utility parameter after completing the fourth iteration (14 search points), as compared with the initial search point, shows utility improvement of approximately 45%. Thus, it can be well-recognized that the two-level heuristic algorithm to optimize the project's harmonization model performs well.

Table 8.1. Performance illustration of the harmonization algorithm (for a beta-distribution p.d.f.)

№ of search steps	*C*	*D*	*R*	*№ v of iteration*	*Feasibility*	*Utility $U(C,D,R)$*	*Value $U^{(v)}$ after the v-th iteration*
0	250	95	1.000	1	Feasible	2.50	2.50
1	246	95	0.996	1	Feasible	2.90	2.90
2	242	95	0.922	1	Feasible	3.02	3.02
3	238	95	0.793	1	Feasible	2.13	3.02
4	242	93	0.861	1	Feasible	3.41	3.41
5	242	91	0.723	1	Feasible	3.03	3.41
6	244	93	0.895	2	Feasible	3.55	3.55
7	246	93	0.912	2	Feasible	3.52	3.55
8	240	93	0.814	2	Feasible	3.14	3.55
9	244	94	0.936	2	Feasible	3.46	3.55
10	244	92	0.835	2	Feasible	3.45	3.55
11	**245**	**93**	**0.914**	**3**	**Optimal**	**3.64**	**3.64**
12	243	93	0.875	3	Feasible	3.45	3.64
13	245	94	0.951	4	Feasible	3.51	3.64
14	245	92	0.855	4	Feasible	3.55	3.64

Since values $U^{(3)}$ and $U^{(4)}$ coincide, the algorithm terminates after the *fourth* iteration

Table 8.2. Performance illustration of the harmonization algorithm (for a normal distribution p.d.f.)

№ of search steps	*C*	*D*	*R*	№ v of iteration	*Feasibility*	Utility $U(C,D,R)$	Value $U^{(v)}$ after the v-th iteration
0	250	95	1.000	1	Feasible	2.50	2.50
1	246	95	0.989	1	Feasible	2.90	2.90
2	242	95	0.915	1	Feasible	2.95	2.95
3	238	95	0.782	1	Feasible	2.02	2.95
4	242	93	0.829	1	Feasible	3.09	3.09
5	242	91	0.698	1	Non-feasible	-	3.09
6	244	93	0.868	2	Feasible	3.28	3.28
7	246	93	0.885	2	Feasible	3.25	3.28
8	240	93	0.802	2	Feasible	3.02	3.28
9	244	94	0.912	2	Feasible	3.22	3.28
10	244	92	0.811	2	Feasible	3.21	3.28
11	**245**	**93**	**0.893**	**3**	**Optimal**	**3.43**	**3.43**
12	243	93	0.847	3	Feasible	3.17	3.43
13	245	94	0.921	4	Feasible	3.21	3.43
14	245	92	0.839	4	Feasible	3.39	3.43

Since values $U^{(3)}$ and $U^{(4)}$ coincide, the algorithm terminates after the *fourth* iteration

Table 8.3. Performance illustration of the harmonization algorithm (for a uniform distribution p.d.f.)

№ of search steps	*C*	*D*	*R*	№ v of iteration	*Feasibility*	Utility $U(C,D,R)$	Value $U^{(v)}$ after the v-th iteration
0	250	95	1.000	1	Feasible	2.50	2.50
1	246	95	0.984	1	Feasible	2.90	2.90
2	242	95	0.912	1	Feasible	2.92	2.92
3	238	95	0.765	1	Feasible	1.85	2.92
4	242	93	0.821	1	Feasible	3.01	3.01
5	242	91	0.695	1	Non-feasible	-	3.01
6	244	93	0.864	2	Feasible	3.24	3.24
7	246	93	0.882	2	Feasible	3.22	3.24
8	240	93	0.795	2	Feasible	2.95	3.24
9	244	94	0.910	2	Feasible	3.20	3.24
10	244	92	0.807	2	Feasible	3.17	3.24
11	**245**	**93**	**0.889**	**3**	**Optimal**	**3.39**	**3.39**
12	243	93	0.844	3	Feasible	3.14	3.39
13	245	94	0.918	4	Feasible	3.18	3.39
14	245	92	0.835	4	Feasible	3.35	3.39

Since values $U^{(3)}$ and $U^{(4)}$ coincide, the algorithm terminates after the *fourth* iteration

2. Using the beta-distribution function results in obtaining the highest values for the project's utility parameter. This stems from the obvious fact that the mean value $\mu = 0.6a + 0.4b$ for beta-distribution p.d.f. within the distribution range (a,b) is closer to the lower bound a, than in case of normal and uniform distributions with symmetrical mean values $\mu = 0.5(a+b)$. It goes without saying that lower mean activity – time values result in higher reliability estimates. Since values of the truncated normal distribution concentrate closer to the mean value, than uniformly distributed values, the corresponding project's utility estimates are slightly better for the normal distribution p.d.f. than for the uniform one.

3. Thus, the optimal project's utility can be determined for the following parametrical values:

   $C = 245$, $D = 93$, $R = 0.914$, $U_G = 3.64$ *(beta-distribution)*,

   $C = 245$, $D = 93$, $R = 0.893$, $U_G = 3.43$ *(normal distribution)* and

   $C = 245$, $D = 93$, $R = 0.889$, $U_G = 3.39$ *(uniform distribution)*.

8.5.2 Case of several PERT-COST network projects

The company is faced with carrying out simultaneously three PERT-COST type network projects of equal significance. The initial projects' data is given in [9]. The projects' parameters are as follows:

Project #1

$D_{01} = 80$;	$R_{01} = 0.55$;	$C_{01} = \$280{,}000$;
$D_{001} = 70$;	$R_{001} = 0.70$;	$C_{001} = \$270{,}000$;
$\Delta D_1 = 5$;	$\delta R = 0.1$;	$\Delta C_1 = \$1{,}000$;
$\delta D = 10$;	$\alpha_R = 1.1$;	$\delta C = \$10{,}000$;
$\alpha_D = 1.0$;		$\Delta C = 0.8$;

Project #2

$D_{02} = 130$;	$R_{02} = 0.70$;	$C_{02} = \$280{,}000$;
$D_{002} = 115$;	$R_{002} = 0.90$;	$C_{002} = \$260{,}000$;
$\Delta D_2 = 5$;	$\delta R = 0.1$;	$\Delta C_2 = \$2{,}000$;
$\delta D = 10$;	$\alpha_R = 1.1$;	$\delta C = \$10{,}000$;
$\alpha_D = 1.0$;		$\Delta C = 0.8$;

Project #3

$D_{03} = 150$;	$R_{03} = 0.60$;	$C_{03} = \$286{,}000$;
$D_{003} = 130$;	$R_{003} = 0.80$;	$C_{003} = \$279{,}000$;

$\Delta D_3 = 5$;	$\delta R = 0.1$;	$\Delta C_3 = \$1{,}000$;
$\delta D = 10$;	$\alpha_R = 1.1$;	$\delta C = \$10{,}000$;
$\alpha_D = 1.0$;		$\Delta C = 0.8$;

Thus, $N_{1D} = 3$, $N_{2D} = 4$, $N_{3D} = 5$, and the number of all possible combinations of vectors $\vec{D}$ is 60. The total budget at the company's disposal to carry out the projects is $C = \$835{,}000$. Calculating values ρ_{kD_k} by (8.2.19) results in following:

$\rho_{1,80} = 1.5$;	$\rho_{2,130} = 1.0$;	$\rho_{3,150} = 2.8$;
$\rho_{1,75} = 1.4$;	$\rho_{2,125} = 0.9$;	$\rho_{3,145} = 2.7$;
$\rho_{1,70} = 1.4$;	$\rho_{2,120} = 0.9$;	$n = 2.7$;
	$\rho_{2,115} = 0.8$;	$\rho_{3,135} = 2.6$;
		$\rho_{3,130} = 2.5$;

Here values $R(C_k, D_k)$ are determined by means of the heuristic procedure outlined earlier in *8.1.4*. The beta-distribution p.d.f. has been chosen.

Since the number of possible combinations $\vec{D}$ is high enough, a *CCSA* algorithm to undertake a cyclic coordinate search has been implemented. After solving *Problem A1* we came to the conclusion that only two vectors $\vec{D}$ refer to feasible solutions, namely

$D_1 = 80$; $D_2 = 130$; $D_3 = 150$; and

$D_1 = 80$; $D_2 = 130$; $D_3 = 145$,

while all other combinations belong to a non-feasible zone.

Solving *Problem A2* with relations (8.2.8-8.2.13, 8.2.15, 8.2.34-8.2.35) by means of the LINDO package results in following:

I. For $D_1 = 80$, $D_2 = 130$, $D_3 = 150$, the budget values assigned to each project are $C_1 = \$277{,}000$, $C_2 = \$278{,}000$, $C_3 = \$280{,}000$ with reliability values

$R_1 = R(C_1, D_1) = 0.70$;
$R_2 = R(C_2, D_2) = 0.84$;
$R_3 = R(C_3, D_3) = 0.72$.

Thus, projects' utility values are as follows: $U_1 = 1.89$; $U_2 = 1.70$, $U_3 = 1.80$. The system's utility is $U_S = 5.39$.

II. For $D_1 = 80$, $D_2 = 130$, $D_3 = 145$, the budget values assigned to each project are $C_1 = \$272{,}000$, $C_2 = \$278{,}000$, $C_3 = \$285{,}000$

with reliability values

$R_1 = R(C_1, D_1) = 0.65$;
$R_2 = R(C_2, D_2) = 0.83$;
$R_3 = R(C_3, D_3) = 0.71$.

Thus, projects' utility values are as follows: $U_1 = 1.74$; $U_2 = 1.59$, $U_3 = 1.79$. The resulting system's utility is $U_S = 5.12$.

It can be well-recognized that the second $\vec{D}$-combination results in a lower value of utility U_S than the first triple. This also stems from the obvious fact that if for a certain triple of projects the slowest project displays a higher utility than the slowest project from another triple, then the sum of utilities for the first triple will be usually higher, than for the second one. Note that for both triples under consideration the values of the project's utilities are "smoothed" and balanced by implementing the maximin principle.

Thus, the optimal solution of problem (8.2.8-8.2.16, 8.2.30) is as follows: $D_1 = 80$, $D_2 = 130$, $D_3 = 150$, $C_1 = \$277{,}000$, $C_2 = \$278{,}000$, $C_3 = \$280{,}000$. Those parametrical values, by means of optimal budget assignment $c(i,j)_k$, $1 \le k \le n$, result in $R_1 = 0.70$, $R_2 = 0.84$, $R_3 = 0.72$, with the total system's utility $U_S = 5.39$.

8.5.3 *<u>Practical applications of harmonization models with safety engineering concepts</u>*

We have deliberately chosen the same project which was considered in *8.5.1*, i.e., with the initial data similar to that outlined in [9]. The basic project's parameters given by experts, are as follows:

- project's budget C;
- due date D;
- reliability R, and
- hazardous failure's probability P_{hf}.

Partial utility coefficients are $\alpha_C = 1.0$, $\alpha_D = 0.5$, $\alpha_R = 1$ and $\alpha_P = 2$. Other project's parameters are as follows: $R_0 = 0.7$, $R_{00} = 0.95$, $C_0 = 250$, $C_{00} = 230$, $D_0 = 95$, $D_{00} = 85$, $P_{hf_0} = 10^{-4}$, $P_{hf_{00}} = 10^{-5}$, $\delta_C = 10$, $\delta_D = 2.0$, $\delta_R = 0.1$, $\delta_P = 10^{-5}$ and $\varepsilon = 0.001$. Initial search steps for the first iteration (by using $CCSA$) are $\Delta C = 4.0$ and $\Delta D = 2.0$ while the second iteration is carried out with $\Delta C = 2.0$ and $\Delta D = 1.0$. All further iterations, $v \ge 2$, are carried out with $\Delta C = 1.0$ and $\Delta D = 1.0$. The number of simulation runs to determine R by the PHM is taken equal 2,000. Additional information obtained from experts (see [9]) is as follows:

$D_{\min} = 80$, $D_{\max} = 100$, $A = 10^{-5}$.

Thus, the project under consideration is practically identical to that outlined in *8.5.1*, except for the additional basic safety engineering parameter P_{hf}. The basic value of probability P_{hf} estimated by means of expert information for the project to be carried out with $C = 250$ and $D = 95$, is $P_{hf} = 0.4 \cdot 10^{-4}$.

The performance of the harmonization model's algorithm is illustrated in Table 8.4. In order to optimize the PHM, we have taken the beta-distribution p.d.f. (8.4.1). As outlined above, beta-distribution is widely used for simulating project activities duration.

Table 8.4. Performance illustration of the harmonization algorithm

№ of search steps	*C*	*D*	*R*	P_{hf}	*№ v of iteration*	*Feasibility*	*Utility* $U\left(C,D,R,P_{hf}\right)$	*Value* $U^{(v)}$ *after the v-th iteration*
0	250	95	1.000	$0.400 \cdot 10^{-4}$	1	Feasible	14.50	14.50
1	246	95	0.996	$0.400 \cdot 10^{-4}$	1	Feasible	14.90	14.90
2	242	95	0.922	$0.400 \cdot 10^{-4}$	1	Feasible	15.02	15.02
3	238	95	0.793	$0.400 \cdot 10^{-4}$	1	Feasible	14.13	15.02
4	242	93	0.861	$0.410 \cdot 10^{-4}$	1	Feasible	15.21	15.21
5	242	91	0.723	$0.423 \cdot 10^{-4}$	1	Feasible	14.57	15.21
6	244	93	0.895	$0.410 \cdot 10^{-4}$	2	Feasible	15.35	15.35
7	246	93	0.912	$0.410 \cdot 10^{-4}$	2	Feasible	15.32	15.35
8	240	93	0.814	$0.410 \cdot 10^{-4}$	2	Feasible	14.94	15.35
9	**244**	**94**	**0.936**	$\mathbf{0.405 \cdot 10^{-4}}$	**2**	**Optimal**	**15.36**	**15.36**
10	244	95	0.960	$0.400 \cdot 10^{-4}$	2	Feasible	15.10	15.36
11	244	92	0.838	$0.415 \cdot 10^{-4}$	2	Feasible	13.68	15.36
12	245	94	0.957	$0.405 \cdot 10^{-4}$	3	Feasible	14.90	15.36
13	243	94	0.901	$0.405 \cdot 10^{-4}$	3	Feasible	14.61	15.36
14	244	95	0.960	$0.400 \cdot 10^{-4}$	3	Feasible	15.10	15.36
15	244	93	0.895	$0.410 \cdot 10^{-4}$	3	Feasible	15.35	15.36
Since values $U^{(2)}$ and $U^{(3)}$ coincide, the algorithm terminates after the *third* iteration								

It can be well-recognized that the optimal solution of the regarded harmonization problem differs essentially from the solution obtained in *8.5.1* for the same PERT-COST project but without taking into account safety engineering concepts. Namely, the optimal project's utility value is approximately 4.2 times higher than the optimal utility value determined with basic parameters C, D and R (15.36 versus 3.64). Since all the project's parameters have been pregiven by one and the same expert team, such a drastic difference in utility values has a simple explanation from the point of subjective human judgment: the project management takes on a high priority basis all measures to prevent environment and construction personnel from any damage which may be caused by hazardous accidents in the course of developing the project. Thus, safety engineering concepts benefit from an essentially higher weight in the total utility value, than all other basic parameters.

The optimal project's utility can be obtained for the following parametrical values:

$$C = 244,\ D = 94,\ R = 0.936,\ P_{hf} = 0.405 \cdot 10^{-4}$$

and is equal $U_G = 15.36$.

Chapter 9. Trade-Off Harmonization Models for Safety Engineering Organization Systems

§9.1 Cost-reliability optimization models for fault tree systems with hazardous failures

9.1.1 Introduction

A hierarchical man-machine organization system which in the course of its functioning can be the source of large-scale accidents, e.g., nuclear power facilities, is considered. In the last four decades a large number of scientists (see, e.g., [6, 8-9, 21, 62-64, 86, 113, 119, 155, 163, 185, etc.]) undertook extensive research in the area of *fault tree analysis* (FTA) in order to develop effective techniques to predict and to prevent various failures of high risk safety technology. Fault tree analysis is mainly based on simulation models, and it can be well-recognized via simulation (see, e.g., [6, 8-9, 64, 86]) that the probability of a critical failure P_{cr} at the top level depends mainly on certain primary failures' probabilities at the bottom level. Thus, increasing the reliability of the corresponding elements at the bottom level results in increasing the overall system's technical reliability. Note that most primary failure probabilities can be decreased by introducing corresponding technical alterations which require the layout of expenditure. The latter may be calculated in advance, on the basis of statistical analysis.

Since the required budget to undertake technical alterations and refinements is usually restricted, various trade-off problems of "cost – reliability" type become highly important. However, despite the prevalence of FTA, publications on cost-optimization problems in safety engineering within the last decades are very scanty (see, e.g., [7, 50]). They practically do not cover cases of complicated hierarchical technical systems with primary failures at the lower levels and top critical failures at the upper level. Few publications on reliability optimization problems with cost parameters for large-scale systems boil down to the study of various network communications problems [9, 162], as well as problems of determining optimal levels of component reliabilities and redundancies with respect to multiple objectives [62, 113, 115, 155]. The results obtained cannot be applied to engineering systems with hierarchical structure, as well as to hierarchical fault trees.

The system's model is, thus, based on a hierarchical fault tree with primary failures at the bottom level. The latter are enumerated by $F_\xi, 1 \le \xi \le Q$. Denote the corresponding primary failure probability by P_ξ. Assume that, by introducing possible technical improvements, value P_ξ can be decreased but cannot become less than its lower bound $P_{\xi\min}$. Assume, further, that all primary failures are independent of each other (this assumption has been declared by the system management).

Determine the corresponding "probability step" ΔP_ξ, i.e., a constant reduction, in order to undertake a whole number of steps to reduce the probability of primary failure F_ξ from P_ξ to $P_{\xi \min}$. Denote the corresponding number of steps by N_ξ. Thus, relation $N_\xi \cdot (\Delta P_\xi) = P_\xi - P_{\xi \min}$ holds. Introducing technical improvements to diminish the primary failure probability by one step requires cost expenditures which are pregiven as well. Denote a top critical failure by F_{cr} with its corresponding probability P_{cr}. Assume that value P_{cr} is high enough and has to be reduced in order not to exceed the prescribed upper level P^*_{cr}. The developed cost-optimization problem (the direct problem) is to determine improved primary failures probabilities $P_{\xi f_\xi}$ of F_ξ after undertaking f_ξ consecutive steps (f_ξ being the step index), $1 \le \xi \le Q$, $0 \le f_\xi \le N_\xi$, which require the minimal accumulated costs to undertake all technical improvements, subject to reliability constraint, i.e., the top level critical probability not to exceed the pregiven upper level P^*_{cr}. Another cost-optimization problem - the dual one - centers on minimizing the failure probability subject to the cost constraint, i.e., to the restricted budget to carry out the technical improvements.

For further discussion in this chapter we will require the following additional terms:

FT - the fault tree comprising n hierarchical levels with its logical and probabilistic structure, which enables calculating top critical failure probabilities on the basis of primary failure probabilities;

SM - the simulation model with incoming primary failures at the bottom level and outcome top level failures;

F_ξ - primary failure, $1 \le \xi \le Q$ (Q - number of primary failures);

P_ξ - the probability of F_ξ (pregiven);

$P_{\xi\, min}$ - the minimal possible probability value P_ξ which, due to technical conditions, cannot be diminished (pregiven);

$C(P_\xi, P_{\xi\, min})$ - the cost of technical improvements in order to reduce the primary failure F_ξ by diminishing its probability up to $P_{\xi\, min}$ (pregiven);

ΔP_ξ - the "probability step" for value P_ξ in order to implement a technical improvement and diminish value P_ξ by ΔP_ξ (pregiven);

N_ξ - the number of steps in order to diminish the probability of F_ξ from P_ξ to $P_{\xi\, min}$ (pregiven);

$P_{\xi f} = P_\xi - f \cdot \Delta P_\xi$ - the diminished probability of primary failure F_ξ after undertaking f consecutive steps, $f \le N_\xi$;

$\Delta C_{\xi f}$ - the cost of reducing the probability of F_ξ by one step, from $P_{\xi, f-1}$ to $P_{\xi f}$;

F_{cr} - the critical failure at the top level;

P_{cr} - the probability of F_{cr} (obtained by using SM with pregiven input probabilities P_{ξ};

N - number of simulation runs in order to obtain representative statistics;

$P_{cr.N}$ - probability of F_{cr} obtained by N simulation runs. P_{cr} is taken equal to $P_{cr.N}$.

$P_{cr.}\big| P_{\xi\, min}$ - the calculated probability of F_{cr}, on condition that in the course of simulation, primary failure F_{ξ} is taken with its minimal possible probability $P_{\xi\, min}$ while all other pregiven primary failures probabilities remain unchanged;

$P_{cr}\big| P_{\xi_1\, min}, P_{\xi_2\, min}, ..., P_{\xi_v\, min}$ - probability of F_{cr} (calculated by means of simulation), on condition that in the course of simulating the FT, v specific primary failures $F_{\xi_1}, ..., F_{\xi_v}$ are taken with their minimal possible probabilities $P_{\xi_1\, min}, ..., P_{\xi_v\, min}$. All other primary failures F_{ω}, $\omega \neq \xi_q$, $1 \leq q \leq v$, are taken with their pregiven probabilities P_{ω};

$P_{cr.min}$ - probability of F_{cr}, on condition that in the course of simulation, all probabilities of primary failures are taken to be equal to their minimal possible values $P_{\xi\, min}$; $P_{cr.min}$ can be calculated via SM;

$P_{\xi\, f_{\xi}}$ - diminished primary failure probability P_{ξ}, $1 \leq \xi \leq Q$, after carrying out f_{ξ} "probability steps", $0 \leq f_{\xi} \leq N_{\xi}$;

$P_{cr}\big| \{P_{\xi\, f_{\xi}}\}$ - probability of F_{cr} obtained after N simulation runs with reduced probabilities of primary failures $P_{\xi\, f_{\xi}}$, $1 \leq \xi \leq Q$;

P_{cr}^{*} - the upper permissible level of P_{cr} (pregiven);

ΔC - the dual cost-optimization problem's accuracy tolerance (pregiven);

C^{*} - the restricted budget amount to undertake all technical improvements for the dual problem (pregiven).

9.1.2 The direct cost-optimization problem

The direct problem is as follows:

Determine the optimal set of probabilities of all primary failures $P_{\xi\, f_{\xi}} = P_{\xi} - f_{\xi} \cdot \Delta P_{\xi}$, $1 \leq \xi \leq Q$, $0 \leq f_{\xi} \leq N_{\xi}$, which requires the minimal budget to undertake all technical improvements

$$Min \sum_{\xi=1}^{Q} \sum_{\gamma=1}^{f_\xi} \Delta C_{\xi\gamma} \qquad (9.1.1)$$

subject to

$$P_{cr} \left| \left\{ P_{\xi f_\xi} \right\} \right. \le P_{cr}^{*} , \qquad (9.1.2)$$

$$0 \le f_\xi \le N_\xi , \qquad (9.1.3)$$

$$P_{\xi f_\xi} \ge P_{\xi min} , \ 1 \le \xi \le Q . \qquad (9.1.4)$$

Values $\Delta C_{\xi\gamma}$, $0 \le \gamma \le N_\xi$, are pregiven.

Restriction (9.1.2) means that the probability of the top critical failure with optimal input primary failure probabilities (obtained by means of simulation) must not exceed the prescribed upper level P_{cr}^{*}. Restriction (9.1.4) means that all of the optimal primary failures probabilities $P_{\xi f_\xi}$ must be no less than their minimal possible values $P_{\xi \min}$.

It can be well-recognized that problem (9.1.1-9.1.4) is equivalent to the following problem of integer programming: determine optimal integer values a_ξ, $1 \le \xi \le Q$, $0 \le a_\xi \le N_\xi$ (N_ξ pregiven), which deliver the minimal value to objective (9.1.1) subject to (9.1.2). It can be easily proven that since primary failures are independent of each other, such a problem is NP-complete [66, 176]. Thus, the optimal solution can be obtained only by means of a lookover algorithm that checks the feasibility of each of $M = \prod_{\xi=1}^{Q} N_\xi$ combinations $\{a_\xi\}$. If M is high enough, and considering that examining each combination $(a_1, ..., a_Q)$ by using the algorithm requires N simulation runs, problem (9.1.1-9.1.4) requires so much computational time, that it cannot develop an optimal solution for multilevel technical systems with a large number of primary failures at the bottom level. Thus, only approximate or heuristic algorithms can be applied to solve problem (9.1.1-9.1.4), which is, in essence, a complicated stochastic optimization model.

It goes without saying that if relation

$$P_{cr.min} \le P_{cr}^{*} \qquad (9.1.5)$$

does not hold, problem (9.1.1-9.1.4) cannot obtain even a feasible solution, since chance constraint (9.1.2) is not met for any combination $\left\{ P_{\xi f_\xi} \right\}$ subject to (9.1.3). Thus, we will henceforth assume that relation (9.1.5) does hold.

Note that problem (9.1.1-9.1.4) is equivalent to the following problem:

Determine an optimal subset of certain reduced probabilities

$$P_{\alpha_q f_{\alpha_q}} = P_{\alpha_q} - f_{\alpha_q} \cdot \Delta P_{\alpha_q}$$

of $r \le Q$ primary failures $F_{\alpha_q} \subset \{F_\xi\}$, $1 \le q \le r$, to minimize objective

$$\underset{\left\{P_{\alpha_q f_{\alpha_q}}\right\}}{Min} \sum_{q=1}^{r} \sum_{\gamma=1}^{f_{\alpha_q}} \Delta C_{\alpha_q \gamma}$$

subject to

$$P_{cr} \left| \left\{P_{\alpha_q f_{\alpha_q}}\right\} \le P^*_{cr} \right., \ 1 \le q \le r, \ 0 \le f_{\alpha_q} \le N_{\alpha_q},$$

$$P_{\alpha_q f_{\alpha_q}} \ge P_{\alpha_q min} .$$

Other $Q-r$ primary failures $F_\omega \subset \{F_\xi\} \backslash \{F_{\alpha_q}\}$, thus, obtain $f_\omega = 0$ and do not undergo any technical alterations with $\Delta C_{\omega\gamma} = 0$.

To conclude, problem (9.1.1-9.1.4) is a partial harmonization model with one independent basic parameter R (the system's reliability) and one dependent basic parameter C (the budget which can be assigned for implementing technical improvements for primary elements). Note that since the top critical failure is a hazardous one, value $1-P_{cr}$ is actually the system's reliability R (see Table 5.2) which is forbidden to deteriorate below the prescribed level. Thus, the direct problem (9.1.1-9.1.4) is a partial harmonization model $PHM_1(R) = C$, where C has to be minimized by solving the optimization problem. Restrictions (9.1.3-9.1.4) reflect the specific features of the system's model.

9.1.3 Approximate heuristic algorithm by means of the sensitivity concept

The algorithm outlined below, presents a heuristic procedure to determine an approximate solution of problem (9.1.1-9.1.4). As outlined above, in *5.3.4*, problem (9.1.1-9.1.4) cannot be solved by means of dynamic programming and other similar classical approaches. Thus, we introduce the concept of cost-sensitivity which enables obtaining an approximate heuristic solution. The latter can serve, if necessary, for corrective algorithms, in order to establish an improved solution with better accuracy.

The backbone of the cost-sensitivity algorithm is to redistribute the budget needed to diminish the critical probability P_{cr} by $\Delta P = P_{cr} - P^*_{cr}$ among the essential primary failures which actually affect the top critical failure. All essential primary failures have to be examined, beginning from more cost-sensitive ones. A primary failure F_{ξ_1} is regarded to be more cost-sensitive than F_{ξ_2} if relation

$$\Delta C_{\xi_1 f} < \Delta C_{\xi_2 f} \ , \quad \Delta P_{\xi_1} = \Delta P_{\xi_2} \ , \quad 1 \le f \le min \left\{ N_{\xi_1}, N_{\xi_2} \right\},$$

holds. We start examining all the essential primary failures being ordered beforehand according to their cost-sensitivity. Undertaking a routine search step, i.e., examining a routine essential primary failure F_{ξ_v}, results in determining value $P_{cr} \Big| P_{\xi_1 \min}, P_{\xi_2 \min}, \ldots, P_{\xi_v \min}$ by means of simulation. The search process proceeds until the difference $P_{cr} - P_{cr}^*$ will be exhausted.

The enlarged step-by-step procedure of the algorithm is as follows:

<u>Step 1</u>. Calculate, via SM, values $P_{cr} \Big| P_{\xi\, min}$ for all $\xi, \ 1 \le \xi \le Q$.

<u>Step 2</u>. Compare the relative deviations of the current top critical failure probability from P_{cr} if *only one* primary failure F_ξ is at minimum, while all other F_ξ's remain at their current values

$$\frac{1}{P_{cr}} \left\{ P_{cr} - P_{cr} \Big| P_{\xi\, min} \right\} = W_\xi \,, \qquad (9.1.6)$$

for all primary failures F_ξ.

<u>Step 3</u>. Rearrange values W_ξ in descending order. Denote the new indices (ordinal numbers) of the primary failures according to the new order, by $\eta_1, \eta_2, \ldots, \eta_Q$. Exclude all the primary failures which practically do not correlate with F_{cr}. Denote the number of remaining primary failures by $S \le Q$.

<u>Step 4</u>. Calculate values

$\left[P_{cr} \Big| P_{\eta_1\, min} \right], \left[P_{cr} \Big| P_{\eta_1\, min}, P_{\eta_2\, min} \right], \ldots, \left[P_{cr} \Big| P_{\eta_1\, min}, \ldots, P_{\eta_S\, min} \right]$ by means of fault tree simulation. Determine *the minimal whole number* r, $r \le S$, which satisfies

$$\left[P_{cr} \Big| P_{\eta_1\, min}, \ldots, P_{\eta_r\, min} \right] \le P_{cr}^* \,.$$

Note that, due to relation (9.1.5), number r always exists. Thus,

$$r = \underset{q}{Min} \left\{ \forall q : P_{cr} \Big| P_{\eta_1\, min}, \ldots, P_{\eta_q\, min} \le P_{cr}^* \right\}.$$

Note that Step 4 can be carried out straightforwardly, without singling out non-correlated primary failures at Step 3. However, since the number S of remaining primary failures is usually essentially less than Q, implementing Step 4 for all Q primary failures results in a significant (and non-effective!) increase of the computational time.

Step 5. Consider r primary failures $F_{\eta_1}, F_{\eta_2}, \dots, F_{\eta_r}$. The initial search point has to be determined as a vector in the r - dimensional space, as follows:

$$\overrightarrow{X} = \left(X_1, X_2, \dots, X_r \right), \; P_{\eta_q \min} \le X_q \le P_{\eta_q}, \; 1 \le q \le r,$$

X_q being the probability to be reduced.

For all other primary failures F_w, $w \neq \eta_q$, $1 \le q \le r$, set the corresponding probabilities equal to the initial values P_w. These probabilities remain unchanged and, thus, enter the solution of problem (9.1.1-9.1.4).

For all primary failures $F_{\eta_1}, F_{\eta_2}, \dots, F_{\eta_r}$ separately, calculate relative cost-sensitivity values, i.e., *the average relative decrease of the top critical failure probability by investing a cost unit to undertake technical improvements*

$$\gamma_{\eta_q} = \frac{W_{\eta_q}}{C\left(P_{\eta_q}, P_{\eta_q \min}\right)}, \quad 1 \le q \le r, \tag{9.1.7}$$

where W_{η_q} is calculated by (9.1.6).

Step 6. Reorder cost-sensitivity values γ_{η_q}, $1 \le q \le r$, in descending order. Thus, primary failures F_{η_q} will obtain a new order. Denote the corresponding new indices (ordinal numbers) of r primary failures under consideration by F_{α_q}, $1 \le q \le r$.

It can be well-recognized that if one has invested a certain amount of budget for undertaking technical improvements in order to diminish the probability of a primary failure, and if the budget investments are the same for all primary failures, then the "payback", i.e., the relative decrease of value P_{cr}, is a monotonously decreasing function of the failure's ordinal number α_q, $1 \le q \le r$. Thus, the "straightforward" influence of investing a budget cost unit decreases from F_{α_1} to F_{α_r}. This consideration is used below, at the next step.

Step 7. The suggested procedure to determine the quasi-optimal solution $\left\{P_{\alpha_q}\right\}$ of problem (9.1.1-9.1.4) is as follows: start examining the primary failure F_{α_1}, then turn to the next one F_{α_2},…, etc., until the last primary failure F_{α_r} is analyzed. In the course of examining a routine primary failure F_{α_q}, $1 \le q \le r$, the probabilities of all preceding primary failures, which have been already examined, are set equal to $P_{\alpha_v\, min}$, $1 \le v \le q-1$, while for all next, non-examined values F_{α_v}, their corresponding probabilities are set equal to P_{α_v}, $q+1 \le v \le r$, i.e., to the initial values. The term "examining" means that the probability of a routine primary failure P_{α_q} undergoes sequential diminishing by the corresponding "probability step" ΔP_{α_q}. Thus P_{α_q} is examined first, then $P_{\alpha_q} - \Delta P_{\alpha_q}$, ... , $P_{\alpha_q} - f_{\alpha_q} \cdot \Delta P_{\alpha_q}$, ... , and so forth through $P_{\alpha_q\, min}$.

For each diminished probability solution

$$P_{\alpha_q f_{\alpha_q}}, \quad 1 \le q \le r, \quad 1 \le f_{\alpha_q} \le N_{\alpha_q},$$

calculate by means of simulation the probability of the critical failure F_{cr}

$$P_{cr} \Big| P_{\alpha_1\, min}, ..., P_{\alpha_{q-1}\, min}, P_{\alpha_q f_{\alpha_q}} \left(= P_{\alpha_q} - f_{\alpha_q} \cdot \Delta P_{\alpha_q}\right), P_{\alpha_{q+1}}, ..., P_{\alpha_r}, \tag{9.1.8}$$

and compare value P_{cr} with its upper prescribed level P_{cr}^*. The process terminates at the first r-dimensional vector

$$\overrightarrow{X} = \left[\left\{P_{\alpha_v\, min}\right\}, P_{\alpha_q f_{\alpha_q}}, \left\{P_{\alpha_w}\right\}\right], \; 1 \le q \le r, \; 1 \le v \le q-1, \; q+1 \le w \le r, \tag{9.1.9}$$

for which value (9.1.8) satisfies $P_{cr} \le P_{cr}^*$.

Thus, vector (9.1.9) is taken as the approximate solution of problem (9.1.1-9.1.4) with the budget to undertake the required technical improvements equal to

$$C\left(\overrightarrow{X}\right) = \sum_{v=1}^{q-1} C\left(P_{\alpha_v}, P_{\alpha_v\, min}\right) + \sum_{d=1}^{f_{\alpha_q}} \Delta C_{\alpha_q d}, \quad q \subset \{1, r\}. \tag{9.1.10}$$

The idea of the algorithm is to spend as little budget as possible in order to meet the chance constraint (9.1.2). Note that primary failures F_{α_w}, $q+1 \le w \le r$, do not change their initial probabilities and, thus, do not require cost investments.

In certain cases a very cost-sensitive primary failure may have less practical influence on the top critical failure. This means that for a fault tree, essentiality, i.e., the influence on the top critical failure, is more important than cost-sensitivity. We have first to single out all essential primary failures, and only afterwards rearrange the latter according to their cost-sensitivity. That means that Steps 3 and 6 of the algorithm cannot be unified.

Note, in conclusion, that the validity of the outlined above algorithm is based on the assumption of constant cost increase $\Delta C_{\xi f}$ when the failure probability is diminished by a constant probability step ΔP_ξ. This assumption has been accepted in the course of implementing the algorithm on a real technical system with a possible top level hazardous failure [8-9, 86].

9.1.4 *The dual problem*

The dual problem will be formulated and solved for the cost-optimization problem as follows:

Determine the optimal set of probabilities of primary failures $P_{\xi f_\xi} = P_\xi - f_\xi \cdot \Delta P_\xi$, $1 \le \xi \le Q$, in order to minimize the probability $P_{cr} \left| \left\{ P_{\xi f_\xi} \right\} \right.$ subject to the restricted budget amount C^* to undertake the corresponding technical improvements, i.e.,

$$\underset{\{P_{\xi f_\xi}\}}{Min} \; P_{cr} \left| \left\{ P_{\xi f_\xi} \right\} \right. \tag{9.1.11}$$

subject to

$$\sum_{\xi=1}^{Q} \sum_{q=1}^{f_\xi} \Delta C_{\xi q} \le C^* , \tag{9.1.12}$$

$$1 \le \xi \le Q, \quad 0 \le f_\xi \le N_\xi . \tag{9.1.13}$$

Restriction (9.1.12) means that the amount of costs to undertake the technical improvements must not exceed the pregiven cost amount C^*. Value $P_{cr} \left| \left\{ P_{\xi f_\xi} \right\} \right.$ is obtained by means of simulation. Note that in case $P_{cr.\min} > P_{cr}^*$ the dual problem has no solution.

It can be well-recognized that problem (9.1.11-9.1.13) is, in essence, a dual problem for the direct one, (9.1.1-9.1.4). The solution outlined below is based on the algorithms to solve the direct problem outlined in *9.1.3*.

The step-by-step algorithm to solve the dual problem (9.1.11-9.1.13) is as follows:

Step 1. Choose the largest total budget value to achieve the smallest P_{cr}

$$C_{max} = \sum_{\xi=1}^{Q} \sum_{q=1}^{N_\xi} \Delta C_{\xi q} , \quad (9.1.14)$$

which enables the minimal possible top critical failure probability $P_{cr.min}$, on condition that in the course of simulation, all probabilities of primary failures are taken to be equal to their minimal possible values $P_{\xi min}$. $P_{cr.min}$ is calculated via SM.

Step 2. Set: $P_{cr.min} \Rightarrow P_{cr.1}$, $P_{cr.} \Rightarrow P_{cr.2}$, where P_{cr} is the current top critical failure probability. Note that at the beginning of the algorithm's work $P_{cr.2}$ is obtained via simulation with initial primary failure probabilities, i.e., no budget costs are required.

Step 3. Calculate $P_{cr.3} = 0.5 \cdot \left(P_{cr.1} + P_{cr.2} \right)$. Note that value $P_{cr.3}$, like values $P_{cr.1}$ and $P_{cr.2}$, represents the probability of a hazardous failure. The corresponding value set for P_ξ, $1 \le \xi \le Q$, are optimal primary failures' probabilities $P_{\xi f_\xi}$ to be determined in the direct cost-optimization problem (9.1.1-9.1.4) with $P^*_{cr} = P_{cr.3}$.

Step 4. Solve the direct cost-optimization problem (9.1.1-9.1.4) for values $P^*_{cr} = P_{cr.1}$, $P^*_{cr} = P_{cr.2}$ and $P^* = P_{cr.3}$. Denote the obtained minimal cost values by C_1, C_2 and C_3, correspondingly. Note that at the beginning of the algorithm's work $C_1 = C_{max}$ and $C_2 = 0$ (current with no added cost).

Step 5. Compare values C_3 and C^*. If $\left| C_3 - C^* \right| < \Delta C$, where ΔC stands for the error accuracy tolerance (see *5.3.1*), go to *9*. Otherwise go to *6*.

Step 6. Examine relation $C_2 \le C^* \le C_3$. If it holds, go to *7*. Otherwise, i.e., in case $C_3 < C^* \le C_1$, go to *8*. Note that relation $C_2 \le C^* \le C_1$ is an evident one since assigning C_1 results in the minimal possible top failure probability.

Step 7. Replace $P_{cr.3} \Rightarrow P_{cr.1}$. Go to *3*.

Step 8. Replace $P_{cr.3} \Rightarrow P_{cr.2}$. Go to *3*.

Step 9. Probabilities $\left\{ P_{\xi f_\xi} \right\}$ obtained from the corresponding cumulative cost value

$$\sum_{\xi=1}^{Q} \sum_{q=1}^{f_\xi} \Delta C_{\xi q} = C_3 \quad (9.1.15)$$

are the optimal values which result in minimizing objective (9.1.11) subject to

(9.1.12).

It can be well-recognized that problem (9.1.11-9.1.13) is solved by using the bisection method [176] in combination with the direct problem (9.1.1-9.1.4) outlined in the previous section.

It can be well-recognized that the dual problem (9.1.11-9.1.13) is a partial harmonization problem as well. In comparison with the direct problem (9.1.1-9.1.4), the independent and dependent basic parameters R and C have switched their places: the dual problem (9.1.11-9.1.13) is nothing else but a $PHM_2(C) = R$, where reliability value $R = 1 - P_{cr}$ has to be maximized (P_{cr} minimized) subject to the prescribed budget C.

Note, in conclusion, that in §9.1 we have not developed the algorithm for solving harmonization problems in order to maximize linear utility values (5.3.5). This has been done deliberately, since undertaking a search for the extremum with only two basic parameters does not meet any difficulties. However, we have developed two major partial harmonization models $PHM_1(R) = C$ and $PHM_2(C) = R$, which can be applied to various hierarchical plants with branching structure. Those PHM cannot be solved by classical methods and require only heuristic solutions.

§9.2 Experimentation

In order to check the fitness of the outlined above models, extensive experimentation has been undertaken [9]. A real complicated multilevel safety engineering OS, namely, an emergency core cooling system of a heavy water cooled nuclear reactor, with a possible hazardous failure at the top level, is considered. The system comprises at the bottom level 24 primary failures which affect elements at higher hierarchical levels and, thus, result in the top critical failure. The simulation model to calculate the top critical failure probability P_{cr} is outlined in [8-9, 86].

The structure of the fault tree is presented in Fig. 9.1. Each element of the fault tree F_{ijm} is formalized by three indices, namely, i, j and m. Value m, $1 \le m \le n$, denotes the index of the hierarchical level, while i stands for the index (ordinal number) of an element E_{im} at that level. Index j denotes the j-th type possible failure which may occur within element's E_{im} work. Since the system's fault tree comprises $n = 8$ hierarchical levels, for all $Q = 24$ primary failures at the bottom level index m is set equal 8. Those primary failures F_{ij8} refer to 16 different elements E_{i8}, $1 \le i \le 16$, while the hazardous top failure refers to the highest level with $m = 1$.

For intermediate, secondary failures F_{ijm} at higher hierarchical levels, i.e., with indices $1 < m < 8$, probabilities P_{ijm} can be calculated by using pregiven extremely complicated "logical lists" which are implemented in the fault tree on the basis of failure signals leaving elements at the bottom level and entering other elements at higher levels.

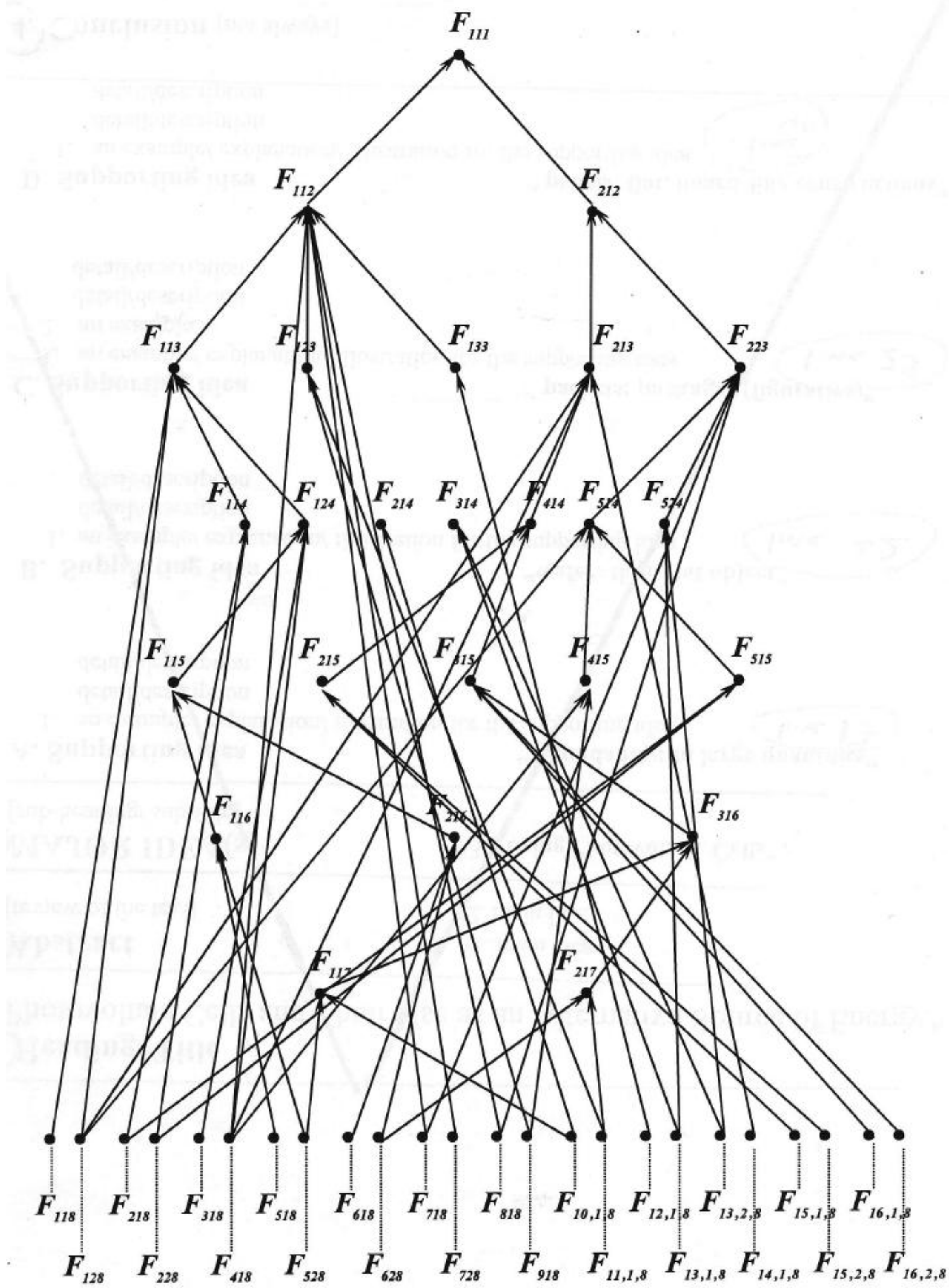

Figure 9.1. <u>Hierarchical structure of the fault tree</u>

Thus, it can be well-recognized that the structure of the hierarchical fault tree determines, in essence, the technical system's branch tree. Note that there is no standby system or redundancy involved in the functioning.

The initial system's data is presented in Table 9.1. Here $P_{\xi \min}$ denotes the minimal possible probability value P_ξ which, due to technical reasons, cannot be further diminished, while $C(P_\xi, P_{\xi \min})$ stands for the cost of technical improvements in order to reduce the primary failure F_ξ by diminishing its probability to occur up to $P_{\xi \min}$.

Table 9.1. The system's initial data (obtained by expert methods)

Primary failures F_ξ, $1 \le \xi \le Q$	Existing probabilities P_ξ	Minimal possible probabilities $P_{\xi\, min}$	$C(P_\xi, P_{\xi\, min})$
F_1	$1.5 \cdot 10^{-3}$	$1.5 \cdot 10^{-4}$	\$ 8,500
F_2	$1.2 \cdot 10^{-3}$	$1.2 \cdot 10^{-4}$	\$ 14,000
F_3	$1.4 \cdot 10^{-3}$	$1.4 \cdot 10^{-4}$	\$ 11,000
F_4	$1.6 \cdot 10^{-3}$	$1.6 \cdot 10^{-4}$	\$ 10,500
F_5	$3.1 \cdot 10^{-3}$	$3.1 \cdot 10^{-4}$	\$ 8,000
F_6	$1.3 \cdot 10^{-3}$	$1.3 \cdot 10^{-4}$	\$ 10,000
F_7	$4.4 \cdot 10^{-3}$	$4.4 \cdot 10^{-4}$	\$ 9,500
F_8	$1.5 \cdot 10^{-3}$	$1.5 \cdot 10^{-4}$	\$ 12,500
F_9	$1.6 \cdot 10^{-3}$	$1.6 \cdot 10^{-4}$	\$ 13,500
F_{10}	$1.8 \cdot 10^{-3}$	$1.8 \cdot 10^{-4}$	\$ 11,500
F_{11}	$3.7 \cdot 10^{-3}$	$3.7 \cdot 10^{-4}$	\$ 12,000
F_{12}	$1.6 \cdot 10^{-3}$	$1.6 \cdot 10^{-4}$	\$ 16,000
F_{13}	$1.5 \cdot 10^{-3}$	$1.5 \cdot 10^{-4}$	\$ 15,000
F_{14}	$1.3 \cdot 10^{-3}$	$1.3 \cdot 10^{-4}$	\$ 12,500
F_{15}	$1.1 \cdot 10^{-3}$	$1.1 \cdot 10^{-4}$	\$ 14,500
F_{16}	$1.3 \cdot 10^{-3}$	$1.3 \cdot 10^{-4}$	\$ 12,500
F_{17}	$1.4 \cdot 10^{-3}$	$1.4 \cdot 10^{-4}$	\$ 9,500
F_{18}	$5.2 \cdot 10^{-3}$	$5.2 \cdot 10^{-4}$	\$ 13,500
F_{19}	$1.2 \cdot 10^{-3}$	$1.2 \cdot 10^{-4}$	\$ 13,000
F_{20}	$1.3 \cdot 10^{-3}$	$1.3 \cdot 10^{-4}$	\$ 9,500
F_{21}	$2.4 \cdot 10^{-3}$	$2.4 \cdot 10^{-4}$	\$ 8,500
F_{22}	$4.5 \cdot 10^{-3}$	$4.5 \cdot 10^{-4}$	\$ 13,500
F_{23}	$1.1 \cdot 10^{-3}$	$1.1 \cdot 10^{-4}$	\$ 14,000
F_{24}	$1.0 \cdot 10^{-3}$	$1.0 \cdot 10^{-4}$	\$ 11,000

The simulation model (SM) developed by means of the fault tree model outlined in [8-9] enables calculating the top critical failure probability P_{cr} on the basis of primary failures' probabilities. The minimal top critical probability value $P_{cr.\min}$ for all primary failures obtaining their minimal possible probability values, has been calculated via SM by means of performing 10^6 simulation runs and equals $P_{cr.\min} = 6 \cdot 10^{-10}$. Value P_{cr} for all primary failures obtaining their existing probability values, calculated by means of simulation as well, is equal $6.09 \cdot 10^{-7}$ and does not guarantee the desired system reliability in terms of accident prevention, while value $P_{cr.\min}$, on the contrary, results in an excellent reliability value.

The management takes nowadays all possible measures to increase the system's reliability R_S. Since the latter satisfies $R_S = 1 - P_{cr}$ and since it is anticipated that the only way to reduce value P_{cr} is by means of diminishing the primary failures' probabilities, the cost-reliability optimization problems center on introducing optimal technical improvements in primary elements to minimize the overall critical failure F_{cr}.

Restrictions for both direct and dual cost-optimization problems, namely, $P_{cr}^* = 2 \cdot 10^{-9}$ and $C^* = \$50{,}000$, are pregiven. Thus, the two problems may be formulated as follows (see 9.1.3):

The direct problem	*The dual problem*
$\underset{\{P_{\xi f_\xi}\}}{Min} \sum_{\xi=1}^{Q} \sum_{q=1}^{f_\xi} \Delta C_{\xi q}$	$\underset{\{P_{\xi f_\xi}\}}{Min} \; P_{cr} \left\| \left\{ P_{\xi f_\xi} \right\} \right.$
subject to $P_{cr} \left\| \left\{ P_{\xi f_\xi} \right\} \right. \le 2 \cdot 10^{-9} = P_{cr}^*$.	subject to $\sum_{\xi=1}^{Q} \sum_{q=1}^{f_\xi} \Delta C_{\xi q} \le \$50{,}000 = C^*$.

In order to solve both problems, we have first to single out the most essential and significant primary failures. Thus, Step 1 of the heuristic algorithm outlined in *9.1.3*, has been implemented. First, values $P_{cr} / P_{\xi \min}$, $1 \le \xi \le 24$, have been calculated by means of simulation (10^6 simulation runs).

Carrying out Steps 2 and 3 of the algorithm, i.e., calculating $Q = 24$ values W_ξ, rearranging the latter in descending order and excluding $(Q - S)$ non-significant primary failures, results in obtaining a sequence of $S = 10$ primary failures F_{11}, F_6, F_2, F_{20}, F_{13}, F_{14}, F_5, F_1, F_{12} and F_{17}. Those primary failures, being improved, have the highest local influence on the top critical failure probability, while other primary failures are essentially less correlated with P_{cr}.

Afterwards, by using Step 4 of the algorithm in *9.1.3*, we obtain the minimal value r enabling satisfaction

$$\left[P_{cr} \middle/ P_{\xi_1 \, min} , \; P_{\xi_2 \, min} , \dots , P_{\xi_r \, min} \right] \leq \; P_{cr}^{*} \; = \; 2 \cdot 10^{-9} , \tag{9.2.1}$$

where $\eta_1 = 11, \; \eta_2 = 6, \; \eta_3 = 2, \; \dots , \; \eta_8 = 1$. A conclusion can be drawn that $r = 8$ primary failures $F_{11}, \; F_6, \; F_2, \; F_{20}, \; F_{13}, \; F_{14}, \; F_5, \; F_1,$ taken with their minimal possible probability values (while all other primary failures do not change their existing probabilities), enable achievement of the prescribed top critical probability P_{cr}^{*} estimated as $2 \cdot 10^{-9}$. Call henceforth those selected primary failures *essential* ones.

At Steps 5 and 6 of the algorithm the cost-sensitivity of essential primary failures has been checked by means of

$$\gamma_{\xi} \;\; = \;\; \frac{P_{cr} - P_{cr} / P_{\xi \, min}}{P_{cr} \; \cdot C\left(P_{\xi} \, , \, P_{\xi \, min} \right)} \, . \tag{9.2.2}$$

The calculated values γ_{ξ} are as follows:

$$\gamma_{11} \;\; = \;\; \frac{12{,}000 \times 6.09 \cdot 10^{-7}}{2.95 \cdot 10^{-7}} \;\; = \;\; 0.040366;$$

$$\gamma_{6} \;\; = \;\; \frac{10{,}000 \times 6.09 \cdot 10^{-7}}{2.89 \cdot 10^{-7}} \;\; = \;\; 0.047461;$$

$$\gamma_{2} \;\; = \;\; \frac{14{,}000 \times 6.09 \cdot 10^{-7}}{2.47 \cdot 10^{-7}} \;\; = \;\; 0.028970;$$

$$\gamma_{20} \;\; = \;\; \frac{9{,}500 \times 6.09 \cdot 10^{-7}}{1.87 \cdot 10^{-7}} \;\; = \;\; 0.032322;$$

$$\gamma_{13} \;\; = \;\; \frac{15{,}000 \times 6.09 \cdot 10^{-7}}{1.29 \cdot 10^{-7}} \;\; = \;\; 0.014121;$$

$$\gamma_{14} \;\; = \;\; \frac{12{,}500 \times 6.09 \cdot 10^{-7}}{1.13 \cdot 10^{-7}} \;\; = \;\; 0.014844;$$

$$\gamma_{5} \;\; = \;\; \frac{8{,}000 \times 6.09 \cdot 10^{-7}}{0.94 \cdot 10^{-7}} \;\; = \;\; 0.019294;$$

$$\gamma_{1} \;\; = \;\; \frac{8{,}500 \times 6.09 \cdot 10^{-7}}{0.45 \cdot 10^{-7}} \;\; = \;\; 0.008693.$$

Reordering values γ_{ξ} in descending order results in the following sequence:

$$\gamma_6,\ \gamma_{11},\ \gamma_{20},\ \gamma_2,\ \gamma_5,\ \gamma_{14},\ \gamma_{13},\ \gamma_1. \tag{9.2.3}$$

Further on, the "probability steps" ΔP_ξ and the costs $\Delta C_{\xi f}$ of reducing the probability of F_ξ by one step, namely, from $P_{\xi,f-1}$ to $P_{\xi f}$, have been determined. On the basis of experts' decision it has been determined that the most cost-sensitive essential failures F_ξ, namely, F_6, F_{11}, F_{20} and F_2, should comprise three equal "probability steps" with the corresponding equal values $\Delta C_{\xi f}$ for $f = 1,2,3$. Thus, for those primary failures $N_\xi = 3$, and ΔC_ξ (we omit index f in order to simplify the terms) are equal to $1/3 \cdot C\left(P_\xi, P_{\xi \min}\right)$.

Thus,

$$\Delta P_6 = \frac{1.3\cdot 10^{-3} - 1.3\cdot 10^{-4}}{3} = 3.9\cdot 10^{-4};$$

$$\Delta P_{11} = \frac{3.7\cdot 10^{-3} - 3.7\cdot 10^{-4}}{3} = 1.11\cdot 10^{-3};$$

$$\Delta P_{20} = \frac{1.3\cdot 10^{-3} - 1.3\cdot 10^{-4}}{3} = 3.9\cdot 10^{-4};$$

$$\Delta P_2 = \frac{1.2\cdot 10^{-3} - 1.2\cdot 10^{-4}}{3} = 3.6\cdot 10^{-4}.$$

The corresponding "cost steps" are as follows:

$$\Delta C_6 = \frac{10{,}000}{3} = \$\ 3{,}333;$$

$$\Delta C_{11} = \frac{12{,}000}{3} = \$\ 4{,}000;$$

$$\Delta C_{20} = \frac{9{,}500}{3} = \$\ 3{,}167;$$

$$\Delta C_2 = \frac{14{,}000}{3} = \$\ 4{,}667.$$

As to the second subset of less cost-sensitive primary failures F_5, F_{14}, F_{13} and F_1, it has been decided to determine "probability steps" and "cost-steps" by subdividing the whole scale into four equal parts. Thus, $N_5 = N_{14} = N_{13} = N_1 = 4$, and

$$\Delta P_5 \quad = \quad \frac{3.1 \cdot 10^{-3} - 3.1 \cdot 10^{-4}}{4} \quad = \quad 0.7 \cdot 10^{-3};$$

$$\Delta P_{14} \quad = \quad \frac{1.3 \cdot 10^{-3} - 1.3 \cdot 10^{-4}}{4} \quad = \quad 2.92 \cdot 10^{-4};$$

$$\Delta P_{13} \quad = \quad \frac{1.5 \cdot 10^{-3} - 1.5 \cdot 10^{-4}}{4} \quad = \quad 3.37 \cdot 10^{-4};$$

$$\Delta P_1 \quad = \quad \frac{1.5 \cdot 10^{-3} - 1.5 \cdot 10^{-4}}{4} \quad = \quad 3.37 \cdot 10^{-4}.$$

For the "cost steps" $\Delta C_\xi = \frac{1}{4} \cdot C\left(P_\xi, P_{\xi \min}\right)$, $\xi = 5, 14, 13, 1,$ we obtain:

$$\Delta C_5 \quad = \quad \frac{8{,}000}{4} \quad = \quad \$\ 2{,}000;$$

$$\Delta C_{14} \quad = \quad \frac{12{,}500}{4} \quad = \quad \$\ 3{,}125;$$

$$\Delta C_{13} \quad = \quad \frac{15{,}000}{4} \quad = \quad \$\ 3{,}750;$$

$$\Delta C_1 \quad = \quad \frac{8{,}500}{4} \quad = \quad \$\ 2{,}125.$$

Obtaining values ΔP_ξ and ΔC_ξ enables realizing the trade-off cost-optimization problem, which is a combination of the direct and the dual problems.

After determining the "cost-steps" and "probability steps" by subdividing the whole scale into four equal parts for F_5, F_{14}, F_{13} and F_1, and into three parts for F_6, F_{11}, F_{20} and F_2, the direct cost-optimization problem has been solved by carrying out Step 7 of the algorithm (see *9.1.3*).

The initial data to undertake cost-optimization is presented in Table 9.2.

Each element $E(\xi, f)$ of the presented matrix in the table comprises two values:

a) value

$$P_{\xi f} = P_\xi - f \cdot \Delta P_\xi, \tag{9.2.4}$$

where

$$f \;=\; \begin{cases} 0,\,1,\,2,\,3 & \text{for} \quad \xi = 6,\,11,\,20,\,2 \\ 0,\,1,\,2,\,3,\,4 & \text{for} \quad \xi = 5,\,14,\,13,\,1 \end{cases} \;, \text{ and} \tag{9.2.5}$$

b) value

$$C_{\xi f} \;=\; C\left(P_{\eta}, P_{\eta\, min}\right) + \sum_{d=1}^{f} \Delta C_{\xi d} \;, \tag{9.2.6}$$

where η are all indices which precede ξ in sequence (9.2.3). For technical reasons, we have slightly rounded off several values $P_{\xi f}$.

Table 9.2. The initial data for the cost-optimization problem

ξ	$E_{\xi f}$	$f=0$	$f=1$	$f=2$	$f=3$	$f=4$
6	$P_{6,f}$	$1.3 \cdot 10^{-3}$	$9.1 \cdot 10^{-4}$	$5.2 \cdot 10^{-4}$	$1.3 \cdot 10^{-4}$	
	$C_{6,f}$ (\$)	0	3,333	6,666	10,000	
11	$P_{11,f}$	$3.7 \cdot 10^{-3}$	$2.6 \cdot 10^{-3}$	$1.5 \cdot 10^{-3}$	$3.7 \cdot 10^{-4}$	
	$C_{11,f}$ (\$)	10,000	14,000	18,000	22,000	
20	$P_{20,f}$	$1.3 \cdot 10^{-3}$	$9.1 \cdot 10^{-4}$	$5.2 \cdot 10^{-4}$	$1.3 \cdot 10^{-4}$	
	$C_{20,f}$ (\$)	22,000	25,170	28,340	31,500	
2	$P_{2,f}$	$1.2 \cdot 10^{-3}$	$8.4 \cdot 10^{-4}$	$4.8 \cdot 10^{-4}$	$1.2 \cdot 10^{-4}$	
	$C_{2,f}$ (\$)	31,500	36,170	40,840	45,500	
5	$P_{5,f}$	$3.1 \cdot 10^{-3}$	$2.4 \cdot 10^{-3}$	$1.7 \cdot 10^{-3}$	$1.0 \cdot 10^{-3}$	$3.1 \cdot 10^{-4}$
	$C_{5,f}$ (\$)	45,500	47,500	49,500	51,500	53,500
14	$P_{14,f}$	$1.3 \cdot 10^{-3}$	$1.0 \cdot 10^{-3}$	$7.0 \cdot 10^{-4}$	$4.0 \cdot 10^{-4}$	$1.3 \cdot 10^{-4}$
	$C_{14,f}$ (\$)	53,500	56,620	59,740	62,860	66,000
13	$P_{13,f}$	$1.5 \cdot 10^{-3}$	$1.2 \cdot 10^{-3}$	$8.0 \cdot 10^{-4}$	$4.8 \cdot 10^{-4}$	$1.5 \cdot 10^{-4}$
	$C_{13,f}$ (\$)	66,000	69,750	73,500	77,250	81,000
1	$P_{1,f}$	$1.5 \cdot 10^{-3}$	$1.2 \cdot 10^{-3}$	$8.0 \cdot 10^{-4}$	$4.8 \cdot 10^{-4}$	$1.5 \cdot 10^{-4}$
	$C_{1,f}$ (\$)	81,000	83,120	85,240	87,360	89,500

Thus, value $C_{\xi f}$ is an accumulated value which incorporates the costs of all possible technical amendments for primary failures which are more cost-sensitive than F_{ξ} and, thus, precede the latter in sequence (9.2.3), together with the costs of technical amendments which have been spent for the primary failure F_{ξ} itself.

For example, value $C_{22} = \$40{,}840$ incorporates all costs for reducing primary failure probabilities P_6 (from $1.3 \cdot 10^{-3}$ to $1.3 \cdot 10^{-4}$), P_{11} (from $3.7 \cdot 10^{-3}$ to $3.7 \cdot 10^{-4}$), P_{20} (from

$1.3 \cdot 10^{-3}$ to $1.3 \cdot 10^{-4}$), and the costs for reducing the probability P_2 by two "probability steps" of length $\Delta P_2 = 3.6 \cdot 10^{-4}$.

Both problems - the direct and the dual ones - have been solved by carrying out Step 7 of the algorithm. Examining step-by-step the subset of cost-sensitive primary failures (9.2.3) by diminishing their failure probabilities by ΔP_ξ while accumulating the costs of the corresponding technical amendments, is shown in Table 9.3. Table 9.3 differs from Table 9.2 by substituting value $P_{\xi f}$ (the first value of element $E_{\xi f}$) for another one, namely

$$P_{cr} \big| P_{\eta_1 \min}, P_{\eta_2 \min}, \dots, P_{\eta_q \min}, \; P_{\xi f} = P_\xi - f \cdot \Delta P_\xi, \; \{P_\gamma\}, \qquad (9.2.7)$$

where η_r, $1 \le r \le q$, are indices which precede ξ in sequence (9.2.3), while all γ are indices which either stand after ξ in sequence (9.2.3) or aren't included in (9.2.3) at all. Values (9.2.7) have been calculated by means of simulation by undertaking 10^6 simulation runs for each value, while all input values $P_{\xi f}$ have been taken from Table 9.2.

Table 9.3. Consecutive accumulated examination of possible technical amendments for cost-sensitive primary failures

ξ	$E_{\xi f}$	f=0	f=1	f=2	f=3	f=4
6	P_{cr}	$6.09 \cdot 10^{-7}$	$5.01 \cdot 10^{-7}$	$4.047 \cdot 10^{-7}$	$3.2 \cdot 10^{-7}$	
	$C_{6,f}$ (\$)	0	3,333	6,666	10,000	
11	P_{cr}	$3.2 \cdot 10^{-7}$	$2.56 \cdot 10^{-7}$	$1.98 \cdot 10^{-7}$	$1.45 \cdot 10^{-7}$	
	$C_{11,f}$ (\$)	10,000	14,000	18,000	22,000	
20	P_{cr}	$1.45 \cdot 10^{-7}$	$1.239 \cdot 10^{-7}$	$1.025 \cdot 10^{-7}$	$8.1 \cdot 10^{-8}$	
	$C_{20,f}$ (\$)	22,000	25,170	28,340	31,500	
2	P_{cr}	$8.1 \cdot 10^{-8}$	$5.47 \cdot 10^{-8}$	$3.28 \cdot 10^{-8}$	$1.5 \cdot 10^{-8}$	
	$C_{2,f}$ (\$)	31,500	36,170	40,840	45,500	
5	P_{cr}	$1.5 \cdot 10^{-8}$	$1.41 \cdot 10^{-8}$	$1.32 \cdot 10^{-8}$	$1.22 \cdot 10^{-8}$	$1.13 \cdot 10^{-8}$
	$C_{5,f}$ (\$)	45,500	47,500	49,500	51,500	53,500
14	P_{cr}	$1.13 \cdot 10^{-8}$	$9.96 \cdot 10^{-9}$	$8.7 \cdot 10^{-9}$	$7.46 \cdot 10^{-9}$	$6.43 \cdot 10^{-9}$
	$C_{14,f}$ (\$)	53,500	56,620	59,740	62,860	66,000
13	P_{cr}	$6.43 \cdot 10^{-9}$	$5.35 \cdot 10^{-9}$	$4.01 \cdot 10^{-9}$	$3.04 \cdot 10^{-9}$	$2.11 \cdot 10^{-9}$
	$C_{13,f}$ (\$)	66,000	69,750	73,500	77,250	81,000
1	P_{cr}	$2.11 \cdot 10^{-9}$	$2.01 \cdot 10^{-9}$	$1.86 \cdot 10^{-9}$	$1.75 \cdot 10^{-9}$	$1.63 \cdot 10^{-9}$
	$C_{1,f}$ (\$)	81,000	83,120	85,240	87,360	89,500

A careful examination of Table 9.3 enables solution of both direct and dual optimization problems as outlined above. Namely, to obtain $P_{cr} \approx 2 \cdot 10^{-9}$, the management has to refine primary failure elements F_6, F_{11}, F_{20}, F_2, F_5, F_{14}, F_{13} by diminishing their existing probabilities from P_ξ to $P_{\xi \min}$. As to the primary failure F_1, its probability has to be diminished by one step. The minimal budget to implement the necessary technical amendments is $\$\ 83{,}120$. This is the solution of the considered direct problem.

It can be well-recognized that assigning a budget equal to $\$\ 50{,}000$ dedicated to undertaking possible technical refinements in the system, enables obtaining the minimal top critical failure probability $1.3 \cdot 10^{-8}$ with the following primary failures' amendments:

a) primary failure probabilities P_6, P_{11}, P_{20} and P_2, have to be reduced up to $P_{6\,min}$, $P_{11\,min}$, $P_{20\,min}$, $P_{2\,min}$;

b) primary failure probability P_5 has to be reduced by two steps, i.e., by $1.4 \cdot 10^{-3}$.

Note that the outlined above solution of the dual problem is in fact nothing else but a simplified modification of the bisection method presented in 9.1.4.

To implement the bisection method outlined in 9.1.4, set $P_{cr.\min} = 1.63 \cdot 10^{-9} = p_1$, $P_{cr} = 6.09 \cdot 10^{-7} = p_2$, $C_1 = 89{,}500$ and $C_2 = 0$, according to Steps 1, 2 of the dual problem's algorithm. By calculating $P_{cr.3} = 3.053 \cdot 10^{-7}$ at Step 3 and solving the direct problem at Step 4, we obtain $C_3 \cong 11{,}000$. Since relation $C_3 < C^* < C_1$ holds, we replace $P_{cr.3}$ by p_2 at Step 8, again calculate $P_{cr.3} = 0.5 \cdot \left(1.63 \cdot 10^{-9} + 3.053 \cdot 10^{-7}\right) = 1.53 \cdot 10^{-7}$, solve the direct problem in order to obtain a new $C_3 \cong 21{,}500$, etc. It can be well-recognized that values $\{C_3\}$ in the course of carrying out the iterative bisection method by using Table 9.3, converge to value $P_{cr.3} \cong 1.3 \cdot 10^{-8}$ with the error accuracy not exceeding $\Delta C = 1{,}000$.

It can be well-recognized that Tables 9.2-9.3 may be modified by means of implementing the concept of technical improvements discussed in §9.2. Thus, by means of checking 8 essential and sensitive primary failures in (9.2.3), we may determine 28 technical improvements outlined in Table 9.4.

Table 9.4. Description of technical improvements

№ of technical improvement $(TI)_k$	*Corresponding primary failure*	*Description of the technical improvement*	*Capital investments* ΔC_k *(in $)*
1	F_6	Diminish the probability of primary failure F_6 by the *first* probability step	*3,333*
2	F_6	Diminish the probability of primary failure F_6 by the *second* probability step	*3,333*
3	F_6	Diminish the probability of primary failure F_6 by the *third* probability step	*3,333*
4	F_{11}	Diminish the probability of primary failure F_{11} by the *first* probability step	*4,000*
5	F_{11}	Diminish the probability of primary failure F_{11} by the *second* probability step	*4,000*
6	F_{11}	Diminish the probability of primary failure F_{11} by the *third* probability step	*4,000*
7	F_{20}	Diminish the probability of primary failure F_{20} by the *first* probability step	*3,167*
8	F_{20}	Diminish the probability of primary failure F_{20} by the *second* probability step	*3,167*
9	F_{20}	Diminish the probability of primary failure F_{20} by the *third* probability step	*3,167*
10	F_2	Diminish the probability of primary failure F_2 by the *first* probability step	*4,667*
11	F_2	Diminish the probability of primary failure F_2 by the *second* probability step	*4,667*
12	F_2	Diminish the probability of primary failure F_2 by the *third* probability step	*4,667*
13	F_5	Diminish the probability of primary failure F_5 by the *first* probability step	*2,000*
14	F_5	Diminish the probability of primary failure F_5 by the *second* probability step	*2,000*
15	F_5	Diminish the probability of primary failure F_5 by the *third* probability step	*2,000*
16	F_5	Diminish the probability of primary failure F_5 by the *fourth* probability step	*2,000*
17	F_{14}	Diminish the probability of primary failure F_{14} by the *first* probability step	*3,125*
18	F_{14}	Diminish the probability of primary failure F_{14} by the *second* probability step	*3,125*

№ of technical improvement $(TI)_k$	Corresponding primary failure	Description of the technical improvement	Capital investments ΔC_k (in $)
19	F_{14}	Diminish the probability of primary failure F_{14} by the *third* probability step	*3,125*
20	F_{14}	Diminish the probability of primary failure F_{14} by the *fourth* probability step	*3,125*
21	F_{13}	Diminish the probability of primary failure F_{13} by the *first* probability step	*3,750*
22	F_{13}	Diminish the probability of primary failure F_{13} by the *second* probability step	*3,750*
23	F_{13}	Diminish the probability of primary failure F_{13} by the *third* probability step	*3,750*
24	F_{13}	Diminish the probability of primary failure F_{13} by the *fourth* probability step	*3,750*
25	F_1	Diminish the probability of primary failure F_1 by the *first* probability step	*2,125*
26	F_1	Diminish the probability of primary failure F_1 by the *second* probability step	*2,125*
27	F_1	Diminish the probability of primary failure F_1 by the *third* probability step	*2,125*
28	F_1	Diminish the probability of primary failure F_1 by the *fourth* probability step	*2,125*

Table 9.4 enables developing an "accumulated" Table 9.5 where each ordinary number k of the technical improvements TI_k corresponds with both the value of accumulated capital investments $\sum_{\xi=1}^{\eta} \Delta C_\xi$ and the monotonously increasing value of the system's reliability $R_S\{TI_\xi, 1 \le \xi \le k\} = 1 - P_{cr}\{TI_\xi, 1 \le \xi \le k\}$. Both values are obtained for all technical improvements preceding and comprising TI_k, i.e., $\{TI_\xi, 1 \le \xi \le k\}$, to be implemented into the technical system under consideration. Note that value $\sum_{\xi=1}^{\eta} \Delta C_\xi$ is calculated from Table 9.4 on the basis of column (4), while $1 - P_{cr}\{TI_\xi\} = R_S\{TI_\xi\}$ is calculated by means of the simulation model, on the basis of Table 9.2. Thus, Table 9.5 provides a graceful demonstration of the solution process and can be regarded as an efficient tool for solving practical cost-optimization problems on the basis of cost-reliability sensitivity concepts. Both Tables 9.3 and 9.5 can be recommended for practical reasons. It goes without saying that using both Tables 9.3 and 9.5 results in obtaining the same solution of both regarded here cost-reliability problems.

Table 9.5. Accumulated cost- and reliability values (basic values: $C_0 = 0$, $R_0 = 0.999999391$)

Value k	$\sum_{\xi=1}^{k} \Delta C_\xi$ (in \$)	$R_S\{TI_\xi, 1 \le \xi \le k\}$
1	3,333	0.999999499
2	6,666	0.9999995953
3	10,000	0.999999680
4	14,000	0.999999744
5	18,000	0.999999802
6	22,000	0.999999855
7	25,170	0.9999998761
8	28,340	0.9999998975
9	31,500	0.9999999190
10	36,170	0.9999999453
11	40,840	0.9999999672
12	45,500	0.9999999850
13	47,500	0.9999999859
14	49,500	0.9999999868
15	51,500	0.9999999878
16	53,500	0.9999999887
17	56,620	0.9999999904
18	59,740	0.9999999913
19	62,860	0.99999999254
20	66,000	0.99999999357
21	69,750	0.99999999465
22	73,500	0.99999999599
23	77,250	0.99999999696
24	81,000	0.99999999789
25	83,120	0.99999999799
26	85,240	0.99999999814
27	87,360	0.99999999825
28	89,500	0.99999999837

Chapter 10. Cost-Reliability Optimization Models for Maintenance Systems in Safety Engineering

§10.1 Cost-reliability harmonization models

10.1.1 The system's description

We will consider a complicated man-machine organization system functioning under random disturbances. The device's reliability, i.e., its probability to avoid critical failures within a sufficiently long period of time, has to be extremely high since critical failures present a definite threat to people's safety, to the environment, etc., and may result in an accident or a major hazardous condition. Thus, increasing the device's reliability is considered to be an important problem of safety engineering, on assumption that the existing reliability value proves to be insufficient.

Consider, further [9], that there exist N technical improvements (TI) to increase the system's reliability. For each k-th TI, $1 \le k \le N$, investing ΔC_k cost expenditures results in increasing the device's reliability by ΔR_k. Assume that those parameters are obtained by means of simulation model SM and do not depend on the number of technical improvements which have already been implemented. Thus, the result of a routine k-th technical improvement does not depend on other $\{TI\}$.

To outline the models under consideration we will require the following additional terms:

TS - complicated multi-level technical system with hazardous failures at the upper level;

N - the number of possible technical improvements TI_k, $1 \le k \le N$;

ΔC_k - cost value required to carry out TI_k;

SM - simulation model to calculate the system's reliability value;

ΔR_k - additional reliability value obtained as a result of undertaking TI_k (to be calculated by means of simulation model SM);

P_ℓ - additional non-basic parameter, $1 \le \ell \le m$;

m - the number of non-basic parameters;

α_C - the budget's partial utility;

α_R - the reliability's partial utility;

α_{P_ℓ} - partial utility of parameter P_ℓ;

R^* - the system's reliability level to avoid hazardous failures (pregiven);

C^* - maximal additional budget (pregiven);

$\Delta P_{\ell k}$ - additional value of parameter P_ℓ obtained as a result of undertaking TI_k,

$1 \le \ell \le m,\ 1 \le k \le N$;

$\Delta U_{\ell k}$ - additional system's utility obtained as a result of undertaking TI_k, $1 \le k \le N$ (to be calculated by means of simulation model SM);

R_0 - system's reliability value prior to undertaking amendments (pregiven);

$\Delta R\left\{\xi_1, ..., \xi_Q\right\}$ - increase of the system's reliability due to implementing Q different technical improvements $\left\{TI_{\xi_q}\right\}$, $1 \le q \le Q$, $\xi_q \le N$ (calculated by means of a simulation model SM).

The problems to be considered below present simplified particular cases of the general theory of harmonization models outlined in Chapter 5. However, an effective and simple to use heuristic approach based on cost-sensitivity, can be suggested. To our opinion, the developed models can be applied to a broad spectrum of technical devices in the framework of safety engineering [9, 39-40]. The problems are as follows:

10.1.2 *The direct cost-reliability problem*

Determine the optimal set of technical improvements TI_{ξ_q}, $1 \le q \le Q \le N$, $\xi_q \le N$, which requires the minimal amount of costs to undertake the TI in order to increase the device's reliability by not less than $R^* - R_0$ (see *Notation* in *10.1.1*), i.e.,

$$\underset{\{\xi_q\}}{Min}\left\{\sum_{q=1}^{Q} \Delta C_{\xi_q}\right\} \tag{10.1.1}$$

subject to

$$R_0 + \sum_{q=1}^{Q} \Delta R_{\xi_q} \ \ge \ R^*. \tag{10.1.2}$$

10.1.3 *The dual cost-reliability problem*

Determine the optimal set of TI_{ξ_q}, $1 \le q \le Q \le N$, $\xi_q \le N$, in order to maximize the system's reliability subject to the restricted amount of costs ΔC to undertake the corresponding TI, i.e.,

$$\underset{\{\xi_q\}}{Max}\left\{\sum_{q=1}^{Q} \Delta R_{\xi_q}\right\} \tag{10.1.3}$$

subject to

$$\sum_{q=1}^{Q} \Delta C_{\xi_q} \le \Delta C . \qquad (10.1.4)$$

Since all TI are independent of each other, both problems (10.1.1-10.1.2) and (10.1.3-10.1.4) are NP-complete [66, 176], and, an optimal solution can be obtained only by means of implementing an algorithm (mainly by means of dynamic programming) that checks the feasibility of all possible combinations of Q elements from N, while Q itself changes from 1 to N. If N is high enough, the corresponding algorithm requires a lot of computational time, according to the justification outlined in §5.3. We suggest using a heuristic procedure based on cost-sensitivity. Note, that if relation

$$\sum_{i=1}^{Q} \Delta R_i \ge R^* - R_0 \qquad (10.1.5)$$

does not hold, the direct problem (10.1.1-10.1.2) has no solution. As to the dual problem, the corresponding restriction

$$\sum_{i=1}^{Q} \Delta C_i \le C^* \qquad (10.1.6)$$

results in a trivial solution $Q = N$, i.e., all technical investments have to be implemented.

10.1.4 The direct problem's solution (Algorithm I)

In order to proceed, we will introduce a new definition. Call henceforth *the cost-reliability of a technical improvement* the ratio $\gamma = \Delta R / \Delta C$. It can be well-recognized that if TI_{k_1} has a higher cost-reliability than TI_{k_2}, investing one and the same cost expenditure results in a higher increase of the reliability parameter in case of implementing the TI_{k_1} than TI_{k_2}. This consideration is used below, in the step-by-step heuristic algorithm:

Step 1. Calculate cost-reliability values γ_k for all TI_k, $1 \le k \le N$.

Step 2. Reorder values γ_k in descending order. Thus, values γ_k, $1 \le k \le N$, will obtain a new order. Denote the corresponding new indices (ordinal numbers) of technical improvements by TI_{ξ_q}, $1 \le q \le N$.

Step 3. Determine the minimal value Q which satisfies

$$Q = Min \left[V : \sum_{q=1}^{V} \Delta R_{\xi_q} \ge R^* - R_0 \right]. \qquad (10.1.7)$$

Step 4. Determine the quasi-optimal indices of the chosen technical improvements:

TI_{ξ_1} , TI_{ξ_2} , ... , TI_{ξ_Q} .

The idea of Algorithm I is to spend as little budget as possible in order to meet constraint (10.1.2).

10.1.5 The dual problem's solution (Algorithm II)

The corresponding step-by-step heuristic algorithm is as follows:

Steps 1 and 2 fully coincide with the corresponding stages of Algorithm I.

Step 3. Determine the maximal value Q which satisfies

$$Q = Max\left[V : \sum_{q=1}^{V} \Delta C_{\xi_q} \leq C^* \right]. \tag{10.1.8}$$

Step 4 fully coincides with Step 4 of *Algorithm I*.

It can be well-recognized that introducing the concept of cost-reliability enables a simple and effective solution of various cost-optimization problems in safety engineering. Practical results of applications to real industrial establishments are outlined below.

Similarly to models described in Chapter 5, both optimization problem (10.1.1-10.1.2) and (10.1.3-10.1.4) are partial harmonization problems. Problem (10.1.1-10.1.2) is a $PHM_1(R) = C$ with one independent basic parameter - system's reliability value R, and one dependent parameter - the budget to be assigned for undertaking technical improvements. $PHM_1(R) = C$ centers on minimizing C subject to the prescribed reliability. Problem (10.1.3-10.1.4) is a $PHM_2(C) = R$ which centers on maximizing the reliability value R subject to restricted budget value C. Both problems are more generalized than the PHM problems outlined in the previous section, although they are solved by using the same cost-sensitivity heuristic method.

10.1.6 Cost-reliability harmonization model with two basic independent parameters

We will consider an interesting case (and for certain multi-level OS an important one!) of a system with possible hazardous failures at the upper level. Two independent basic parameters are imbedded in the model: budget C to carry out technical improvements, and the system's reliability value R. In order to simplify the problem assume that, similarly to the model outlined above, in (10.1.2-10.1.5), all technical improvements are additive, i.e., additional system's reliability $\Delta R\{\xi_1,..., \xi_Q\}$ obtained by implementing $\left\{ TI_{\xi_q} \right\}$, $1 \leq q \leq Q$, is equal $\sum_{q=1}^{Q} \Delta R_{\xi_q}$.

Set the "weight" of increasing the device's reliability (per reliability unit) by α_r, and let the corresponding weight of cost investments per cost unit be α_c. The harmonization model is an extension of the cost-reliability model outlined above. The problem is as follows:

Determine the optimal set of $\left\{ TI_{\xi_q} \right\}$, $1 \le q \le Q$, $\xi_q \le N$, in order to maximize the harmonization objective

$$J = \underset{\{\xi_q\}}{Max} \left\{ \sum_{q=1}^{Q} \left[\alpha_r \cdot \Delta R_{\xi_q} - \alpha_c \cdot \Delta C_{\xi_q} \right] \right\} \tag{10.1.9}$$

subject to

$$\sum_{q=1}^{Q} \Delta C_{\xi_q} \le C^*, \tag{10.1.10}$$

$$\sum_{q=1}^{Q} \Delta R_{\xi_q} \ge R^* - R_0 = \Delta R. \tag{10.1.11}$$

Note that since the costs ΔC_{ξ_q} to be invested in the course of undertaking TI_{ξ_q} decrease the system's utility, i.e., decrease objective (10.1.9), they have to be taken with a negative sign, while increasing the system's reliability results in increasing the quality of the system as a whole.

Model (10.1.9-10.1.11) unlike all previous harmonization models in safety engineering, comprises two restrictions since for both basic parameters C and R their corresponding upper and lower bounds are pregiven.

Model (10.1.9-10.1.11) is a complicated NP-complete problem which requires only heuristic solutions, since using classical precise optimization algorithms meets unavoidable computational difficulties. We suggest to solve the problem by implementing the idea of cost-sensitivity, based on introducing values $\gamma_k = \frac{\Delta R_k}{\Delta C_k}$, $1 \le k \le N$, and, later on, reordering TI_k, $1 \le k \le N$, in the descending order of values γ_k. Thus, sequence TI_{ξ_q}, $1 \le q \le N$, is obtained.

To develop a heuristic procedure, we will modify objective (10.1.9) as follows:

$$J = \underset{\{\xi_q\}}{Max} \left\{ \sum_{q=1}^{Q} \left[\left(\frac{\alpha_r \cdot \Delta R_{\xi_q}}{\alpha_c \cdot \Delta C_{\xi_q}} - 1 \right) \cdot \alpha_c \cdot \Delta C_{\xi_q} \right] \right\} = \underset{\{\xi_q\}}{Max} \left\{ \sum_{q=1}^{Q} \left[\left(\eta \cdot \gamma_{\xi_q} - 1 \right) \cdot \alpha_c \cdot \Delta C_{\xi_q} \right] \right\}, \tag{10.1.12}$$

where $\eta = \frac{\alpha_r}{\alpha_c}$ is a constant value which does not depend on the TI index.

Since values γ_{ξ_q} are monotonously decreasing, the multiplicand $\eta \cdot \gamma_{\xi_q} - 1$, $1 \le q \le N$, may for a certain number q turn negative.

Certain realistic assumptions are imbedded in the model:

1. Since the reliability parameter for a technical device with critical failures usually dominates over other parameters, we will assume that relation $\eta = \frac{\alpha_r}{\alpha_c} > 1$ holds.
2. Assume that for the TI_{ξ_1} with the *highest cost-sensitivity*, relation $\eta \cdot \gamma_{\xi_1} > 1$ holds, otherwise a degenerate conclusion can be drawn that the best compromise for the device under consideration is to not undertake any technical improvements at all.

On the basis of the above assumptions the following step-by-step heuristic algorithm to solve harmonization problem (10.1.9-10.1.11) can be suggested:

Step 1. Determine the maximal N_1 satisfying

$$N_1 = Max\left[V : \sum_{q=1}^{V} \Delta C_{\xi_q} \le C^* \right]. \tag{10.1.13}$$

Step 2. Determine the minimal N_2 satisfying

$$N_2 = Min\left[V : \sum_{q=1}^{V} \Delta R_{\xi_q} \ge \Delta R \right]. \tag{10.1.14}$$

Note that if $N_2 > N_1$, the problem has no solution. In case $N_2 \le N_1$ apply Step 3.

Step 3. Determine the maximal N_3 satisfying

$$N_3 = Max\left\lfloor V : \eta \cdot \gamma_{\xi_V} \ge 1 \right\rfloor \tag{10.1.15}$$

subject to

$$N_3 \le N. \tag{10.1.16}$$

Step 4. Determine value Q satisfying

$$Q = \begin{cases} N_1 & if \quad N_3 \ge N_1 \\ N_3 & if \quad N_2 \le N_3 \le N_1 \\ N_2 & if \quad N_3 \le N_2 \end{cases}. \tag{10.1.17}$$

Step 5. Technical improvements $TI_{\xi_1}, TI_{\xi_2}, \ldots, TI_{\xi_Q}$ are taken as the quasi-optimal set $\{TI\}$ to be implemented, with objective

$$J = \sum_{q=1}^{Q} \left[\left(\eta \cdot \gamma_{\xi_q} - 1 \right) \cdot \alpha_c \cdot \Delta C_{\xi_q} \right] . \qquad (10.1.18)$$

Objective (10.1.18) honors restrictions (10.1.10-10.1.12) and delivers the maximal value for the problem's heuristic solution $\{TI\}_{\xi_q}$, $1 \le q \le Q$.

Note, in conclusion, that the ratio $\eta = \frac{\alpha_r}{\alpha_c}$ may not be a constant value. In case of extremely high reliability values R, i.e., when R practically guarantees avoiding hazardous failures and relation $R >> R^*$ holds, the partial utility value α_R may undergo an essential decrease while value α_C will remain constant. Thus, certain technical difficulties may arise. However, from the principal point of view, the algorithm will not be subject to drastic changes.

In the harmonization model under consideration a straightforward heuristic method to optimize objective (10.1.9) is used. As to partial harmonization models, they do not exist in this case, since there are no dependent basic parameters: both basic parameters are set by means of restrictions (10.1.10-10.1.11) and are pregiven beforehand. No parameter is optimized by means of partial harmonization. Both parameters influence one another: this mutual influence is implemented in the heuristic algorithm by means of analyzing partial utility values α_C and α_R.

10.1.7 <u>Generalized harmonization models in safety engineering with non-basic parameters</u>

The harmonization model under consideration comprises, besides two basic parameters C (the budget to be assigned to undertake technical improvements) and R (the system's reliability to avoid hazardous failures), a variety of non-basic parameters entering the system's utility model as well. Non-basic parameters are, e.g., the probability of completing the production program not later than the pregiven due date, reliability value to avoid non-hazardous failures which nevertheless may cause certain damage to the personnel and/or to the environment, specific design failures, etc. Unlike the outlined above cost-reliability models, all TI are non-additive, i.e., the aggregate increase $\Delta R\{\xi_1,..., \xi_Q\}$ due to simultaneous implementation of $TI_{\xi_1},..., TI_{\xi_Q}$, may not be equal $\sum_{q=1}^{Q} \Delta R_{\xi_q}$. This makes the harmonization problem more complicated.

Referring to the *Notation* outlined in *10.1.1*, two problems can be formulated:

<u>Direct Problem</u>

Determine $Q \le N$ technical improvements TI_{ξ_q}, $1 \le q \le Q$, $\xi_q \le N$, to maximize the system's additional utility

$$\underset{\{\xi_q\}}{Max}\left[\sum_{q=1}^{Q}\Delta U_{s\xi_q}\right] =$$
$$= \underset{\{\xi_q\}}{Max}\left\{\sum_{q=1}^{Q}\left[\alpha_C\cdot\Delta C_{\xi_q} + \left(\sum_{\ell=1}^{m}\alpha_{P_\ell}\cdot\Delta P_{\ell\xi_q}\right)\right] + \alpha_R\cdot\Delta R\left(\xi_1,\xi_2,\ldots,\xi_Q\right)\right\} \qquad (10.1.19)$$

subject to

$$R_0 + \Delta R\left(\xi_1, \xi_2, \ldots, \xi_Q\right) \geq R^*. \qquad (10.1.20)$$

Dual Problem

$$\underset{\{\xi_q\}}{Max}\left[\sum_{q=1}^{Q}\Delta U_{s\xi_q}\right] \qquad (10.1.21)$$

subject to

$$\sum_{q=1}^{Q}\Delta C_{\xi_q} \leq C^*. \qquad (10.1.22)$$

It can be well-recognized that solving both problems (10.1.19-10.1.20) and (10.1.21-10.1.22) by means of precise algorithms results in tremendous and practically unavoidable computational difficulties. We suggest developing enhanced heuristic procedures based on sensitivity analysis. Two basic sensitivity values for each TI_k will be used:

$$\text{reliability-sensitivity} \quad \omega_k = \frac{\Delta U_{sk}}{\Delta R_k}, \qquad (10.1.23)$$

and

$$\text{cost-sensitivity} \quad \eta_k = \frac{\Delta U_{sk}}{\Delta C_k}, \quad 1 \leq k \leq N. \qquad (10.1.24)$$

Note that both values ω_k and η_k, $1 \leq k \leq N$, can be obtained only by means of simulation, since ΔU_{sk} comprises ΔR_k and has to be calculated via simulation with an enormous number of simulation runs (see Chapter 5).

10.1.8 The problems' solution

The enlarged step-by-step *Algorithm I* to solve the direct problem is as follows:

Step 1. Calculate reliability-sensitivity values ω_k for all TI_k, $1 \leq k \leq N$.

Step 2. Reorder values ω_k in descending order. Thus, values ω_k will obtain a new order. Denote the corresponding new indices (ordinal numbers) of technical improvements by TI_{ξ_q}, $1 \le q \le N$.

Step 3. Determine the minimal value Q satisfying

$$Q = Min \left[V: \ \Delta R(\xi_1, ..., \xi_V) \ge R^* - R_0 \right]. \qquad (10.1.25)$$

Step 4. If $\Delta U_{SQ} < 0$ go to the next step. Otherwise go to Step 6.

Step 5. If value Q exceeds the minimal value obtained at Step 3 go to Step 8. Otherwise go to Step 9.

Step 6. If $Q = N$ go to Step 9. Otherwise apply the next step.

Step 7. Counter $Q + 1 \Rightarrow Q$ works. Go to Step 4.

Step 8. $Q - 1 \Rightarrow Q$. Apply the next step.

Step 9. Determine the quasi-optimal indices of the chosen technical improvements to be implemented:

$TI_{\xi_1}, TI_{\xi_2}, ..., TI_{\xi_Q}$.

The step-by-step procedure of *Algorithm II* to solve the dual problem is as follows:

Step 1. Calculate cost-sensitivity values η_k for all TI_k, $1 \le k \le N$.

Step 2. Reorder values η_k in descending order, similarly to Step 2 of *Algorithm I*.

Step 3. Determine the maximal value Q satisfying

$$Q = Max \left[V: \ \sum_{q=1}^{Q} \Delta C_{\xi_q} \le C^* \right]. \qquad (10.1.26)$$

Step 4. If $\Delta U_{SQ} \le 0$ go to the next step. Otherwise apply Step 7.

Step 5. If $Q = 1$ go to Step 7. Otherwise apply the next step.

Step 6. Counter $Q - 1 \Rightarrow Q$ works. Go to Step 4.

Step 7. Determine the quasi-optimal solution of *Algorithm II*, i.e., the quasi-optimal sub-set of $\left\{ TI_{\xi_q} \right\}$, $1 \le q \le Q$.

Algorithms I and *II* cover a broad spectrum of safety engineering problems. A numerical example of solving a direct harmonization problem (10.1.19-10.1.20) with 4 parameters on a real industrial plant is outlined in Chapter 5.

Note that the direct harmonization problem (10.1.19-10.1.20) is based on $(m+1)$ partial harmonization models with R being an independent parameter: $PHM_1(R) = C$, $PHM_{1\ell}(R) = P_\ell$, $1 \le \ell \le m$, which later on enter the utility increment ΔU_s. As to the dual problem (10.1.21-10.1.22), it comprises another $(m+1)$ PHM with budget value C being an independent parameter: $PHM_2(C) = R$, $PHM_{2\ell}(C) = P_\ell$, $1 \le \ell \le m$.

In conclusion, partial utility parameters α_R and α_ℓ, $1 \le \ell \le m$, in practice, are usually piecewise functions depending on the parameters' values. As demonstrated below, this causes certain computational difficulties in solving harmonization problems. However, those difficulties do not inflict principal troubles and can be overcome.

§10.2 Experimentation in harmonizing a safety engineering system

In order to verify the fitness of the outlined above harmonization theory, the problem considered in §10.1, has been extended, namely, additional basic parameters have been included. Two additional basic parameters have been suggested by an expert team [9], namely:

- the system's *weight* W_S depending on implementation of technical improvements TI_ξ, $1 \le \xi \le k$, in primary failures F_ξ. Value W_S increases monotonously by increasing the ordinal number k, $1 \le \xi \le 28$. This, in turn, may result in developing further non-aesthetic features capable of diminishing the system's market price and jeopardizing successful competition with other similar devices;
- the system's non-critical failure value, i.e., the probability value P_{ncr}. The non-critical failure F_{ncr} cannot cause large-scale accidents but may result in a less severe personnel health & safety conditions violations / minor injuries / property damage, etc. Value P_{ncr} always decreases monotonously in the course of increasing number k. It goes without saying that although P_{ncr} is essentially less important than the top critical probability P_{cr}, it should be taken into account by calculating the system's optimal utility value.

Thus, the basic parameters to be taken into consideration in the course of the harmonization process, are as follows:

1. Capital investments C, i.e. the total cost of undertaking TI_ξ, $1 \le \xi \le k$.
2. Additional reliability value $R_S = 1 - P_{cr}$, after implementing $\{TI_\xi, 1 \le \xi \le k\}$.
3. Additional system's weight W_S obtained in the course of undertaking $\{TI_\xi$,

$1 \le \xi \le k \}$.

4. Additional decreased non-critical top failure value P_{ncr} after undertaking the outlined above technical improvements $\{ TI_{\xi}, 1 \le \xi \le k \}$.

The first two basic parameters can be easily examined by means of Table 9.5. As to values W_S and P_{ncr}, they can be obtained only via expert information, by examining Table 9.4.

The summarized expert information is presented in Table 10.1.

Table 10.1. Accumulated weight- and non-critical top failure levels (obtained by means of expert information)

Value k	Increasing $\sum_{\xi=1}^{k} \Delta W_{\xi}$ (in tons)	Decreasing $\sum_{\xi=1}^{k} \Delta P_{ncr\ \xi}$ (in 10^{-4})
1	0.05	2.0
2	0.10	4.0
3	0.15	6.0
4	0.25	8.0
5	0.35	10.0
6	0.45	12.0
7	0.60	13.5
8	0.75	15.0
9	0.90	16.5
10	0.95	18.0
11	1.00	19.5
12	1.05	21.0
13	1.25	22.0
14	1.45	23.0
15	1.65	24.0
16	1.85	25.0
17	2.00	25.8
18	2.15	26.6
19	2.30	27.4
20	2.45	28.2
21	2.65	28.9
22	2.85	29.6
23	3.05	30.3
24	3.25	31.0
25	3.40	31.5
26	3.55	32.0
27	3.70	32.5
28	3.85	33.0

A special emphasis has to be drawn as to the way information in Tables 9.4 and 10.1 has been obtained. The first basic parameter C is an independent parameter and can be preset by the management. Parameter $R_S\{TI_\xi, 1 \le \xi \le k\}$ has been determined by means of the partial harmonization model, after carrying out numerous simulation runs. Parameters ΔW_S and ΔP_{ncr} have been calculated by using the Delphi method, on the basis of expert information. Thus, the problem under consideration can be regarded as a mixed "expert – harmonization" model, where expert information supplements the information obtained through optimizing the partial harmonization model.

Local parametrical utility values α_η, $1 \le \eta \le 4$, have also been pregiven by experts and are as follows:

I. $\alpha_C = -1 \quad \textit{for the cost unit} \quad \delta C = \$1{,}000;$

II. $$\alpha_R = \begin{cases} +10, & \textit{reliability unit} \quad \delta R = 10^{-7}, \quad \textit{for} \quad 0.999999 \le R \le 0.9999999 \\ +\ 3, & \textit{reliability unit} \quad \delta R = 10^{-8}, \quad \textit{for} \quad 0.9999999 \le R \le 0.99999999 \\ +\ 1, & \textit{reliability unit} \quad \delta R = 10^{-9}, \quad \textit{for} \quad 0.99999999 \le R \le 0.999999999 \end{cases};$$

III. $$\alpha_W = \begin{cases} -0.1, & \textit{weight unit} \quad \delta W = 100\ Kg, \quad \textit{for values} \quad 0\ ton \le W \le 1\ ton \\ -0.2, & \textit{weight unit} \quad \delta W = 100\ Kg, \quad \textit{for values} \quad 1\ ton < W \le 2\ ton \\ -0.3, & \textit{weight unit} \quad \delta W = 100\ Kg, \quad \textit{for values} \quad W > 2\ ton \end{cases};$$

IV. $$\alpha_{F_{ncr}} = \begin{cases} +\ \ 1, & \textit{non-critical probability unit} \quad \delta P_{ncr} = 10^{-4}, \quad \Delta P_{ncr} \le 25 \cdot 10^{-4} \\ +0.5, & \textit{non-critical probability unit} \quad \delta P_{ncr} = 10^{-4}, \quad \Delta P_{ncr} > 25 \cdot 10^{-4} \end{cases}.$$

It can be well-recognized that, except for the first independent cost parameter, all other local utility values are of peace-wise type. This makes the harmonization problem more complicated.

The final table to be presented - Table 10.2 - illustrates the harmonization process for the multilevel technical system. Note that in this table, moving in the positive direction (see Chapter 5) of a routine basic parameter results in using a positive sign for that parameter.

Examining Table 10.2 leads to the conclusion that the maximal system's utility is provided by value $k = 12$. This, in turn, corresponds to the following technical improvements to be undertaken:

- primary failure probabilities P_6, P_{11}, P_{20} and P_2 have to be reduced up to their minimal possible values $P_{6\ min}$, $P_{11\ min}$, $P_{20\ min}$ and $P_{2\ min}$, correspondingly. Other primary failure elements remain unchanged.

Table 10.2. Illustration of the harmonization process

k	$-\sum_{\xi=1}^{\eta} \alpha_C \times \times \Delta C_\xi$	$+\sum_{\xi=1}^{k} [\alpha_R \times \times (R_s - R_0)]$	$-\sum_{\xi=1}^{k} (\alpha_W \times \times W_\xi)$	$+\sum_{\xi=1}^{k} \alpha_{F_{ncr}} \times \times \Delta P_{ncr}$	$\sum_{\eta=1}^{4} \alpha_\eta U_\eta$	*Feasibility*
1	- 3.333	+10.8	-0.05	+ 2.00	+ 9.417	feasible
2	- 6.666	+20.4	-0.10	+ 4.00	+17.634	feasible
3	-10.000	+28.9	-0.15	+ 6.00	+24.750	feasible
4	-14.000	+35.3	-0.25	+ 8.00	+29.250	feasible
5	-18.000	+41.1	-0.35	+10.00	+32.750	feasible
6	-22.000	+46.4	-0.45	+12.00	+35.950	feasible
7	-25.170	+48.5	-0.60	+13.50	+36.230	feasible
8	-28.340	+50.6	-0.75	+15.00	+36.510	feasible
9	-31.500	+57.2	-0.90	+16.50	+41.300	feasible
10	-36.170	+65.0	-0.95	+18.00	+45.880	feasible
11	-40.840	+71.6	-1.00	+19.50	+49.260	feasible
12	**-45.500**	**+77.0**	**-1.10**	**+21.00**	**+51.400**	**optimal**
13	-47.500	+77.6	-1.50	+22.00	+50.600	feasible
14	-49.500	+78.6	-1.90	+23.00	+50.200	feasible
15	-51.500	+79.6	-2.30	+24.00	+49.800	feasible
16	-53.500	+80.5	-2.70	+25.00	+49.300	feasible
17	-56.620	+81.8	-3.00	+25.40	+47.180	feasible
18	-59.740	+82.7	-3.45	+25.80	+45.310	feasible
19	-62.860	+83.8	-3.90	+26.20	+43.240	feasible
20	-66.000	+84.8	-4.35	+26.60	+41.050	feasible
21	-69.750	+85.8	-4.95	+26.95	+38.050	feasible
22	-73.500	+87.1	-5.55	+27.40	+35.450	feasible
23	-77.250	+88.1	-6.15	+27.65	+32.350	feasible
24	-81.000	+89.1	-6.75	+28.00	+29.350	feasible
25	-83.120	+89.3	-7.20	+28.25	+27.330	feasible
26	-85.240	+89.4	-7.65	+28.50	+25.010	feasible
27	-87.360	+89.4	-8.10	+28.75	+22.690	feasible
28	-89.500	+89.5	-8.55	+29.00	+20.450	feasible

It can be well-recognized that the solution of the harmonization problem differs essentially from the solution obtained in §9.1 when solving the local direct cost-reliability optimization problem. The latter covers less basic characteristics and parameters than the harmonization model being based on new system concepts.

Chapter 11. "Look-Ahead" Heuristics in Safety Engineering Maintenance Systems

§11.1 Introduction

In the recent four decades extensive research has been undertaken in order to develop general search strategies designated as "look-ahead" approaches. The idea is to undertake decision-making by guessing the next operation to perform (usually by means of a heuristic). Note that "look-ahead" strategies are usually applied not to a row of values, but to a problem to be optimized. Thus, the term "the next operation" signifies the step, i.e., the decision-making, which brings us closer to the problem's goal to be optimized. If the decision-making does not result in progress towards the goal, then the step can be retracted and another decision-making can be tried.

Various scientists (see, e.g., [186]) have developed a variety of algorithms in order to improve the objectives of problems related to trees defining the search space, as well as to branch- and bounds constraints. D. Golenko-Ginzburg and A. Gonik [108] used "look-ahead" techniques to improve the general job-shop schedule.

In our current research we consider another area, namely, to optimize one of the basic system's parameters, e.g. the system's reliability, by means of constrained technical improvements subject to the restricted budget assigned for the parameter's amendment.

We will consider henceforth the system's reliability although other basic parameters may be taken into account as well. The outlined below "look-ahead" techniques are combined with the cost-sensitivity approach [9]. This results in effective heuristic algorithms.

§11.2 The system's description

A complicated technical device functioning under random disturbances is considered. The device's reliability, i.e., its probability to avoid critical failures within a sufficiently long period of time, has to be extremely high since critical failures present a definite threat to people's safety, to the environment, etc., and may result in an accident or a major hazardous condition. Thus, increasing the device's reliability is considered to be an important problem of Safety Engineering, on assumption that the existing reliability value proves to be insufficient.

There exist N technical improvements (TI) to increase the device's reliability. For each k-th TI, $1 \le k \le N$, investing ΔC_k cost expenditures results in increasing the device's reliability by ΔR_k. Those parameters are obtained by means of simulation model SM and do not depend on the number of technical improvements which have already been implemented. Thus, the result of a routine k-th technical improvement does not depend on other $\{TI\}$.

We have previously formulated and solved [9, 21, 23] two cost-reliability problems as follows:

The direct problem

Determine the optimal set of TI_{ξ_q}, $1 \le q \le Q \le N$, $\xi_q \le N$, in order to maximize the device's reliability subject to the restricted amount of costs ΔC to undertake the corresponding TI.

The dual problem

Determine the optimal set of technical improvements TI_{ξ_q}, $1 \le q \le Q \le N$, $\xi_q \le N$, which requires the minimal amount of costs to undertake the TI in order to increase the device's reliability by not less than ΔR.

However, the above formulated problems are not generalized. This is because all technical improvements TI_k, $1 \le k \le N$, are not considered to be compound. In real practice, however, technical improvements have a stepwise structure, i.e., each TI_k consists of consecutively realized steps $TI_{k1}, TI_{k2}, ..., TI_{kn_k}$. Here each n_k denotes the number of steps for TI_k.

Assume that for each routine TI_k undertaking step TI_{km} can be realized on condition that all $(m-1)$ previous steps $TI_{k\xi}$, $1 \le \xi \le m-1$, have been accomplished before. Thus, each step TI_{km} can be realized if and only if the previous step $TI_{k,m-1}$ has been finished.

Assume, further, that each technical improvement TI_k can be interrupted after undertaking a routine step TI_{km}, $1 \le m \le n_k$, i.e., only part of the steps may be implemented. Each step TI_{km} is characterized by two cost-reliability parameters:

- cost ΔC_{km} to undertake TI_{km};
- additional reliability value ΔR_{km} which increases the total device reliability level R by implementing step TI_{km}. Value ΔR_{km} can be calculated by means of SM.

The problem under consideration is as follows:

Given a restricted cost value C, single out a set of optimal steps $\{TI_{km}\}$, $1 \le k \le N$, $m \le \xi_k$, $0 \le \xi_k \le n_k$, in order to maximize the total additional reliability $\sum_{k=1}^{N} \sum_{m=0}^{\xi_k} \Delta R_{km}$ subject to the restricted total cost investments $\sum_{k=1}^{N} \sum_{m=0}^{\xi_k} \Delta C_{km} \le C$.

Note that TI_{k0} denotes that the k-th technical improvement has not been implemented at all, i.e., not a single step of the TI_k sequence has been operated as yet.

The problem can be solved by means of integer programming involving a rather complicated algorithm. Essentially simpler quasi-optimal results may be obtained by using heuristic approaches, namely, by implementing sensitivity analysis in the corresponding optimization algorithm, as outlined in [9]. On the basis of cost-sensitivity analysis several heuristic quasi-optimal algorithms have been developed. Note that the general idea of cost-sensitivity is based on analyzing the ratio $\gamma_{km} = \frac{\Delta R_{km}}{\Delta C_{km}}$. The developed algorithms are an essential extension of the direct and dual cost-sensitivity models outlined in [9, 21, 23].

§11.3 Notation

Let us introduce the following terms:

$\{TI_k\}$ - the system's technical improvements cell;

TI_k - the k-th technical improvement to increase the system's reliability, $1 \le k \le N$;

N - the number of possible technical improvements;

R^* - the minimal acceptable system's reliability value to avoid hazardous failures (pregiven);

C - the restricted budget to undertake technical improvements (pregiven);

R_0 - system's reliability value prior to undertaking amendments (pregiven);

SM - simulation model to estimate the system's reliability;

TI_{km} - the m-th improvement step entering a compound TI_k, $1 \le m \le n_k$;

n_k - the number of improvement steps to implement the k-th stepwise technical improvement;

ΔC_{km} - the cost expenditures to implement TI_{km};

ΔR_{km} - increase of the system's reliability level due to implementing TI_{km};

γ_{km} - cost-reliability value of improvement step TI_{km};

R_i - accumulated reliability value at a routine iteration i;

C_i - available total budget value at a routine iteration i;

$N_i \le N$ - number of remaining technical improvements at a routine iteration i;

$\{TI_k\}_i$ - "truncated", i.e., remaining technical improvements cell at a routine iteration i, before singling out the next quadruple for the solution area; $\{TI_k\}_i$ differs from $\{TI_k\}$ by excluding TI_{km} which have been previously singled out on prior iterations from the solution area;

$\{TI^{I}_{km}\}$ - technical improvements TI_{km} which have to be determined for the solution area (see §11.5) by implementing *Algorithm I*;

$\{TI^{II}_{km}\}$ - technical improvements TI_{km} which have to be singled out by implementing *Algorithm II* (see §11.5);

$A_{ki} \subset A_k$ - the k-th remaining area A_k of the cell $\{TI_k\}_i$ at the routine iteration i. Note that in some cases A_{ki} may be empty. All $\{A_{ki}\}$ are enumerated in an arbitrary order which remains unchanged until the cost-reliability *Algorithm* terminates. Each array A_{ki} comprises $4 \cdot n_{ki}$ quadruples;

n_{ki} - number of technical improvements steps in array A_{ki} at the i-th iteration;

$\delta C = \dfrac{C}{\sum_{k=1}^{n}\sum_{m=1}^{n_k} \Delta C_{km}}$ - the budget supplement rate in order to carry out all $\{TI_{km}\}$;

R_A - the total increase of the system's reliability by implementing the cost-reliability algorithm;

C_A - the total budget expenses by implementing the cost-reliability algorithm;

n_{kA} - number of improvement steps of the k-th TI which have been actually singled out in the course of implementing the algorithm;

$\eta_A = \dfrac{R_A}{C_A}$ - cost-reliability sensitivity of the algorithm.

§11.4 The problems' formulation

The *direct cost-reliability problem* is as follows:

Given values R_0, C, $\{\Delta C_{km}\}$, $\{\Delta R_{km}\}$, $1 \le k \le N$, $1 \le m \le n_k$, determine values ξ_k, $1 \le \xi_k \le n_k$, in order to maximize the system's reliability level

$$\underset{\{\xi_k\}}{Max} \left\{ R_0 + \sum_{k=1}^{N}\sum_{m=0}^{\xi_k} \Delta R_{km} \right\} \qquad (11.4.1)$$

subject to

$$\sum_{k=1}^{N}\sum_{m=0}^{\xi_k} \Delta C_{km} \le C , \qquad (11.4.2)$$

$$\Delta C_{k0} = 0 , \qquad (11.4.3)$$

$$\Delta R_{k0} = 0 . \qquad (11.4.4)$$

The *dual cost-reliability problem* is as follows:

Given values R_0, R^*, $\{\Delta C_{km}\}$, $\{\Delta R_{km}\}$, $1 \le k \le N$, $1 \le m \le n_k$, determine values ξ_k, $1 \le \xi_k \le n_k$, in order to minimize the cost expenditures

$$\underset{\{\xi_k\}}{Min} \left\{ \sum_{k=1}^{N} \sum_{m=0}^{\xi_k} \Delta C_{km} \right\} \tag{11.4.5}$$

subject to (11.4.3-11.4.4) and

$$R_0 + \sum_{k=1}^{N} \sum_{m=0}^{\xi_k} \Delta R_{km} \ge R^* . \tag{11.4.6}$$

Thus, solving the direct problem delivers the maximal reliability increase subject to restricted budget, while the dual problem boil down to minimizing the budget for undertaking technical improvements subject to system's reliability level restricted from below.

§11.5 Simplified heuristic algorithms for optimizing the direct cost-reliability problem

As outlined above, Problem (11.4.1-11.4.4) may be solved by means of cost-sensitivity analysis. The general idea is as follows. Call henceforth the *cost-reliability of a technical improvement* the ratio $\gamma = \Delta R / \Delta C$. It can be well-recognized that if TI_{k_1} has a higher cost-reliability than TI_{k_2}, investing one and the same cost expenditure results in a higher increase of the reliability parameter in case of implementing the TI_{k_1} than TI_{k_2}. This consideration is used below, in the case of compound technical improvements.

Note that for any technical improvement with a stepwise structure the corresponding quasi-optimal steps to be singled out have to be implemented consecutively, one after another. Thus, to solve the problem of singling out the next step by analyzing a set of several unfinished TI_k, $1 \le k \le N$, one has:

- to examine for all $\{TI_k\}$ their first non-realized steps $\{TI_{k1}\}$, $1 \le k \le N$;
- later on to calculate their corresponding values $\{\gamma_{k1}\}$, and
- to choose step $TI_{\eta 1}$ *which delivers the maximal value* to γ.

The chosen step has to be supplied with expenditure costs while the remaining budget C has to be updated. Afterwards the chosen step $TI_{\eta 1}$, being cancelled in TI_η, enters the accumulated quasi-optimal set (the algorithm's solution), while all non-implemented improvement steps in TI_η are shifted to the left in order for the second step to obtain the first position. The solution process proceeds (by examining the remaining TI_k by means of γ) until either the budget volume C decreases and cannot be assigned to any starting step of a non-accomplished TI_k, or all steps in $\{TI_k\}$ are supplied with expenditure costs in case $\sum_{k=1}^{N}\sum_{m=1}^{n_k} \Delta C_{km} \le C$ (a trivial solution).

The enlarged step-by-step procedure of the algorithm is as follows:

Step 1 (subsidiary). Enumerate N technical improvements $\{TI\}$ in an arbitrary order. Form N corresponding arrays A_k, $1 \le k \le N$, each array A_k comprising $4 \cdot n_{ki}$ consecutive values, i.e., n_k quadruples. Each m-th quadruple in the k-th array comprises values k, m, ΔR_{km} and ΔC_{km}, correspondingly, i.e., all parameters characterizing step improvement TI_{km}.

Step 2. Calculate N values

$$\gamma_k = \frac{\Delta R_{km_k}}{\Delta C_{km_k}}, \; 1 \le k \le N, \tag{11.5.1}$$

where m_k denotes the first quadruple of the k-th array. At the beginning of the algorithm's functioning all $m_k = 1$.

Step 3. Determine

$$\gamma_\omega = \underset{k}{Max}\, \gamma_k . \tag{11.5.2}$$

If more than one ω holds for (11.5.2), take one of them with the maximal $\Delta C_{\omega m_\omega}$.

Step 4. Send quadruple (ω, m_ω, $\Delta R_{\omega m_\omega}$, $\Delta C_{\omega m_\omega}$) to the solution array, which comprises a set of quadruples.

Step 5. Update $C - \Delta C_{\omega m_\omega} \to C$, $n_\omega - 1 \to n_\omega$, $N - 1 \to N$.

Step 6. Cancel quadruple (ω, m_ω, $\Delta R_{\omega m_\omega}$, $\Delta C_{\omega m_\omega}$) and shift to the left by four values all the remaining quadruples of A_ω.

Step 7. Check whether array A_ω appears to be empty or not. If empty, go to the next step. Otherwise apply Step 9.

Step 8. Modify Step 2 in order to prevent implementing value ω in calculating (11.4.1).

Step 9. Check whether relation $C \ge \underset{k}{Min}\, \Delta C_{km_k}$ holds or not. If yes, go to Step 2. Note that for an empty array A_ω examining the possibility of choosing the next quasi-optimal quadruple is not carried out (see Step 8). In case $C < \underset{k}{Min}\, \Delta C_{km_k}$ apply the next step.

Step 10. The solution process terminates. All the quadruples entering the solution array form the quasi-optimal improvement steps $\{k, m, \Delta R_{km}, \Delta C_{km}\}$.

It can be well-recognized that the quasi-optimal reliability level after implementing the above algorithm (call it henceforth *Algorithm I*) may be calculated as

$$R = R_0 + \sum_{k=1}^{N} \sum_{m=0}^{\xi_k} \Delta R_{km} \ . \tag{11.5.3}$$

Algorithm I has a simple modification which we will designate henceforth *Algorithm II*. The latter resembles *Algorithm I*, besides one essential detail. When calculating values γ_k (see Step 2) we use the following modification

$$\gamma_k = \frac{\sum_{\alpha=m_k}^{n_k} \Delta R_{k\alpha}}{\sum_{\alpha=m_k}^{n_k} \Delta C_{k\alpha}}, \ 1 \le k \le N \ . \tag{11.5.4}$$

Relation (11.5.4) means that cost-reliability is calculated not for each local improvement step, but for the entire technical improvement TI_k *as a whole*. It can be well-recognized that the numerator in (11.5.4) is equal to the sum of local reliability steps ΔR_{km} for all remaining improvement steps TI_{km}, while the denumerator stands for the sum of the corresponding cost investment steps ΔC_{km}. Thus, using γ_k calculated by (11.5.4) prevents missing high cost-sensitivity steps at the end of A_k in case of steps with smaller cost-sensitivity at the beginning of array A_k.

The structure of *Algorithm II* is similar to the outlined above step-by-step procedure of *Algorithm I*, besides substituting (11.5.1) for (11.5.4). All other steps remain unchanged.

§11.6 Generalized cost-optimization algorithm based on switching procedures

It can be well-recognized that, unlike *Algorithm I*, *Algorithm II* is based on "look-ahead" cost-sensitivity and, thus, provides better results in cases when the total budget C covers a majority of improvement steps for most TI. However, in case when the ratio

$$\delta C = \frac{C}{\sum_{k=1}^{n}\sum_{m=1}^{n_k}\Delta C_{km}} << 1 \tag{11.6.1}$$

is very small, the pregiven budget C can usually cover only the first improvement steps. This, in turn, means that there is actually no need in "look-ahead" sensitivity analysis. In order to develop a more generalized algorithm which comprises cases of any δC values we present *Algorithm III* which implements both *Algorithms I* and *II* simultaneously on the basis of a switching procedure.

The general idea of *Algorithm III* is as follows: at each consecutive iteration i to single out the next TI_{km}, i.e., the forthcoming iteration to be undertaken, both *Algorithms I* and *II* are implemented independently for the remaining TI cell. For both algorithms the accumulated sum of reliability values $\sum_k\left[\sum_m R\{TI^I_{km}\}\right]$ and $\sum_k\left[\sum_m R\{TI^{II}_{km}\}\right]$ is calculated and later on compared with each other. The algorithm which results in higher accumulated reliability, is chosen to provide the next technical improvement step TI_{km}, and the corresponding quadruple is send to the solution area. After up-dating the remaining technical improvements cell $\{TI_{km}\}_i$ the competing procedure among *Algorithms I* and *II* repeats anew, until the budget value C would be exhausted. Being more complicated than both local *Algorithms I* and *II*, *Algorithm III* provides better results.

The enlarged step-by-step procedure of *Algorithm III* is as follows:

<u>Step 1</u> coincides with the corresponding steps in *Algorithms I* and *II*.

<u>Step 2</u>. After each routine i-th iteration check whether unification $\bigcup A_{ki}$ appears to be empty or not. If empty, go to Step 12. Otherwise apply the next step.

<u>Step 3</u>. Check whether relation $C \geq \underset{k}{Min}\{\Delta C_{km_{ki}}\}$ holds, where m_{ki} denotes the first quadruple of the k-th array. If yes, go to the next step. Otherwise apply Step 12.

<u>Step 4</u>. Determine (for the purpose of forecasting) by means of *Algorithm I* all iterations

$\{TI^{I}_{km}\}$ until the algorithm terminates.

Step 5. Calculate values

$$R_i + \sum_k \sum_m \{\Delta R^I_{km}\} = R^I , \tag{11.6.2}$$

$$C_i - \sum_k \sum_m \{\Delta C^I_{km}\} = C^I , \tag{11.6.3}$$

until the budget value C^I ceases to cover future technical improvements. Here

$$\Delta R^I_{km} = \Delta R\{TI^I_{km}\},\ \Delta C^I_{km} = \Delta C\{TI^I_{km}\}.$$

Step 6. Determine by means of *Algorithm II* all iterations $\{TI^{II}_{km}\}$ in order to "look-ahead" the fitness of the algorithm.

Step 7. Similarly to Step 5, calculate values

$$R_i + \sum_k \sum_m \Delta R\{TI^{II}_{km}\} = R^{II} , \tag{11.6.4}$$

$$C_i - \sum_k \sum_m \Delta C\{TI^{II}_{km}\} = C^{II} . \tag{11.6.5}$$

Step 8. Compare values R^I and R^{II}. If $R^I > R^{II}$ go to Step 10. In case $R^I < R^{II}$ apply the next step. In case $R^I = R^{II}$ compare values C^I and C^{II}. If $C^I > C^{II}$ go to Step 10. Otherwise apply the next step.

Step 9. Send quadruple $(\omega, m_\omega, \Delta R_{\omega m_w}, \Delta C_{\omega m_w})$ which has been determined in the course of i-th iteration by implementing *Algorithm II*, to the solution area. Thus, the i-th routine iteration of *Algorithm III* is accomplished. Go to Step 11.

Step 10. The step is similar to Step 9, with the exception of substituting *Algorithm II* by *Algorithm I*.

Step 11. Update the information in arrays $\{A_{ki}\}$ by diminishing value N_i by one, shifting all quadruples in Array $A_{\omega i}$ by 4 to the left, updating values

$$R_i + \Delta R_{\omega m_\omega} \Rightarrow R_{i+1} , \tag{11.6.6}$$

$$C_i - \Delta C_{\omega m_\omega} \Rightarrow C_{i+1}, \tag{11.6.7}$$

$$i + 1 \Rightarrow i. \tag{11.6.8}$$

Go to Step 2.

Step 12. The work of *Algorithm III* terminates.

Note that if implementing two different algorithms results in an equal increase of the system's total reliability R, the comparative efficiency of both algorithms can be assessed by comparing the algorithms' cost-reliability sensitivity to be calculated as

$$\eta_A = \frac{\sum_{k=1}^{N} \sum_{m=1}^{n_{kA}} \Delta R_{km}}{\sum_{k=1}^{N} \sum_{m=1}^{n_{kA}} \Delta C_{km}}\ . \tag{11.6.9}$$

Theorem

For any technical improvements cell $\{TI_{km}\}$ *with fixed total budget* C *relation*

$$R^{***} \geq \max\left(R^*, R^{**}\right), \tag{11.6.10}$$

where R^*, R^{**}, R^{***} *denote the accumulated reliability values obtained by means of Algorithms I, II, and III, correspondingly, holds.*

Proof

Assume that for a certain $\{TI_{km}\}$ relation (11.6.10) is not true. This may originate from two cases:

Case 1. $R^{***} < R^*$.

Case 2. $R^{***} < R^{**}$.

Examine *Case 1* first. Denote by $n\left(R^*, R^{***}\right) \geq 0$ the number of first coinciding iterations $\{TI_{km}\}$ when implementing *Algorithms I* and *III* independently. Since $R^{***} \neq R^*$ there has to be a decision point after $n\left(R^*, R^{***}\right)$ first iterations when the concurrence ceases to hold. Since in the course of implementing *Algorithm III* we choose at each decision point the iteration which results in the forecasted path with the maximal accumulated reliability, value R^*, being equal to $\sum_k \sum_m TI_{km}^{I}$, has to be less than value $\sum_k \sum_m TI_{km}^{III}$ of that forecasted path. From the other hand, in the course of implementing *Algorithm III* at each routine decision point the forecasted value R^{***} cannot diminish but only increase. Thus, at the end of implementing *Algorithm III* the actual value R^{***} has to exceed R^* in contradiction with the definition of *Case 1*. For *Case 2*, the logical analysis is similar to that outlined above. Thus, for both cases our assumption is false, and relation (11.6.10) holds. ■

§11.7 The dual problem's solution

The step-by-step procedure of the heuristic algorithm to solve problem (11.4.3-11.4.6) [call it henceforth *Algorithm IV*] is as follows:

Steps 1-2 of *Algorithm IV* fully coincide with Steps 1-2 of *Algorithms I* or *II* (see §11.5).

Step 3 resembles the corresponding step of *Algorithms I, II* with one exception: choosing the maximal $\Delta C_{\omega m_\omega}$ in case of several ω satisfying (11.5.2) has to be substituted for the "*minimal*" $\Delta C_{\omega m_\omega}$. This is done deliberately since minimizing the expenditure costs in (11.4.5) centers on preferring the minimal cost steps in the course of implementing *Algorithm IV*.

Step 4 coincides with the corresponding step in *Algorithms I* or *II*.

Step 5 has to be formulated as follows: Update the accumulated reliability

$$R + \Delta R_{\omega m_\omega} \to R \tag{11.7.1}$$

and accumulate the cost expenditures

$$C + \Delta C_{\omega m_\omega} \to C. \tag{11.7.2}$$

At the beginning of the *Algorithm IV* value C has to be set equal to zero, while

Steps 6-8 coincide with *Algorithms I, II*.

Step 9 has to be formulated as follows: Check relation

$$R + R \geq R^*. \tag{11.7.3}$$

If (11.7.3) does not hold, return to Step 2. Otherwise apply the next step.

Step 10 coincides with the corresponding step in *Algorithms I* or *II*.

It can be well-recognized that the minimized cost expenditures after implementing *Algorithm IV* may be calculated as

$$Min\ C = \sum_{k=1}^{N} \sum_{m=0}^{\xi_k} \Delta C_{km} \ . \tag{11.7.4}$$

Note that when implementing *Algorithm IV* on the basis of the direct *Algorithm III* (see §11.6), the principal structure of the dual algorithm remains similar to the structure outlined above. Each improvement step of the solution array has to be determined by the look-ahead switching procedure.

§11.8 Experimentation

In order to check the fitness of the algorithms' cost-reliability sensitivity, extensive experimentation has been undertaken. A technical improvements cell has been simulated as follows:

a) the number N of technical improvements has been simulated as a whole number uniformly distributed within the lower and upper bounds $a_N = 5$, $b_N = 25$;

b) for each k-th technical improvement, $1 \le k \le N$, the number of consecutive improvement steps n_k has been simulated as a whole number uniformly distributed within the lower and upper bounds $a_k = 3$, $b_k = 15$;

c) each cost value ΔC_{km}, $1 \le k \le N$, $1 \le m \le n_k$, has been simulated as a whole number uniformly distributed within the distribution area $a_{cmk} = 200$, $b_{cmk} = 400$;

d) four different distribution bounds have been considered for local reliability values ΔR_{km}, $1 \le k \le N$, $1 \le m \le n_k$, namely: ΔR_{km} is a whole number uniformly distributed within lower and upper bounds:

1. $a_{Rkm} = 20$, $b_{Rkm} = 40$;

2. $a_{Rkm} = 40$, $b_{Rkm} = 80$;

3. $a_{Rkm} = 60$, $b_{Rkm} = 120$;

4. $a_{Rkm} = 80$, $b_{Rkm} = 160$.

All values ΔC_{km} and ΔR_{km} are expressed in conditional terms.

All distribution bounds do not depend on values N, n_k, k and m.

It can be well-recognized that each distribution results in four different cost-sensitivity values $\gamma_{km} = \dfrac{\Delta R_{km}}{\Delta C_{km}}$.

1) $\gamma_1 = 0.1$; 2) $\gamma_2 = 0.2$; 3) $\gamma_3 = 0.3$; 4) $\gamma_4 = 0.4$.

e) for each simulated combination of N, n_k, ΔC_{km} and γ_{km} 9 different levels of budget supplement to carry out all technical improvements $\{TI_{km}\}$ have been examined by means of (11.6.1)

1) $\delta = 0.1$; 2) $\delta = 0.2$; 3) $\delta = 0.3$; 4) $\delta = 0.4$; 5) $\delta = 0.5$; 6) $\delta = 0.6$; 7) $\delta = 0.7$; 8) $\delta = 0.8$; 9) $\delta = 0.9$

For $4 \times 9 = 36$ combinations of δ and γ 10,000 initial data models have been simulated, and later on algorithms' cost-reliability sensitivity rates η_A have been calculated by using each of the algorithms under comparison. Then, average sensitivity rates $\bar{\eta}_A$ have been calculated for each algorithm and each combination of δ and γ_{km}. The results are presented in Table 11.1.

Table 11.1. Comparison of algorithms' cost-reliability sensitivity for various δ and γ_{km}

$\bar{\eta}_{A\,III}$	$\bar{\eta}_{A\,II}$	$\bar{\eta}_{A\,I}$	δ	γ_{km}
0.1236	0.1163	0.1225	0.10	0.1
0.1169	0.1133	0.1140	0.20	0.1
0.1132	0.1109	0.1097	0.30	0.1
0.1105	0.1089	0.1069	0.40	0.1
0.1083	0.1072	0.1048	0.50	0.1
0.1064	0.1064	0.1031	0.60	0.1
0.1046	0.1042	0.1017	0.70	0.1
0.1029	0.1026	0.1005	0.80	0.1
0.1010	0.1010	0.0994	0.90	0.1
0.2491	0.2345	0.2469	0.10	0.2
0.2356	0.2284	0.2299	0.20	0.2
0.2282	0.2236	0.2213	0.30	0.2
0.2228	0.2197	0.2157	0.40	0.2
0.2184	0.2163	0.2114	0.50	0.2

0.2146	0.2146	0.2080	0.60	0.2
0.2110	0.2101	0.2052	0.70	0.2
0.2075	0.2070	0.2027	0.80	0.2
0.2038	0.2037	0.2005	0.90	0.2
0.3747	0.3527	0.3718	0.10	0.3
0.3543	0.3436	0.3458	0.20	0.3
0.3432	0.3363	0.3327	0.30	0.3
0.3351	0.3305	0.3242	0.40	0.3
0.3285	0.3253	0.3180	0.50	0.3
0.3227	0.3206	0.3129	0.60	0.3
0.3174	0.3160	0.3087	0.70	0.3
0.3121	0.3114	0.3050	0.80	0.3
0.3066	0.3063	0.3016	0.90	0.3
0.5004	0.4713	0.4964	0.10	0.4
0.4730	0.4586	0.4619	0.20	0.4
0.4584	0.4492	0.4446	0.30	0.4
0.4475	0.4414	0.4331	0.40	0.4
0.4388	0.4345	0.4246	0.50	0.4
0.4311	0.4282	0.4180	0.60	0.4
0.4240	0.4221	0.4123	0.70	0.4
0.4170	0.4160	0.4074	0.80	0.4
0.4096	0.4093	0.4030	0.90	0.4

The following conclusions can be drawn from the Chapter:

1. *Algorithm III* proves to be more effective than both other cost-reliability algorithms under comparison. For any combination of δ and γ_{km} its average cost-reliability sensitivity rate is higher than by implementing *Algorithms I* and *II*.

2. If the total budget C does not cover the $\{TI\}$ cell requirement by more than 20% ($\delta \le 0.2$) *Algorithm I* proves to be more effective than *Algorithm II* since there is practically no need in "look-ahead" sensitivity analysis. For $\delta > 0.2$ "look-ahead" techniques start to be useful, and *Algorithm II* becomes more effective than *Algorithm I*.

3. For a pair of different values of γ_{km} the ratio of corresponding algorithms' average cost-reliability rates is close to be equal the ratio of values γ_{km} themselves. This is true for any value δ and for all the algorithms under comparison.

4. The basic advantage of the suggested algorithms is that it makes possible to implement technical improvements of compound nature, when those improvements have a stepwise consecutive structure.

5. By comparing the several newly developed cost-reliability algorithms, it can be well-recognized that algorithms based on the "look-ahead" cost-sensitivity lookover, are more efficient than an algorithm which does not comprise forecasting techniques.

6. It has been proven theoretically that *Algorithm III* provides for the direct cost-reliability Problem's solution results being at least not worse than by implementing other algorithms. As to the algorithm's cost-reliability sensitivity rate, extensive simulation shows that *Algorithm III* provides always better results than *Algorithms I* and *II*.

7. All the suggested algorithms can be easily programmed on PC and are simple in usage. They can be applied both to the direct and the dual cost-reliability models.

8. Besides cost-reliability problems, the results obtained may be used for optimizing any system's parameter by means of constrained technical improvements subject to restricted budget assigned for the parameter's amendment.

Chapter 12. Optimization Models for Building Projects

§12.1 Cost-optimization model for several construction projects under random disturbances

12.1.1 Introduction

Large-scale construction (building) projects are nowadays one of the most essential parts of the modern technical progress. Such projects are usually monitored by two-level project management (PM) companies. At the upper level (the company level) PM managers transfer to the lower level (the project level) various goal parameters for the projects' contractors. The latter actually determine and implement all the planning, control and scheduling procedures for the corresponding individual building projects in order to amend the project's situation.

It can be well-recognized [5, 57, 107, 168] that to-day a building company does not determine either any on-line control or scheduling techniques for the subordinated projects; neither does the company undertake even quasi-optimal resource reallocation among the projects. This is because those techniques do not exist as yet. Each contractor undertakes individual decision-making in order to optimize (or, better to say, to refine) his own project's parameters, independently on other company's projects. Such actions, being useful for a single project, may result in heavy financial losses for a building company as a whole. This is because building resources are usually restricted and, thus, projects are not independent. For those projects the unification of local optimums may be very far from a global one.

The goal of §12.1 is to determine both planning and control procedures (including resource reallocation) *for the company level*. Those actions are input parameters for the lower level, where only scheduling procedures are left at the contractors' disposal [107].

In our opinion, such a two-level model, being a novel one, will help the building companies to avoid financial losses. Since building resources are usually restricted, a clever heuristic resource maneuvering has to be introduced.

12.1.2 The problem's description

A building company which simultaneously monitors several contractors is considered. Each contractor operates a building project (house, factory, hospital, etc.) consisting of a chain of operations to be processed in an individual definite technological sequence.

Each project's operation utilizes qualified resources of various specialties, i.e., several non-consumable resources with fixed capacities (manpower, scrappers, etc.). For any operation in progress, all the resources remain unchanged until the operation terminates.

Each type of resources at the company's disposal is in limited supply, is predetermined and the resource limit remains unchanged at the same level throughout the project's realization, i.e., until the last project is actually completed. Thus, due to the limited resource levels, project's operations may have to wait in lines for resource supply, in order to proceed functioning. Since for each operation its duration is a random variable with given density function, a deterministic schedule of the moments when operations actually start cannot be pre-determined. Note that such simultaneously realized projects with consecutive operations cover a broad variety of different management structures, especially aggregated building projects [138, 173].

The system under consideration is based on a single storehouse where all the resources are stored and, if necessary, send to a certain contractor. After finishing an operation all the resources undertaking that operation are returned to the storehouse. Assume that all building projects start functioning at $t=0$. Assume, further, that the company externally receives from the customers in advance, i.e. before moment $t=0$, the cost of all finished building projects. This cost covers the following expenses:

I. The cost of hiring and maintaining all the resources within all the projects' realization, i.e., starting from $t=0$ and finishing at the moment the last project will be finished.

II. Delay penalties. The company has to pay for each i-th project a certain penalty C_i per time unit within the project's duration, i.e., from $t=0$ until the finishing time F_i, when the project is finished and is delivered to the customer.

The problem is as follows:

- to determine *beforehand* the optimal total resource capacities of all types of resources at the building company's disposal within the projects' realization, and
- to determine random values of the moments the projects' operations actually start (*in the course of the projects' realization*, conditioned on our decision rules and based on the total predefined resource values),

- in order to *minimize* the average of the company's expenses (including the penalties to the customers). We introduce average values since the durations of the projects' realizations are random values.

The problem is solved via a heuristic algorithm by a combination of the cyclic coordinate descent method [9, 133] (at the upper level) and a simulation scheduling model (at the lower level). Resource allocation between the projects waiting in lines is carried out via an embedded newly developed decision rule. This rule enables to support projects with high delay penalties at the expense of projects with low penalties in the course of the projects' realization.

12.1.3 Notation

Let us introduce the following terms:

n - number of building project $BP_i, 1 \le i \le n$, to be realized simultaneously;

$O_{i\ell}$ - the ℓ-th operation of the i-th project in the form of a consecutive chain, $1 \le \ell \le m_i$;

m_i - number of operations in project BP_i;

$t_{i\ell}$ - random duration of operation $O_{i\ell}$;

$\bar{t}_{i\ell}$ - average value of $t_{i\ell}$ (pregiven);

$V_{i\ell}$ - variance of $t_{i\ell}$ (pregiven);

R_k - the total capacity of the k-th type of resources, $1 \le k \le d$, at the disposal of the building company (a deterministic value to be optimized);

d - number of resources;

$r_{i\ell k}$ - the k-th resource capacity required by operation $O_{i\ell}$ (pregiven);

$S_{i\ell}$ - the moment operation $O_{i\ell}$ actually starts (a random value, to be determined by the simulation model via a decision rule in the course of realizing the projects);

$F_{i\ell} = S_{i\ell} + t_{i\ell}$ - the moment operation $O_{i\ell}$ terminates (a random value);

C_i - penalty the building company pays per time unit within the BP_i's duration (pregiven, a constant value);

$F_{i\ell}$ - the moment operation $O_{i\ell}$ terminates (a random value);

F_i - the moment project BP_i terminates, $F_i = S_{im_i} + t_{im_i}$ (a random value);

F - the moment the last project terminates, $F = \underset{i}{Max}\, F_i$;

$W_k\{S_{i\ell}, t\}$ - the summarized capacity of the k-th resource assigned to operations at moment t, on condition that operations $O_{i\ell}$ starts at moments $S_{i\ell}, 1 \le k \le d$;

$R_k(t) = R_k - W_k(S_{i\ell}, t)$ - free available resources of k-th type at moment t;

$\vec{R}_k^*(t) \subset \vec{R}_k(t)$ - a part of free available resources to be set aside for the tense project;

S_k - the cost of hiring, maintaining and utilizing the k-th resource unit at the time unit, $1 \le k \le d$, (pregiven, a constant value);

ΔR_k - the positive search step value to optimize variable R_k, $1 \le k \le d$ (pregiven);

ε - the relative accuracy value to obtain an optimal solution (pregiven);

$R_{k\,\min}$ - the minimal possible level for the total capacity R_k, $1 \le k \le d$ (pregiven by experts);

$R_{k\,\max}$ - the maximal possible level for value R_k, $1 \le k \le d$ (pregiven);

$Q^{(sim)}$ - simulated system's expenses by one simulation run;

$\overline{Q}$ - the system's average total expenses.

Note that the following relations hold:

$$R_{k\ \min} \geq \underset{i}{Max}\ \underset{\ell}{Max}\ r_{i\ell k}\,, \tag{12.1.1}$$

$$R_{k\ \max} \leq \sum_i \left\{ \underset{1\leq \ell \leq m_i}{Max}\ r_{i\ell k} \right\}, \tag{12.1.2}$$

$$R_{k\ \min} \leq R_k \leq R_{k\ \max}\,,\ 1 \leq i \leq n\,,\ 1 \leq \ell \leq m_i\,,\ 1 \leq k \leq d\,. \tag{12.1.3}$$

Restriction (12.1.1) is evident since otherwise some of the projects cannot be realized at all. If (12.1.2) does not hold a certain part of resources will not participate in the projects' realization.

12.1.4 The problem's formulation

The problem is to determine both optimal deterministic values $R_k, 1 \leq k \leq d$, (before the projects' realization) and random values $S_{i\ell}$ (in the course of the projects' realization and conditioned on our decisions), $1 \leq i \leq n$, $1 \leq \ell \leq m_i$, to minimize the average of the total expenses:

$$\underset{[R_k, S_{i\ell}]}{Min}\ \overline{Q} = Min\ E\left\{ \sum_{i=1}^{n} C_i F_i + F \cdot \sum_{k=1}^{d} S_k R_k \right\}. \tag{12.1.4}$$

It can be well-recognized that the first summond denotes the total amount of penalties paid to the customers while the second one denotes the total expenses of hiring and maintaining building resources. Note that both summonds are random values since F, F_i, $1 \leq i \leq n$, are random as well.

12.1.5 The problem's solution

Problem (12.1.4) is an extremely complicated problem which cannot be solved by using regular methods. Thus, we will use heuristic approaches.

The suggested heuristic algorithm to solve the problem comprises two levels. At the lower level (the internal cycle) the simulation model undertakes numerous simulation runs in order to manage the project's realization. At the upper level (the external cycle) the heuristic search sub-algorithm undertakes cycle coordinate optimization in order to obtain the optimal vector $\vec{R}_k$. The procedure of the optimization is based on optimizing objectives cyclically with respect to coordinate variables $R_1, R_2, ..., R_d$. Coordinate R_1 is optimized first, then R_2, and so forth through R_d. The cyclic coordinate search algorithm

(CCSA) [9, 133] is widely known and is very efficient in various optimization problems with complicated, mostly non-linear restrictions.

At the lower level a decision-making simulation model has to be implemented. The input data of the simulation model is the vector of total resource capacities $\vec{R}_k$, $1 \le k \le d$, which is determined in the course of the coordinate descent algorithm's work. Thus, in the course of a routine simulation run, vector $\{R_k\}$ is fixed and remains unchanged. It goes without saying that vector $\vec{R}_k$ satisfies (12.1.1-12.1.3).

The main task of the simulation model is to determine (in the course of a simulation run) random starting moments $S_{i\ell}$ of all operations $O_{i\ell}$, $1 \le i \le n$, $1 \le \ell \le m_i$, entering the model.

A routine simulation run starts functioning at $t = 0$ and terminates with the completion of the last project. The simulation model comprises three sub-models as follows:

<u>Sub-model I</u> actually governs most of the procedures to be undertaken in the course of the projects' realization, namely:

- determines essential moments (decision points) when projects may be supplied with free available resources. A routine essential moment usually coincides either with the moment $F_{i\ell}$ an operation is finished and additional resources become available, or when a subset of new operations $O_{i\ell}$ becomes ready to be processed;
- singles out (at a routine decision point) all the operations that are ready to be processed;
- returns the utilized non-consumable resources to the company's store (at the moment an operation is finished);
- determines the remaining projects at each routine decision point;
- determines the completion moment for each project.

<u>Sub-model II</u> comprises decision rules to reallocate free available resources at each decision moment t among operations waiting for resources. In order to determine decision rules penalty values C_i have to be taken into account.

<u>Sub-model III</u> supplies the chosen operations $O_{i\ell}$ with resources by means of *Sub-model II* and later on simulates the corresponding durations $t_{i\ell}$ (for projects which are waiting in lines to be provided with resources).

Note that realizing *Sub-models II* and *III* results in complicated procedures with new sophisticated logistics.

The enlarged procedure of the problem's solution is presented in Fig. 12.1.

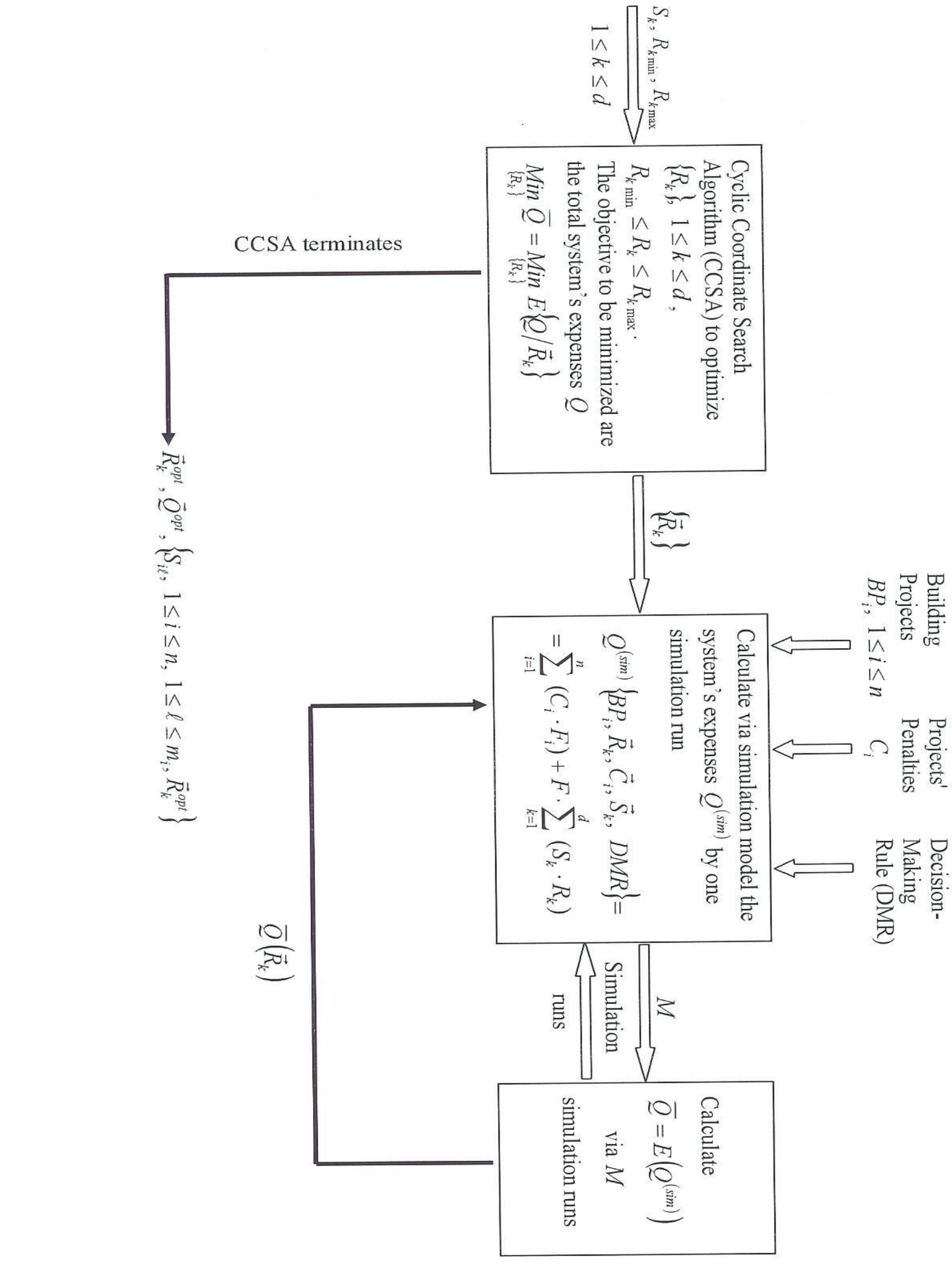

Figure 12.1. The enlarged procedure of the problem's solution

12.1.6 Decision-making in the simulation model

The basic idea of decision-making within the simulation model's realization is to choose projects' operations $O_{i\ell}$ to be supplied with resource from the line of projects ready to be operated. Determining values $S_{i\ell}$ is carried out at essential moments t (see *12.1.5*) via a newly developed heuristic decision-making rule.

Given at a routine decision point $t \ge 0$:

- f project - operations $O_{i_1\ell_1}$, $O_{i_2\ell_2}, \cdots, O_{i_f\ell_f}$ ready to be processed, $1 \le i_g \le n$, $1 \le \ell_g \le m_{ig}$, $1 \le g \le f$; call henceforth subset $\{O_{i_g\ell_g}\}$ subset B.
- $\{R_k\}$ - total resource capacities $R_1, \cdots, R_d$ determined in the course of coordinate optimization at the upper level. Vector $\vec{R}_k$ remains unchanged within a routine simulation run;
- $r_{i\ell_k}$ - the k-th resource capacity required by $O_{i\ell}$ (pregiven and unchanged in the course of the problem's solution);
- $\bar{t}_{i\ell}, V_{i\ell}$ - parameters of $O_{i\ell}$, $1 \le i \le n$, $1 \le \ell \le m_i$, (pregiven and unchanged within the problem's solution);
- $R_k(t)$ - free available resources from the storehouse, $1 \le k \le d$, at decision point t (a random vector conditioned on our decisions);
- cost values C_i, $1 \le i \le n$,

- the decision rule boils down to determine integer values $\xi_{i_g\ell_g}$, $1 \le g \le f$, where

$$\xi_{i_g\ell_g} = \begin{cases} 0 & \text{if} \quad \text{project's operation } O_{i_g\ell_g} \text{ will not obtain resources at moment } t; \\ 1 & \quad\quad \text{otherwise.} \end{cases} \tag{12.1.5}$$

We have developed an entirely new decision-making rule by integrating together tense heuristic decision rules [68], forecasting heuristic rules [85] and the classical zero-one programming model [176].

In order to undertake decision-making we need to solve a subsidiary problem to be outlined below.

12.1.7 Subsidiary problem (Problem A)

Problem A which has to be solved at any decision point t, is as follows:

For all project's operations $O_{j_h\ell_h} \subset \{O_{i_\ell}\} \backslash \{O_{i_g\ell_g}\}$ which at the moment t are under way, calculate the mean values of their termination moments $F_{j_h\ell_h}$ on condition that the latter exceed value t.

To solve that problem, one has to determine for a pregiven random value $t_{j_h \ell_h}$ and the actually determined value $S_{j_h \ell_h}$ (conditioned on our past decision making) the mean value of the termination moment $F_{j_h \ell_h}$ on condition that $F_{j_h \ell_h} > t$.

Note, that in the general case, we deal with a classical statistical problem of calculating the conditional mean value

$$E\{\alpha/\alpha > \Delta\} = \frac{1}{1 - F_\alpha(\Delta)} \int_{\Delta}^{\alpha} x\, f_\alpha(x) dx, \tag{12.1.6}$$

where $F_\alpha(x)$ is the cumulative probability function

$$F_\alpha(x) = \int_{-\infty}^{x} f_\alpha(y) dy. \tag{12.1.7}$$

We have determined values $E(\alpha/\alpha > \Delta)$ analytically for three distributions of random processing time of the operations [85]:

a) normal distribution with mean $\bar{t}_{i\ell}$ and variance $V_{i\ell}$;

b) uniform distribution in the interval $\left[\bar{t}_{i\ell} - 3\sqrt{V_{i\ell}},\ \bar{t}_{i\ell} + 3\sqrt{V_{i\ell}}\right]$;

c) exponential distribution with value $\lambda = 1/\bar{t}_{i\ell}$.

For the exponential distribution with the p.d.f. $f_\alpha(x) = \lambda e^{-\lambda x}$ using (12.1.6-12.1.7) results in

$$E(\alpha/\alpha > \Delta) = \Delta + \frac{1}{\lambda}; \tag{12.1.8}$$

In the case of a uniform distribution in $[a, b]$, i.e., for p.d.f. $f_\alpha(x) = \dfrac{1}{b - a}$,

$$E(\alpha/\alpha > \Delta) = \frac{b + \Delta}{2}. \tag{12.1.9}$$

For the normal distribution with parameters (a, σ^2), i.e., for p.d.f. $f_\alpha(x) = \dfrac{1}{\sqrt{2\pi}} e^{-\frac{(x - a^2)}{2\sigma^2}}$, we obtain

$$E\left(\alpha/\alpha > \Delta\right) = a + \frac{\sigma e^{-(\Delta-a)^2 / 2\sigma^2}}{\sqrt{2\pi}\left[1-\phi\left(\frac{\Delta-a}{\sigma}\right)\right]}, \tag{12.1.10}$$

where $\phi(x) = \frac{1}{\sqrt{2\pi}} \int\limits_{-\infty}^{x} e^{-\frac{y^2}{2}} dy$.

Thus, to obtain value

$$E\left(F_{j_h b_h} \Big/ S_{j_h b_h}, \overline{F}_{j_h b_h} > t\right), \tag{12.1.11}$$

we need to substitute in (12.1.6) values Δ for $t - S_{j_h b_h}$ and $F_{j_h b_h}$ for $t_{j_h b_h}$, while the p.d.f. $f_\alpha(x)$ depends on probability laws (12.1.8-12.1.10).

12.1.8 *Step-wise decision-making heuristic procedure*

Determining values $\xi_{i_g \ell_g}$ (see *12.1.6*) can be realized by means of the following step-wise procedure:

Step 1. Determine the indices of the building projects BP_i to be realized at moment t in the descending order of their penalty values C_i.

Project PB_1 is called the tense project since it defines often the bottleneck of the system. Subset B has to be reordered in the same manner.

Step 2. At a routine decision moment t carry out a check: does the tense project enter subset B or not? If yes, apply the next step. Otherwise go to Step 13.

Step 3. Carry out a check: is it possible to supply at moment t the tense project with needed resources from the company store or not? In other words, do relations

$$R_k(t) \ge r_{1\ell_1 k}, \ 1 \le k \le d, \tag{12.1.12}$$

hold, where ℓ_1 denotes the index of operation the tense project has to undergo at moment t? If (12.1.12) holds, apply the next step. Otherwise go to Step 7.

Step 4. Exclude the tense project form subset B and update vector $\vec{R}_k(t)$ of free available resources

$$R_k(t) - r_{1\ell_1 k} \Rightarrow R_k(t), \ 1 \le k \le m. \tag{12.1.13}$$

Step 5. Solve the zero-one programming problem to be formulated as follows: determine integer values $\xi_{i_g \ell_g}$, $1 \le g \le f-1$, to maximize the objective

$$\underset{\{\xi_{i_g\ell_g}\}}{Max}\left\{\sum_{g=1}^{f-1}\left[C_{i_g}\cdot\bar{t}_{i_g l_g}\cdot\xi_{i_g\ell_g}\right]\right\} \tag{12.1.14}$$

subject to

$$\sum_{g=1}^{f-1}\left\{\xi_{i_g\ell_g}\cdot r_{i_g\ell_g k}\right\}\le R_k\left(t\right),\ 1\le k\le m, \tag{12.1.15}$$

where $\xi_{i_g\ell_g}$ is defined by (12.1.5).

Problem (12.1.5, 12.1.14-12.1.15) is a classical zero-one integer programming problem. Its solution is outlined in many books on operation research, e.g., in [176]. Note that maximizing objective (12.1.14) results in the policy as follows: the project management takes all measures to operate first projects which being realized, decrease more essentially the total penalty the building company has to pay for the projects' prolongation. Only afterwards does the management take care of projects with smaller penalties.

Step 6. After feeding-in-resources for the chosen building projects go to *Sub-model I* (see *12.1.5*), and the projects' realization proceeds until the next decision point will be reached.

Step 7. Examine all projects' operations $O_{j_h b_h}$ which at moment t are under way. Let the number of those operations be q.

For each operation $O_{j_h b_h}$, $1\le h\le q$, solve subsidiary *Problem A* in order to calculate the conditional mean values of the operations' termination moments. Denote those mean values by $\left\{\overline{F}_{j_h b_h}\right\}$.

Step 8. Choose value $\overline{F}_{j_\gamma b_\gamma}=\min\limits_{1\le h\le q}\overline{F}_{j_h b_h}$.

Step 9. Carry out a check: is the amount of available resources from the company's store $\vec{R}_k\left(t\right)$ *together* with additional resources $\vec{r}_{j_\gamma b_\gamma k}$ which will become available from $O_{j_\gamma b_\gamma}$ at moment $t^*=\overline{F}_{j_\gamma b_\gamma}>t$, be enough for operating the tense project BP_1 at moment t^*? In other words, will relations

$$R_k\left(t\right)+r_{j_\gamma b_\gamma k}>r_{1\ell_1 k},\ 1\le k\le d, \tag{12.1.16}$$

hold?

If yes, apply the next step. Otherwise go to Step 12.

Step 10. Determine resource vector

$$R_k^*\left(t\right)=r_{1\ell_1 k}-r_{j_\gamma b_\gamma k},\ 1\le k\le d, \tag{12.1.17}$$

to be stored and set aside for the tense project.

Step 11. Update free available resources

$$R_k^*(t) - R_k^*(t) \Rightarrow R_k(t), \;\; 1 \le k \le d \,. \qquad (12.1.18)$$

Go to Step 5. Later on apply *Sub-model I*.

Note that applying Step 5 from Step 11 differs essentially from applying Step 5 from Step 4. In the latter case the tense project is supplied with resources and starts processing at moment t. If (12.1.16) does not hold, the tense project remains in the line until operation $O_{j_\gamma b_\gamma}$ terminates.

Step 12. Applying this step means that, as it stands now, it is impossible to proceed functioning the building projects from sub-set B. Therefore we suggest not supplying resources for any project operation $\{O_{j_g \ell_g}\}$ until the next decision moment. Go to *Sub-model I*.

Step 13. Applying this step means that the tense project BP_1 is under way and is realized in the course of processing operation O_{1b_h}. Examine the results of Step 7 in order to determine the mean value $\bar{F}_{1b_h}$ of the operation's finishing time. Since value $\bar{F}_{1b_h}$ is calculated at moment t, we will denote this value by $\mathrm{T}_1(t) > t$.

Step 14. Using the results of Step 7 for all other building projects (besides the tense one), enumerate all time values $\bar{F}_{j_h b_h}$ in descending order.

Step 15. Carry out a check: does at least one project exist with the corresponding conditional mean value $\bar{F}_{j_h b_h}$ satisfying

$$t < \bar{F}_{j_h b_h} < \mathrm{T}_1(t)? \qquad (12.1.19)$$

If yes, then the forthcoming tense building project has to be examined not at moment t, but at the closest next decision moment $F_{j_h b_h}$. If (12.1.19) holds Step 16 has to be applied. Otherwise go to Step 17.

Step 16 is similar to Step 5 with only one exception: integer values $\xi_{i_g \ell_g}$ in zero-one programming model are determined for all f projects' operations $O_{i_g \ell_g}$ with non-updated free available resources $\vec{R}_k(t)$. After solving the modified problem (12.1.5, 12.1.14-12.1.15) go to Step 6.

Step 17 is applied in case when the tense project terminates (according to the conditional forecasting) *before* all other projects which are under way. Here a check has to be carried out as follows: does at least one project exist in subset B (i.e., among projects ready to be operated and seeking for resources) which after starting at moment t, will terminate before the moment $\mathrm{T}_1(t)$ which has been calculated at Step 13. Thus, inequalities

$$t + \bar{t}_{i_g \ell_g} < \mathrm{T}_1(t), \;\; 1 \le g \le f \qquad (12.1.20)$$

have to be checked for all f projects entering subset B.

If at least one project satisfying (12.1.20) does exist, apply the next step. Otherwise go to Step 24.

Step 18. Single out all projects entering B and satisfying (12.1.20) and enumerate them anew. Let them be projects-operations

$$O_{v_s w_s},\ 1 \le s \le f^* \le f\ , \tag{12.1.21}$$

where f^* is the number of projects satisfying (12.1.20).

Step 19. Solve zero-one programming model as follows: determine integer values $\xi_{v_s w_s},\ 1 \le s \le f^*$, to maximize objective

$$\underset{\{\xi_{v_s w_s}\}}{Max} \left\{ \sum_{s=1}^{f^*} \left[C_{v_s} \cdot \bar{t}_{v_s w_s} \cdot \xi_{v_s w_s} \right] \right\} \tag{12.1.22}$$

subject to

$$\sum_{s=1}^{f^*} \left\{ \xi_{v_s w_s} \cdot r_{v_s w_s k} \right\} \le R_k(t), \tag{12.1.23}$$

$\xi_{v_s w_s}$ being defined by (12.1.5).

Step 20. If problem (12.1.5, 12.1.22-12.1.23) has at least one non-zero solution, supply the corresponding projects' operations with needed resources. Projects' operations $O_{v_s w_s}$ which failed to obtain resources, are sent back to subset B.

Step 21. Update the free available resources $R_k(t),\ 1 \le k \le d$, as follows:

$$R_k(t) - \sum_{s=1}^{f^*} \left[\xi_{v_s w_s} \cdot r_{vsw_s k} \right] \Rightarrow R_k(t),\ 1 \le k \le d\ . \tag{12.1.24}$$

Step 22. Enumerate the remaining projects' operations in subset B anew, i.e., in decreasing order of their penalties C_i. Define those operations by $O_{x_s z_s},\ 1 \le s \le f^{**}$.

Step 23. Solve one-zero programming problem (12.1.5, 12.1.22-12.1.23) for f^{**} remaining operations $O_{x_s z_s}$ and updated free resources (12.1.24). If the problem's solution includes non-zero integer values, supply the corresponding operations with needed resources. Go to Step 6, i.e., proceed with the building system's functioning at moment t until the next decision moment.

Step 24. Applying this step means that, according to our "look ahead" forecasting, the tense project's termination moment will be the next adjacent decision moment, i.e., the closest one to moment t. That means that even at moment $\mathrm{T}_1(t)$ the tense project cannot be supplied with resources.

Take all the conditional mean values of termination moments for all projects' operation which are under way. Supplement this list with mean values of termination moments for all project's operations waiting in the line for resources at moment t, (i.e. entering subset B) *in case that they will be supplied with resources at that moment*. Thus, the joint list includes q mean values $\bar{F}_{j_h b_h}$, $1 \le h \le q$, and f mean values $t + \bar{t}_{i_g \ell_g}$, $1 \le g \le h$. Enumerate the joint list in descending order. To simplify the terminology, define the operations entering the list by O_η, $1 \le \eta \le q + f$, their corresponding penalty values - by C_η, resource vectors - by $\vec{r}_\eta$, the mean values of the operations' termination moments - by $\bar{\mathrm{T}}_\eta$ (where the minimal value $\bar{\mathrm{T}}_{q+f} = \mathrm{T}_1(t)$ refers to the tense project).

Step 25. Analyse sequence $\{\bar{\mathrm{T}}_\eta\}$, element after element, starting from $\eta = f + q - 1$, *in the reverse direction*, i.e. by diminishing η by one consecutively. Undertake a check: is the operation O_η under way or does it enter subset B? If O_η is under way, apply the next step. Otherwise go to Step 27.

Step 26 is similar to Step 5 and results in solving the zero-one programming problem as follows: determine integer values $\xi_{i_g \ell_g}$, $1 \le g \le f$, to maximize the objective

$$\underset{\{\xi_{i_g \ell_g}\}}{Max} \left\{ \sum_{g=1}^{f} C_{i_g} \cdot \bar{t}_{i_g l_g} \cdot \xi_{i_g \ell_g} \right\}$$

subject to

$$\sum_{g=1}^{f-1} \left\{ \xi_{i_g \ell_g} \cdot r_{i_g \ell_g k} \right\} \le R_k(t), \ 1 \le k \le m, \tag{12.1.25}$$

$\xi_{i_g \ell_g}$ being defined by (12.1.5).

Supply the corresponding operations with resources (in case of non-zero solutions) and go to Step 6.

Step 27. This step is applied in the case when O_η enters subset B and is waiting for required resources. If relation

$$\vec{R}_k(t) > \vec{r}_\eta \tag{12.1.26}$$

holds, i.e., operation O_η can be supplied at moment t with available resources, apply the next step. Otherwise go to Step 29.

Step 28. Update the free available resources $\vec{R}_k(t)$

$$\vec{R}_k(t) - \vec{r}_\eta \Rightarrow R_k(t). \tag{12.1.27}$$

Step 29. Update

$$B \equiv \{O_{i_g \ell_g}\} \backslash O_\eta \Rightarrow B, \qquad (12.1.28)$$

$$f - 1 \Rightarrow f, \qquad (12.1.29)$$

and the counter η

$$\eta - 1 \Rightarrow \eta. \qquad (12.1.30)$$

If $\eta = 0$ go to Step 6. Otherwise go to Step 24, i.e., proceed examining $\{O_\eta\}$ in the reverse direction.

It can be well recognized that the main purpose of Steps 24-29 is to bring the decision moment which happens next after moment $\mathrm{T}_1(t)$, as close as possible to that moment. That is because at moment $\mathrm{T}_1(t)$ the tense project's operation terminates and the project cannot be supplied with resources at that moment. Thus, the next decision moment is needed as soon as possible. To conclude, the general goal of our heuristic is to push the tense project through the building process. After the tense project terminates, another remaining project with the maximal penalty value C_i will take his place.

12.1.9 Cyclic coordinate search algorithm (CCSA)

As mentioned above, the suggested heuristic algorithm to solve the problem comprises two levels. At the lower level (the internal cycle) the simulation model undertakes numerous simulation runs in order to manage the projects' realization. At the upper level (the external cycle) the heuristic search sub-algorithm undertakes cyclic coordinate optimization in order to obtain the optimal vector $\vec{R}_k$. The procedure of the optimization is based on optimizing objective (12.1.4) cyclically with respect to coordinate variables $R_1, R_2, \cdots, R_d$. Coordinate R_1 is optimized first, then R_2, and so forth through R_d. The coordinate descent method is widely known [133] and is very efficient in various optimization problems with complicated, mostly non-linear, objectives. The enlarged step-by-step procedure of the optimizing sub-algorithm is as follows:

Step 1. Determine the initial search point $X^0 = \{R_k^0\}$ by taking deliberately overstating values, e.g., $R_k^0 = R_{k\max}$, $1 \le k \le d$. It can be well-recognized that setting $X^0 = \{R_{k\max}\}$ results in a feasible solution.

Step 2. Fix the initial values $\{R_k\}, \vec{R}_k = \vec{X}^0$, and start diminishing value R_1 by ΔR_1 consecutively, i.e., $R_1 - r \cdot \Delta R_1 \Rightarrow R_1$, $r = 1, 2, \cdots$, while all other coordinates $R_2, R_3, \cdots, R_d$ are fixed and remain unchanged. Each newly determined search point $(R_1 - r \cdot \Delta R_1, R_2, \cdots, R_d)$ has to be examined via simulation in order to verify that checking a new search point results in decreasing objective (12.1.4).

In order to formalize the procedure of verification via a simulation model, we suggest to undertake M simulation runs in order to obtain representative statistics $(M \div 500-1000)$.

Step 3. Proceed examining the monotonous decrease of estimate Q in the course of diminishing consecutively the first coordinate R_1, until either:

1. The diminished value R_1 reaches its lower bound $R_{1\min}$, or
2. The monotonous decrease of objective (12.1.4) ceases to hold for $R_{1\min} \le R_1 \le R_{1\max}$.

In any case value R_1 which corresponds to the minimal value of Q, is fixed, and we start diminishing the second coordinate, R_2 by step ΔR_2 (with fixed values R_1 (newly obtained), $R_3, \cdots, R_d$). The process proceeds for other coordinates, etc., until the last coordinate, R_d, is examined.

Note that in the course of undertaking a coordinate search each successive search results always in decreasing objective (12.1.4). Otherwise, i.e., if a routine search step does not result in decreasing (12.1.4), the corresponding routine coordinate R_k is fixed and the next, the $(k+1)$-th coordinate R_{k+1}, starts to be examined.

Step 4. Obtaining a new search vector $\{\vec{R}_k\}$ in the course of optimizing *all the coordinates separately*, results in realizing the first iteration to determine the quasi-optimal values $\{R_k\}$. All search steps ΔR_k have to be diminished (mostly by dividing by two), and we proceed to minimize (12.1.4) cyclically with respect to the new coordinate variables beginning from R_1.

Step 5. For all next iterations in the course of the coordinate optimization, a search is realized for each routine coordinate $R_k, 1 \le k \le d$, in two opposite directions, namely $R_k - \Delta R_k$ and $R_k + \Delta R_k$, to determine the direction of objective's (12.1.4, 12.1.14-12.1.15) decline. The direction which results in the highest objective's decrease, has to be chosen. The search process proceeds in that direction until the objective's decrease ceases to hold.

Step 6. After undertaking a routine search iteration $v, v=1,2,\cdots,$ the objective (12.1.4), Q^v, referring to that iteration, has to be compared with the results of the previous, $(v-1)$-th iteration, by calculating

$$\Delta^{(v)} = \frac{Q^{(v-1)} - Q^{(v)}}{Q^{(v-1)}}. \tag{12.1.31}$$

Thus, at least two iterations have to be undertaken.

Step 7. If relation $\Delta^{(v)} < \varepsilon$ holds, i.e., if the relative difference between two adjacent iterations $Q^{(v-1)}$ and Q^v becomes less than the pregiven tolerance $\varepsilon > 0$, the algorithm terminates. Otherwise, Step 2 has to be applied.

12.1.10 *Conclusions*

The following conclusions can be drawn from the study:

1. The construction industry is very large, complex, and different from other industries. The industry needs much investment and involves various types of stakeholders and participants. A construction process is a continuous one, usually spread over a number of years. Modern construction projects are usually monitored by two-level PM companies.

   It can be well-recognized that to-day a building company does not determine either any on-line control or scheduling techniques for the subordinated projects; neither does the company undertake even quasi-optimal resource reallocation among the projects. This is because those techniques do not exist as yet. Each contractor undertakes individual decision-making in order to optimize (or, better to say, to refine) his own project's parameters, independently on other company's projects. Such actions, being useful for a single project, may result in heavy financial losses for a building company as a whole. This is because building resources are usually restricted and, thus, projects are not independent. For those projects the unification of local optimums may be very far from a global one.

2. The goal of §12.1 is to determine both planning, control and scheduling procedures, including resource reallocation, at the company level. Those actions are input parameters for the lower level, where only scheduling procedures are left at the contractor's disposal. The objective is to minimize the average of the building company's expenses.

3. The problem is solved by means of a heuristic algorithm through a combination of a cyclic coordinate descent method at the upper level and a decision-making simulation model at the lower level.

4. Resource reallocation between the projects waiting in lines is carried out via a newly developed decision rule.

5. The novelty of the research can be defined by:

   - the problem's formulation which is actually a generalization of formerly developed decision-making simulation models, and
   - by introducing a more effective decision-making rule than the existing ones. The latter are based on one performance rule while our approach is based on a combination of several most effective preference rules together with forecasting models and a classical zero-one programming problem.

6. The efficiency of the developed model has been verified by means of numerical examples and by comparison with other existing heuristics [68, 138, 173].

7. In our opinion, the results obtained can be used in future by a variety of large building and construction companies, as well as by governmental agencies.

§12.2 Monitoring several building projects using costly resource divisions

12.2.1 Introduction

In this discussion we will consider a simultaneously realized group of extremely important building (construction) projects which are monitored by a specialized company. Those projects may result in creating new hi-tech devices, mountain or sea tunnels, new railroads, etc. Practically for nearly all building systems of this type, due to random disturbances from the environment, breakdowns of equipment, a variety of random human factors affecting the work of personnel, etc., those operations experience random deviations from the average speed. Due to random influences, various scheduling models, including resource delivery schedules, become very important, since for such building projects various resource types might prove to be extremely costly.

Most of the existing techniques for industrial scheduling is based on the so-called priority rules (see, e.g., [68, 70, 138, 147, 173]). The objective is mostly to minimize the makespan (the schedule time) according to the starting times of operations obtained by using priority rules. It can be well recognized that one of the most fruitful approach in industrial scheduling is the idea of pairwise comparison [68, 138, 147]. The latter is usually used for choosing activities for a processor from the line of activities ready to be operated on that processor. If at a certain moment several projects are waiting to be operated on a certain processor, a pairwise comparison between the first two competitive projects is arranged. The winner competes with the next project in the line, etc., until only one winner will be left. The latter has to be chosen for the processor.

The idea of §12.2 is to expand this approach to the case of several building projects with different priority indices and random time durations. The competition between two projects which at a certain moment are seeking one and the same division, is based on comparing two different options:

Option A. The first project is chosen to be realized on the division and the second project will be realized after the first project will be finished.

Option B. The second project is chosen for the division and the first one waits until the second project will be processed.

The idea of such a comparison is to calculate for each alternative option the projects' delivery performances, i.e., the probabilities for both projects to meet their due dates on time. The option which ensures the maximal delivery performance for the *couple of projects*, i.e., for the projects' unification, has to be chosen.

12.2.2 The system's description

A building company comprising n simultaneously realized construction projects and m resource divisions (special laboratories, coal harvesters, railroad machines, various auxiliary engineering plants, proving grounds, etc.) is considered. Each building project consists of an individual chain of several stages, each of which needs to be realized during an uninterrupted period by a pregiven division. Each division can process at most one project at a time. A project cannot be realized at the same time by more than one division. Each stage of each project is carried out under random disturbances and, thus, has a random duration. Due to the number of projects and the restricted divisions' capacities, there may be projects which at a certain moment are waiting in a line ready to be realized by one and the same division. For each project its due date to be accomplished and delivered to the customer is pregiven. Each project has its priority index which signifies the importance of the project.

A priority index has to be set for each project by the design office, i.e., by managers which are responsible for the project's delivery performance. The initial data for the i-th project, $1 \le i \le n$, is given in the form of a matrix row where each ℓ-th element, $1 \le \ell \le m$, corresponds to the ℓ-th stage $S_{i\ell}$ of that project and comprises three values: $\bar{t}_{i\ell}$, $V_{i\ell}$ and $m_{i\ell}$. Here $\bar{t}_{i\ell}$ is the average value of random duration $t_{i\ell}$ of the project's stage, $V_{i\ell}$ is the variance of $t_{i\ell}$ and $m_{i\ell}$, $1 \le m_{i\ell} \le m$, is the ordinal number of the division which has to process stage $S_{i\ell}$. Thus, the system's initial data is given in the form of an $(n \cdot m)$ - matrix $W = \left\| \bar{t}_{i\ell},\ V_{i\ell},\ m_{i\ell} \right\|$, $1 \le i \le n$, $1 \le \ell \le m$, where each project has its individual route via the company's divisions. Note that if a project is structured from activities in the form of a network model, values $\bar{t}_{i\ell}$ and $V_{i\ell}$ can be determined beforehand by simulating the subnetwork which corresponds to stage $S_{i\ell}$. An essential number of simulation runs has to be carried out in order to obtain a representative statistics to calculate $\bar{t}_{i\ell}$ and $V_{i\ell}$. If the project is not given in the form of a network model, values $\bar{t}_{i\ell}$ and $V_{i\ell}$ can be set by practitioners by using various expert methods, e.g., the Delphi method.

The problem is to determine starting time values for each project to be passed on each division. Those values are not calculated beforehand and are random values conditioned on the model's decision-making in the course of the projects' realization. The objective is to maximize the weighted function of the projects' delivery performances, i.e., of their probabilities to meet the corresponding due dates on time. Decision-making is carried out via pairwise comparison, by examining the projects' delivery performances together with their priority indices, and is used for choosing projects from a line.

12.2.3 Notation and the problem's formulation

Let us introduce the following terms [99]:

n - number of building projects in the company;

m - number of resource divisions entering the company;

$S_{i\ell}$ - ℓ-th stage of the i-th project, $1 \le \ell \le m$, $1 \le i \le n$;

m_i - number of stages of the i-th project, $1 \le m_i \le m$;

$t_{i\ell}$ - random processing time of $S_{i\ell}$;

$\bar{t}_{i\ell}$ - expected value of $t_{i\ell}$ (pregiven);

$V_{i\ell}$ - variance of $t_{i\ell}$ (pregiven);

$m_{i\ell}$ - index of the division on which $S_{i\ell}$ is realized, $1 \le m_{i\ell} \le m$ (pregiven);

$\| \bar{t}_{i\ell}, V_{i\ell}, m_{i\ell} \|$ - initial data matrix;

ς_i - priority index of the i-th project to indicate the level of the project's importance (pregiven); if project i_1 has a higher importance than project i_2, relation $\varsigma_{i_1} > \varsigma_{i_2}$ holds;

E_i - the earliest possible time moment to start realizing project i (pregiven);

D_i - due date for the i-th project to be accomplished (pregiven);

$T_{i\ell}$ - time moment stage $S_{i\ell}$ starts (a random value conditioned on the model's decisions);

$F_{i\ell}$ - the actual moment stage $S_{i\ell}$ is accomplished;

F_i - the actual time for the i-th project to be accomplished.

The problem is to determine values $T_{i\ell}$, $1 \le i \le n$, $1 \le \ell \le m$, to maximize the objective

$$I = \underset{\{T_{i\ell}\}}{Max} \sum_{i=1}^{n} \left[\varsigma_i \cdot Pr\left\{ F_i \le D_i \right\} \right] \tag{12.2.1}$$

subject to

$$T_{i1} \ge E_i, \ 1 \le i \le n, \tag{12.2.2}$$

where $\Pr\{F_i \le D_i\}$ is the i-th project's delivery performance. Note that maximizing objective (12.2.1) results in the policy as follows: the management takes all measures to accomplish first projects with higher priorities; only afterwards it takes care of other projects. The problem cannot be solved in the general case and allows only a heuristic solution. The latter is based on the combination of a simulation model and a heuristic decision-making rule. Decision-making, i.e., determining values $T_{i\ell}$, is carried out at *essential moments* when either one of the divisions is free for service or a certain project is ready to be processed. Note that if a project is ready to be processed on a certain division *which is free for service* and there is no line for that division, the project is passed to that division. Otherwise, i.e., in the case of a line of competitive projects for one and the same division, a competition is arranged based on the idea of pairwise comparison. We will assume henceforth for simplicity that all values $t_{i\ell}$ have a normal distribution with parameters $\bar{t}_{i\ell}$ and $V_{i\ell}$.

12.2.4 Decision-making rules for projects with random operations

In [68, 80, 99] we suggest two types of forecasting for scheduling various OS with random operations:

- short-term forecasting is used to forecast the moment a certain operation, a group of operations or a project is finished. For that purpose, the average processing time $\bar{t}_{i\ell}$ is suggested;
- long-term forecasting is used to calculate at a certain moment t the probability of meeting the due date on time. Using the Central Limit Theorem, we obtain

$$p_{ti} = \Phi \left\{ \frac{D_i - t - \sum_{s=\ell}^{m_i} \bar{t}_{is}}{\sqrt{\sum_{s=\ell}^{m_i} V_{is}}} \right\}, \tag{12.2.3}$$

where $\Phi(x) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{x} e^{-\frac{t^2}{2}} dt$ is a standard normal distribution and stages $S_{i\ell}, S_{i,\ell+1}, \ldots, S_{im_i}$ have not yet been operated at moment t. An assumption is introduced that those future stages will not wait in lines.

If, at a certain moment t, q projects with ordinal numbers $i_1, i_2, \ldots, i_q$ are waiting in the line and are ready to be passed to the k-th division to realize the corresponding stages $S_{i_1\ell_1}, S_{i_2\ell_2}, \ldots, S_{i_q\ell_q}$, we suggest the idea of pairwise comparison, which has been effectively used in various scheduling problems [68, 80, 138, 173]. Note that relation $k = m_{i_1\ell_1} = m_{i_2\ell_2} = \ldots = m_{i_q\ell_q}$ holds. Arrange a pairwise comparison between the first two competitive projects with indices i_1 and i_2 as follows. If

$$\bar{t}_{i_1\ell_1} + Max \left\{ \sum_{s=\ell_1+1}^{m_{i_1}} \bar{t}_{i_1 s}, \sum_{s=\ell_2}^{m_{i_2}} \bar{t}_{i_2 s} \right\} < \bar{t}_{i_2\ell_2} + Max \left\{ \sum_{s=\ell_2+1}^{m_{i_2}} \bar{t}_{i_2 s}, \sum_{s=\ell_1}^{m_{i_1}} \bar{t}_{i_1 s} \right\} \tag{12.2.4}$$

holds, the first project wins the competition. The winner competes with the next project with index i_3, etc., until only one winner is left. The latter has to be chosen from the line for that division. If relation (12.2.4) holds, that means that if the first project is operated first and the second one afterwards, we will accomplish both projects earlier, than by choosing the second project first for the division. Relation (12.2.4) realizes a short-term forecasting by using average values $\bar{t}_{i\ell}$. Such decision-making is effective when the objective is to minimize the average makespan value, i.e., the actual time period for all projects to be realized

$$Min\ E\ \left\{ \underset{i,\ell}{Max}\ F_{i\ell}\ -\underset{i,\ell}{Min}\ T_{i\ell} \right\}. \tag{12.2.5}$$

If, besides average values $\bar{t}_{i\ell}$, other parameters (e.g., cost parameters, etc.) have to be taken into account, we suggest undertaking the pairwise comparison by using long-term forecasting. Four values are calculated:

1. Probability performance $p_{ti_1} = p_1(t)$ for the first project to be accomplished on time on condition that the project is chosen for the division at moment t.
2. Probability performance $p_{ti_2} = p_2(t)$ for the second project to be accomplished on time on condition that the project is passed to the division at moment t.
3. Probability performance $p_{t+\bar{t}_{i_1\ell_1},\, i_2} = p_3(t)$ for the second project to be accomplished on time on condition that the first project is passed to the division first, i.e., at moment t, and later on, at the time moment $t+\bar{t}_{i_1\ell_1}$, the second project will start to be processed by the division.
4. Probability performance $p_{t+\bar{t}_{i_2\ell_2},\, i_1} = p_4(t)$ for the first project to be accomplished on time on condition that the second project will be chosen first, at moment t, for the division, and later on, at moment $t+\bar{t}_{i_2\ell_2}$, the first project will be passed to the division.

Those four conditional probabilities satisfy [80]

$$p_1(t) = \Phi \left\{ \frac{D_{i_1} - t - \sum_{s=\ell_1}^{m_{i_1}} \bar{t}_{i_1 s}}{\sqrt{\sum_{s=\ell_1}^{m_{i_1}} V_{i_1 s}}} \right\}, \tag{12.2.6}$$

$$p_2(t) = \Phi \left\{ \frac{D_{i_2} - t - \sum_{s=\ell_2}^{m_{i_2}} \bar{t}_{i_2 s}}{\sqrt{\sum_{s=\ell_2}^{m_{i_2}} V_{i_2 s}}} \right\}, \tag{12.2.7}$$

$$p_3(t) = \Phi\left\{\frac{D_{i_2} - t - \bar{t}_{i_1\ell_1} - \sum_{s=\ell_2}^{m_{i_2}} \bar{t}_{i_2\ell_2}}{\sqrt{\sum_{s=\ell_2}^{m_{i_2}} V_{i_2 s}}}\right\}, \tag{12.2.8}$$

$$p_4(t) = \Phi\left\{\frac{D_{i_1} - t - \bar{t}_{i_2\ell_2} - \sum_{s=\ell_1}^{m_{i_1}} \bar{t}_{i_1 s}}{\sqrt{\sum_{s=\ell_1}^{m_{i_1}} V_{i_1 s}}}\right\}. \tag{12.2.9}$$

After calculating values $p_1(t)$, $p_2(t)$, $p_3(t)$ and $p_4(t)$, decision-making is carried out by analyzing them. A decision-making rule for the case of projects with different priorities will be outlined below.

12.2.5 Decision-making rules for projects with priority indices

The developed heuristic decision-making rule to choose the project from the line for a division comprises both short-term and long-term forecasting procedures. If $q \le n$ projects with indices $i_1, i_2, \ldots, i_q$ are seeking the k-th resource division at moment t in the line, we suggest the procedure as follows. A pairwise comparison between the first two projects has to be arranged. Two competitive options are examined:

Option A. The first project is passed to the k-th division at moment t, and the second project afterwards, at moment $t + \bar{t}_{i_1\ell_1}$.

Option B. The second project is passed to the k-th division at moment t and later on, at moment $t + \bar{t}_{i_2\ell_2}$, the first project will start to be carried out.

Using the assumption that later on, in the course of carrying out the remaining stages of both projects, the latter will not wait in lines, we can calculate objective (12.2.1) *for two projects only*, by examining *Options A* and *B* separately. Thus, we finally obtain two comparative values I_A and I_B satisfying:

$$I_A = \varsigma_{i_1} \cdot p_1(t) + \varsigma_{i_2} \cdot p_3(t), \tag{12.2.10}$$

$$I_B = \varsigma_{i_1} \cdot p_4(t) + \varsigma_{i_2} \cdot p_2(t). \tag{12.2.11}$$

Since objective (12.2.1) has to be maximized, the option which delivers the maximum to value I, has to be accepted. Thus, if $I_A \ge I_B$, project i_1 is the winner. Otherwise the i_2

-th project wins the competition. The winner competes with the i_3-th project, etc., until only one winner will be left. The latter has to be passed to be k-th resource division at moment t for further realization.

Decision-making rules (12.2.10-12.2.11) have to be implemented at any essential moment, when a certain division is ready to realize a project and if there is a line of projects (i.e., more than one project) seeking for that resource division.

12.2.6 *Simulation model*

The suggested heuristic algorithm to solve problem (12.2.1-12.2.2) comprises a simulation model together with decision-making rules (12.2.10-12.2.11). The simulation model:

- determines (within a routine simulation run) the system's essential moments;
- determines the lists of projects which are waiting in lines;
- carries out decision-making to choose a project from the line;
- passes the project to the division in case if there is only one project seeking for that division;
- simulates the processing time $t_{i\ell}$ of stage $S_{i\ell}$ at moment t, when the project has been chosen for the division; thus, the actual random moment $F_{i\ell} = t + t_{i\ell}$ to be accomplished is simulated as well;
- carries out a sample of simulation runs to obtain representative statistics.

12.2.7 *Experimentation*

In order to evaluate the performance of the heuristic algorithm an example has been chosen. Six simultaneously realized projects have to be processed on five different divisions. The initial data matrix is given in Table 12.1. The projects' parameters D and ς are presented in Table 12.2. For each project its delivery performance, i.e., the probability of meeting the due date on time, has been calculated on the basis of 500 simulation runs. The results are presented in Table 12.3. The following conclusions can be drawn from the table:

1. An evident correlation between the projects' priority indices and the corresponding delivery performance rates can be recognized. Lower priority indices correspond to lower delivery performance rates, and vice versa. This fully coincides with the general idea of problem (12.2.1-12.2.2).

2. The obtained delivery performance rates are reliable enough for practical industrial problems.

Table 12.1 The initial data matrix (6 projects, 5 divisions)

Projects	Stage 1	Stage 2	Stage 3	Stage 4	Stage 5
1	(150, 100, 1)	(150, 100, 5)	(120, 100, 4)	(120, 100, 3)	(120, 150, 2)
2	(160, 900, 1)	(220, 900, 2)	(480, 900, 3)	(120, 900, 5)	(130, 400, 4)
3	(120, 90, 5)	(480, 1600, 2)	(110, 90, 1)	(130, 400, 3)	(120, 900, 4)
4	(160, 900, 4)	(120, 900, 3)	(180, 600, 5)	(160, 100, 2)	(160, 100, 1)
5	(150, 1000, 5)	(120, 400, 3)	(120, 900, 2)	(100, 100, 1)	(130, 500, 4)
6	(450, 1000, 5)	(140, 900, 4)	(160, 100, 2)	(160, 400, 3)	(120, 900, 1)

Table 12.2 The projects' parameters

Projects	Due date D	Priority index ς
1	1200	25
2	980	30
3	1750	15
4	1100	25
5	1900	20
6	1200	30

Table 12.3 Delivery performance values

Projects	Delivery performance values
1	0.785
2	0.960
3	0.642
4	0.801
5	0.725
6	0.944

It can be well-recognized that the decision models based on the idea of pairwise comparison amongst competing projects are effective and easy in usage. Simulation results, as well as applications on a real design office, show that the pairwise approach is a very useful procedure. The developed decision-making model (12.2.10-12.2.11) can be widely used for solving the general problem (12.2.1-12.2.2) for several projects with different priority indices. One has only to undertake decision-making at each decision point. The model can be applied for a broad variety of industrial projects, including various flexible manufacturing systems.

§12.3 Long-term innovative construction projects with alternative outcomes and branching nodes

12.3.1 Introduction

In Chapter 6 we have presented various decision-tree models which have great usefulness in practice, when operating long-term innovative construction projects under random disturbances, e.g., constructing a major Arctic pipeline [45]. Long-term innovative construction projects (LTICP) are characterized by a high level of indeterminacy, and with various types of branching nodes in key events. Those nodes may be the result of unpredictable outcomes of future pioneering hi-tech experiments, geological surveys with possible alternative outcomes, etc.

What is the essence of our philosophy [25, 83-84] when controlling an innovative project with uncertainty and being, at the outset, something which is basically indeterminate? Many examples from high performance practice show that under such circumstances, the control system should not work to a predetermined plan, but should be inherently adaptable, seeking at each decision node to assess *the best route forward*, reconfiguring the ultimate goals, if appropriate.

Note that the sub-problem of determining *the best route* may be very difficult and complicated, especially for systems with a high level of indeterminacy. Solving this sub-problem usually results in solving the general control problem.

Our philosophy in project planning and control with indeterminacy centers on avoiding predetermining the initial network model; moreover, in certain cases the structure of such a model may be indeterminate. At the initial stage of the project's realization, the network may be restricted to a source node and several alternative sink nodes (goals) together with some milestones (a decision-tree model). Such a restricted project is called an aggregated project. Various activities are usually of random duration. Such a stochastic alternative network is renewed permanently over time, including changes in the ultimate goals. At each decision node, our techniques enable us to choose the optimal outcome. Decision-making is repeatedly introduced for the renewed network at every sequentially reached decision node.

We will examine henceforth a LTICP network model with a very high level of uncertainty - a branching network to control a project with two kinds of alternative events: stochastic (uncontrolled) branching of the development of a project, as well as deterministic branching where the outcome direction is chosen by the project's decision maker [45, 67-70].

Note that while the literature on PERT and CPM network techniques is quite vast, the number of publications on *alternative networks* remains very scanty. Various authors, e.g., Elmaghraby [55], introduced the concept of a research and development (R&D) project as a complex of actions and problems towards achieving a definite goal. Several adequate network models for such projects have been considered [39, 45, 168]. Note that those projects, being alternative, remain uncontrollable.

Golenko-Ginzburg [67-70, 83-84] developed the novel controlled alternative activity network (CAAN model) for projects with both random and deterministic alternative outcomes at key nodes. At each routine decision-making node, the developed algorithm, based on lexicographic scanning, singles out all the sub-networks (the so-called joint variants) that correspond to all possible outcomes from that node. The joint variants of the CAAN model are enumerated by introducing a lexicographic order to the set of maximal paths in the CAAN graph. The corresponding lookover algorithm is very simple in usage. Decision-making results in determining the optimal joint variant and following the optimal direction up to the next decision-making node.

We will use the CAAN alternative network model which suits mostly the LTICP.

12.3.2 Formal description of the alternative stochastic model

The alternative CAAN network model [67, 70] is generally a finite compendent directed graph, $G(U,Y)$, with the following properties:

(1) Graph G has one initial event, y_0 (the network entry), for which $\Gamma^{-1}y_0 = \emptyset$ and $\Gamma y_0 \neq \emptyset$.

(2) Graph G contains a set Y' of events y' (called terminal events, or network exits), where $\Gamma y' = \emptyset$, $\Gamma^{-1}y' \neq \emptyset$ and $|Y'| \geq 2$.

(3) The set of events Y of graph G is not uniform and consists of events of type $\tilde{\chi} \in \tilde{X}$ (classical PERT model) and of more complex logical types, $\tilde{\alpha} \in \tilde{A}$, $\tilde{\beta} \in \tilde{B}$, and $\tilde{\gamma} \in \tilde{\Gamma}$, being represented in the below Table 12.4:

Table 12.4. Logical possibilities of alternative network model events

Designation of an event in the model	Logical relations at the event's receiver	Logical relations at the event's emitter
$\tilde{\chi}$	and	and
$\tilde{\alpha}$	and	exclusive "or"
$\tilde{\beta}$	exclusive "or"	and
$\tilde{\gamma}$	exclusive "or"	exclusive "or"

(4) The set of arcs U of graph G is split into a subset U' of arcs corresponding to the actual functioning of the alternative network, and subset U'' of arcs representing the

logical interconnections between actual and imaginary functions.

(5) Vector W_{kl} of values characterizing actual work is constructed preliminary for every arc, $U_{kl} \in U'$, representing an actual activity. Among such values are the time of the activity duration t_{kl}; the required cost C_{kl}; and other components of this vector. The vector's components $\omega_{kl}^{(\rho)}$ ($\rho = 1 - k$, k being the vector' dimension) can be represented, depending on the degree of indeterminacy, either by determined estimations or by random values with a given distribution function, $f\left(\omega_{kl}^{(\rho)}\right)$, on the interval $\left[\alpha\left(\omega_{kl}^{(\rho)}\right), \beta\left(\omega_{kl}^{(\rho)}\right)\right]$, where $\alpha\left(\omega_{kl}^{(\rho)}\right)$ and $\beta\left(\omega_{kl}^{(\rho)}\right)$ are boundary estimations of the ρ-th component of vector W_{kl}.

(6) For the stochastic alternative model of a combined type, the set of alternative events, $\tilde{A} \cup \tilde{\Gamma}$, is split into subsets $\overline{A}$ - alternative events that show the branching of determined variants, and $\overline{\overline{A}}$ - alternative events that represent the situations of branching stochastic variants, where $\tilde{A} \cup \tilde{\Gamma} = \overline{A} \cup \overline{\overline{A}}$.

(7) When the network event is of alternative nature, it is assigned a set of estimations of corresponding local variant probabilities. In other words, a nonnegative number, $\overline{\overline{p}}_{ij} \leq 1$, such that $\sum_{j=1}^{n_i} \overline{\overline{p}}_{ij} = 1$ (where $\overline{\overline{p}}_{ij}$ is the *a priori* probability of transferring from i to j and n_i stands for the number of local variants appearing in event i), is related to each alternative path starting from event i of type $\tilde{\alpha} \in \overline{\overline{A}}$ or $\tilde{\gamma} \in \overline{\overline{A}}$ and leading to outcome j.

(8) If event i is related to an alternative event of class $\tilde{A}$, the corresponding conditional transfer probability, $\overline{\overline{p}}_{ij}$, is usually assumed to be equal 1. This means that the process of choosing the direction in which the system has to move towards its target is of a determined character; it is the prerogative of the system's controlling device.

Problems of alternative network model analysis and synthesis are solved by applying the principle of network enlarging and obtaining a special graph - the outcome tree [25, 67-70], which is usually designated as $D(A,V)$ and represents a graph that can be constructed by modifying the original model, $G(Y,U)$, as follows:

(a) The set, which consists of the initial event, finite events, and events that are branching points of alternative paths of graph G, is taken as the set of events of graph D. The initial event, $\alpha_0 = y_0$, is called a hanging event.

(b) The set of arcs $V = \{v_{ij}\}$ of graph D is obtained thorough an equivalent transformation of a set of sub-graphs, $\{G_{ij}\}$, extracted from network G according to the following procedure:

- any event α_i, except for the finite ones, α', can be the initial event of sub-graph $G_{ij} = (L_{ij}, U_{ij})$, where $\alpha' \in y_{ij}$ and $\Gamma^{-1}\alpha_i \cap Y_{ij} = \emptyset$;
- $Y_{ij} \subset \tilde{\Gamma}\alpha_i$, where $\tilde{\Gamma}\alpha_i$ stands for the transitive closure of mapping α_i;
- only an α-event of graph G, except for the initial event, $\alpha_0 = y_0$, can be a finite event of sub-graph G_{ij}, and
- no $(\alpha_i, ..., \alpha_j)$-type paths that connect the initial event, α_i, with sub-graph finite event α_j in G_{ij}, contain other α-events of graph G.

(c) every arc, v_{ij}, of outcome tree D is obtained by reducing fragment G_{ij} of network $G(Y,U)$ to one arc beginning at α_i and ending at α_j. In addition, realization probability p_{ij}, fulfilment time t_{ij}, and other parameters equivalent to the corresponding characteristic values for initial fragment G_{ij} are brought into correspondence with the enlarged arc v_{ij}.

If different fragments, G_{ij}, of the model do not intersect, the alternative network is called entirely divisible; all events of the corresponding outcome tree prove to be γ-type events.

We will require a supplementary definition. A *partial variant* is a variant of the network model's realization; it corresponds to a definite direction of its development at an individual stage, characterizes one of the possible ways of reaching the intermediate target, and does not contain alternative situations. The variant of realization of the whole project, which does not contain alternative branchings and is formed by a sequence of partial variants, is called a *full variant*. On the outcome tree, $D(A,V)$, a certain arc, v_{ij}, corresponds to the partial variant, while some path connecting root event α_0 with one of the hanging events, corresponds to the full variant.

The combined outcome tree, $D(A,V)$, can be regarded as a union of purely stochastic outcome trees that reflects some homogenous alternative stochastic network models. The latter are obtained by choosing different directions in the controlled devices. Such stochastic outcome trees, which are all part of the combined outcome tree, $D(A,V)$, are called *joint variants* of realizing the stochastic network model.

The joint variant can be extracted from the original graph, $D(A,V)$, by "fixing" certain directions in interconnected events of type $\bar{\alpha}$ and excluding unfixed directions. In other words, every joint variant can be regarded as a realization variant of the network model. Such a variant has a determined topology, but it contains probability situations and has certain possible stochastic finite states.

Let us examine an outcome tree of a CAAN type alternative project presented on Fig. 12.2. Here $\{\bar{\alpha}\}$ denote decision-making nodes of deterministic nature, where the outcome direction is fully governed by the project's manager. Nodes $\{\bar{\bar{\alpha}}\}$ are of stochastic nature and, as such, are not controlled. Each $\bar{\bar{\alpha}}$-type node comprises several outcome probabilities which form a full group of events.

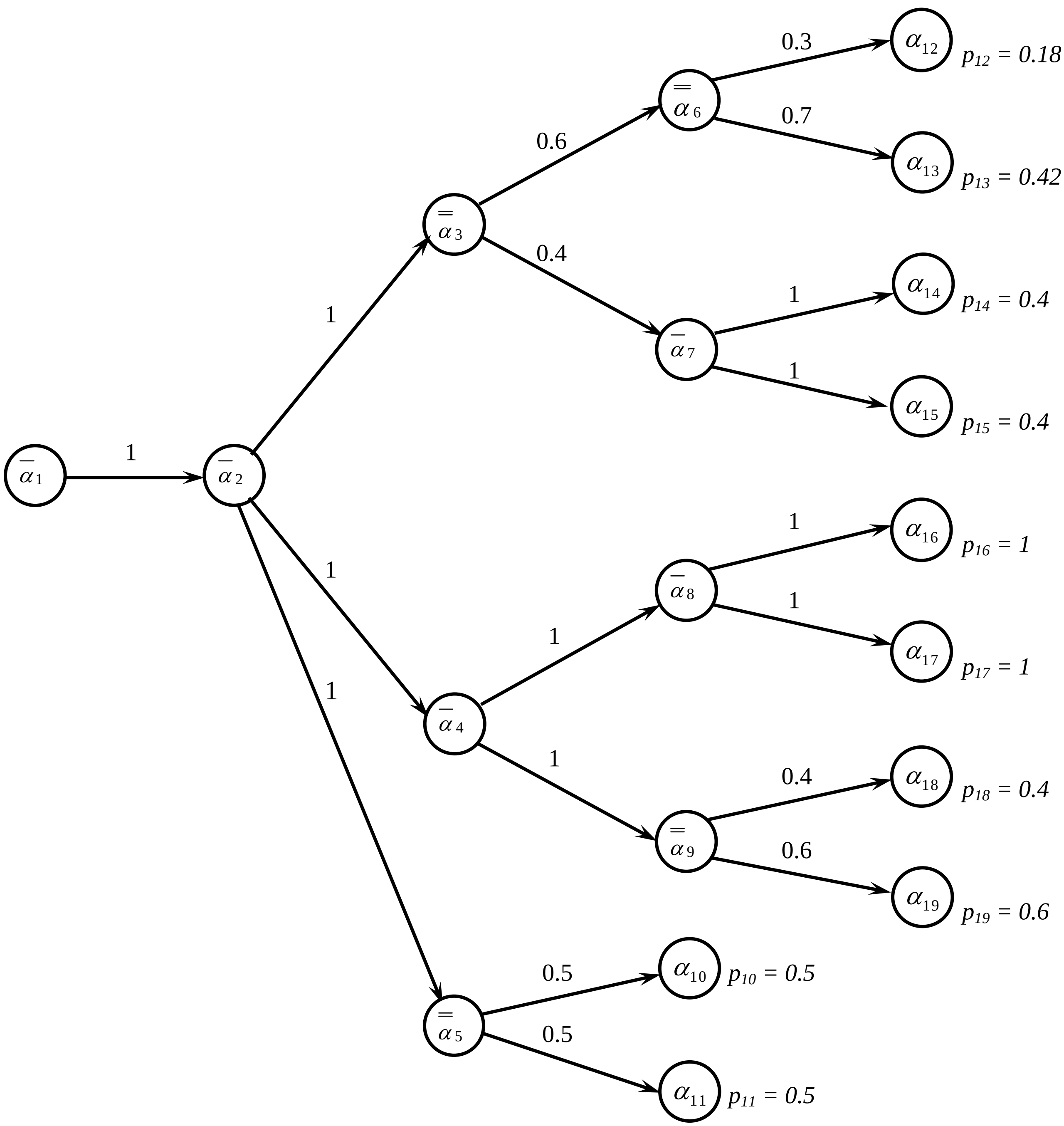

Figure 12.2. *Controlled alternative network project*

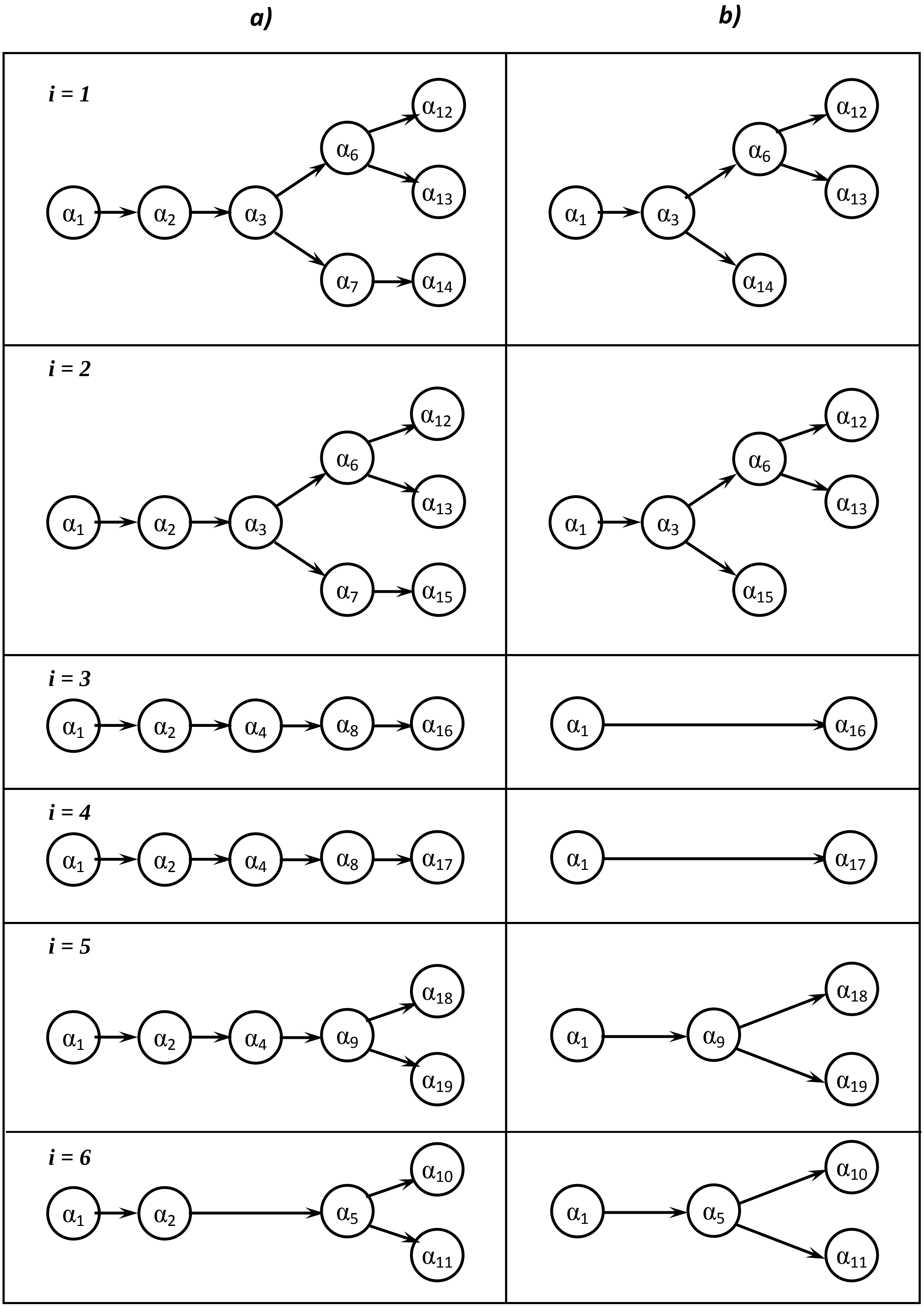

Figure 12.3 *The project's joint variants*

Stage a) on Fig. 12.3 presents six joint variants which can be singled out by analyzing the outcome tree. Note that none of the joint variants comprise alternative deterministic nodes and are determined by choosing non-contradictive directions in nodes $\overline{\alpha}$. Thus, all joint variants are either purely alternative stochastic non-controllable networks of α-type, or non-alternative fragments. Stage b) on Figure 12.3 demonstrates simplified joint variants of both types.

12.3.3 Optimal joint variant

Managing a controlled alternative activity network with two types of branching events means choosing an optimal joint variant which optimizes the project's goal function. For the case of a LTICP we consider several optimality criteria:

A. Since a joint variant can be regarded as a purely stochastic alternative project, we may calculate the entropy level as a measure of indeterminacy for each joint variant. The joint variant with the least entropy level has to be chosen.

B. For the case of an averse risk manager the strategy is as follows. Calculate (for each joint variant) the goal function for the *worst possible probability outcome* (i.e., the worst possible full variant). In other words, we determine the worst goal function value which may be actually (i.e., with probability exceeding zero) achieved in the course of realizing the joint variant. Call such a goal function value for the i-th joint variant $G_{\min}(J_i)$. The joint variant which delivers an optimal value from all $G_{\min}(J_i)$, $1 \le i \le n$, has to be chosen as the optimal one.

C. Calculate for each i-th joint variant J_i, $1 \le i \le n$, the average value of the goal function, i.e., the mathematical expectation given in the form

$$\overline{G}(J_i) = \sum_{j=1}^{n_i} p_j G(F_j), \tag{12.3.1}$$

where p_j denotes the probability of realizing the full variant F_j, and $G(F_j)$ stands for the goal function of that full variant. Joint variant J_ξ satisfying

$$\overline{G}(J_\xi) = \max_{1 \le i \le n} \left\{ \sum_{j=1}^{n_i} p_j G(F_j) \right\}, \tag{12.3.2}$$

has to be preferred as the optimal one.

D. *Criterion D* is contrary to *Criterion B*. We have to calculate for each joint variant the goal function corresponding to the *best goal function outcome* which may be actually obtained in the course of realizing the joint variant. Call it $G_{\max}(J_i)$, $1 \le i \le n$. Joint variant J_η satisfying

$$G_{\max}(J_\eta) = \max_{1 \le i \le n} [G_{\max}(J_i)] \tag{12.3.3}$$

has to be chosen as the optimal one.

The choice of the optimal strategy depends on the nature of LTICP. If the value of the quality of the project's product is extremely important, *Strategy D* has to be preferred. Note that both *Strategies B* and *D* are, in fact, game strategies. In case we are interested in a less nervous progress of the regarded projects, *Strategy C* seems to us to be a better choice.

12.3.4 Numerical example

Let us present a numerical example for the outcome tree appearing in Fig. 12.2 and 12.3. The alternative model of CAAN type comprises 6 joint variants $J_1,...,J_6$ and 10 full variants F_{11}, F_{12}, F_{13}, F_{21}, F_{22}, F_{23}, F_{31}, F_{41}, F_{51}, F_{52}, F_{61}, F_{62} (note that full variants F_{11} and F_{12} coincide with full variants F_{21} and F_{22}). Let the goal function be the project's cost (to be minimized) and preset the local activities' costs as follows: $C_{12}=10$, $C_{23}=6$, $C_{24}=15$, $C_{25}=12$, $C_{36}=14$, $C_{37}=9$, $C_{6,12}=10$, $C_{6,13}=16$, $C_{7,14}=15$, $C_{7,15}=20$, $C_{48}=11$, $C_{49}=13$, $C_{8,16}=15$, $C_{8,17}=8$, $C_{9,18}=10$, $C_{9,19}=18$, $C_{5,10}=12$, $C_{5,11}=36$. Assume, further, that the alternative graph under consideration refers to the LTICP class of projects.

It can be well-recognized from examining Fig. 12.2 and 12.3 that implementing *Strategy A* results in comparing two alternative joint variants J_3 and J_4, both with zero level of entropy. Since

$$C(J_3) = C_{12} + C_{24} + C_{48} + C_{8,16} = 51$$

exceeds

$$C(J_4) = C_{12} + C_{24} + C_{48} + C_{8,17} = 44,$$

joint variant J_4 has to be determined as the optimal one.

Implementing *Strategy B*, i.e., risk-averse decision-making, boils down to calculating the following values:

$$C_{\max}(J_1) = \max \begin{bmatrix} C_{12} + C_{23} + C_{36} + C_{6,12} = 40;\ C_{12} + C_{23} + C_{36} + C_{6,13} = 46; \\ C_{12} + C_{23} + C_{37} + C_{7,14} = 40 \end{bmatrix} = 46;$$

$$C_{\max}(J_2) = \max[40;\ 46;\ 45] = 46;$$

$$C_{\max}(J_3) = C_{12} + C_{24} + C_{48} + C_{8,16} = 51;$$

$$C_{\max}(J_4) = C_{12} + C_{24} + C_{48} + C_{8,17} = 44;$$

$$C_{\max}(J_5) = \max[C_{12} + C_{24} + C_{49} + C_{9,18} = 48;\ C_{12} + C_{24} + C_{49} + C_{9,19} = 56] = 56;$$

$$C_{\max}(J_6) = \max[C_{12} + C_{25} + C_{5,10} = 34;\ C_{12} + C_{25} + C_{5,11} = 58] = 58.$$

Thus, joint variant J_4 which delivers the extreme (the minimal) goal function value if the worst comes to the worst for all joint variants, has to be chosen as the optimal one.

Using the "opposite" *Strategy D*, we may calculate

$$C_{\min}(J_1) = \min[40;\ 46;\ 40] = 40;$$

$$C_{\min}(J_2) = \min[40;\ 46;\ 45] = 40;$$

$$C_{\min}(J_3) = 51;$$

$$C_{\min}(J_4) = 44;$$

$$C_{\min}(J_5) = 48;$$

$$C_{\min}(J_6) = 34.$$

Thus, when implementing risky decision-makings, the result of the procedure is different, namely: joint variant J_6 has to be determined as the optimal one.

When adopting *Strategy C*, the mathematical expectations of the cost to realize the considered joint variants may be calculated as follows (refer again to Fig. 12.2 and 12.3):

$$\overline{C}(J_1) = (10+6+14+10)\cdot 0.18 + (10+6+14+16)\cdot 0.42 + (10+6+9+15)\cdot 0.4 =$$
$$= 7.20 + 19.32 + 16 = 42.52;$$

$$\overline{C}(J_2) = 40\cdot 0.18 + 46\cdot 0.42 + 45\cdot 0.4 = 7.20 + 19.32 + 18 = 44.52;$$

$$\overline{C}(J_3) = 51;$$

$$\overline{C}(J_4) = 44;$$

$$\overline{C}(J_5) = (10+15+13+10)\cdot 0.4 + (10+15+13+10)\cdot 0.6 = 19.2 + 28.8 = 48;$$

$$\overline{C}(J_6) = (10+12+12)\cdot 0.5 + (10+12+36)\cdot 0.5 = 46 \ .$$

Since J_1 results in the minimal mean cost expenses required, it has to be chosen as the optimal one. Thus, adopting different optimality concepts may result in corresponding changing of the joint variant determined as optimal.

12.3.5 Capital investments in long-term alternative projects under random disturbances

It can be well-recognized that in recent years undertaking capital investments and contracting long-term projects which are carried out under random disturbances, has been the subject of lengthy debate and a very sharp criticism (see, e.g., [25, 39]). This is because nowadays it is extremely difficult to implement into commercial agreements both the projects' durations and especially the required volume of the corresponding capital investments. This refers mostly to long-term innovative construction projects based on future geological surveys with a high level of indeterminacy, projects involving implementation of new unique technology, etc. It goes without saying that for LTICP comprising both deterministic and stochastic alternative variants, the challenge of determining with a more or less accuracy the future project's parameters (like cost, duration, reliability attributes, etc.) becomes practically impossible. However, something has to be decided and has to be done immediately, otherwise the losses originating from failure to compete with accelerating technical and technological progress, may prove to be tremendous.

To meet the challenge, we suggest a new step-wise procedure in order to manage LTICP with alternatives of both deterministic and stochastic nature. The main stages of the procedure are as follows:

Stage I. If possible, determine an alternative graph of the future LTICP. The graph has to be similar to that outlined on Fig. 12.2.

Stage II. Determine all the joint variants entering the graph. The corresponding algorithm is outlined in [67-70], and is based on lexicographical simulation.

Stage III. Determine the strategy for recognizing the optimal joint variant. We remind that different conceptual strategies may result in different principles of optimality and indeterminacy and, thus, result in variety of the optimal joint variant identity, as it was demonstrated in the previous *Section*. In our opinion, the majority of LTICP projects may use the average criterion value in order to determine the optimal joint variant, i.e., *Strategy C*.

Stage IV. After determining the optimal joint variant, one may start the contracting process. We suggest undertaking this process sequentially. On the first step the capital investments have to cover the progress of the project from the very beginning until the first branching node of stochastic type. If, for example, we have chosen J_1 presented on Fig. 12.3, as the optimal one, the signed agreements have to cover expenses starting from event $\bar{\alpha}_1$ until the next alternative (branching) node $\bar{\bar{\alpha}}_3$, i.e., the primary capital investments have to cover the realization of fragment $\bar{\alpha}_1 \rightarrow \bar{\alpha}_2 \rightarrow \bar{\bar{\alpha}}_3$. Thus, the corresponding contract has to cover expenses estimated as $C_{12} + C_{23} = 16$.

Stage V. After reaching event $\bar{\bar{\alpha}}_3$ the contract has to be rewritten anew, depending on the realization of the uncontrolled direction ($\bar{\bar{\alpha}}_3 \rightarrow \bar{\bar{\alpha}}_4$ or $\bar{\bar{\alpha}}_3 \rightarrow \bar{\bar{\alpha}}_5$) of the progress of the project.

Stage VI. In the course of the project's realization the joint variant we have chosen before, besides being updated, may undergo other changes as well, both in the structure of the graph itself and in the values of the probability outcomes. Thus the consecutive progress of the project results in consecutive updating the contract's agreement. We do not see another managerial principle applicable to multi-variant alternative projects under consideration. Note that such a form of monitoring enables both on-line and financial control procedures.

12.3.6 Conclusions

The following conclusions can be drawn from the study:

1. Long-Term Innovative Construction Projects may deal with a high level of indeterminacy, as well as with various types of branching nodes in key events. Those nodes may be the result of unpredictable outcomes of future pioneering hi-tech experiments, geological surveys with possible alternative outcomes, etc.
2. We have described and presented an on-line stochastic alternative network model comprising both decision-making nodes with deterministic branching and uncontrollable alternative nodes with probabilistic outcomes.
3. We have demonstrated the possibility of singling out an optimal joint variant from the previously given stochastic alternative network graph. The structure of the optimal joint variant depends on the concept of optimality, as it has been presented by means of the numerical example. A joint variant does not comprise controllable branching events and is, in fact, a purely homogenous alternative stochastic network.
4. We have suggested a new procedure of contracting capital investments for the considered stochastic alternative model. On our opinion, the suggested mechanism may be effectively used in the course of drawing out financial contracts and other agreements in order to supply complicated long-term innovative construction projects of alternative structure.

§12.4 Monitoring building projects by means of target amount rescheduling

It can be well-recognized that all techniques outlined in this Chapter so far, are based on resource reallocations while the projects target amounts are pregiven and remain fixed and predetermined throughout the system's functioning. However, in certain cases, e.g., when the company is specialized on building similar standard apartment houses, the projects' targets may be amended and rescheduled among several projects' contractors. Here each project's target V_i, $1 \le i \le n$, is gauged by a single measure (in square meters). The system's overall target amount V (also gauged in square meters) is fixed, as well as the due date D subject to a chance constraint, i.e., the least permissible probability p of meeting the target on time. Each building project U_i has several possible speeds v_{i1}, v_{i2}, ... , v_{im}, which are subject to random disturbances. The project's output can be measured only at preset inspection (control) points. For each unit, the average costs per time unit for each project and the average cost of performing a single inspection at a control point to observe the actual output at that point, are given.

We present a two-level on-line control model under random disturbances, which centers on minimizing the system's expenses subject to the chance constraint. The suggested two-level heuristic algorithm is based on rescheduling the overall target among the projects both at $t = 0$, when the system starts functioning, and at each emergency point, when it is anticipated that a certain project is unable to meet its local target on time subject to a chance constraint. At any emergency point t the remaining system's target V_t is rescheduled among the projects; thus, new local targets V_{it}, $1 \le i \le n$, $\sum_i V_{it} = V_t$, are determined. New local chance constraint values p_{it} are determined too. Those values enable the system to meet its overall target at the due date subject to the pregiven chance constraint p.

After reassigning to each project U_i its new target V_{it} and the chance constraint value p_{it}, the projects first work independently and are controlled separately. At each k-th control point t_{ik} of project U_i, given the actual amount already produced, decision-making centers on determining both the next control point $t_{i,k+1}$ and the index j of the new speed v_{ij} to proceed with up to that point, $1 \le j \le m$. The on-line control for each project proceeds either until the next emergency point, or until the due date D.

Rescheduling the remaining system's target amount V_t among the projects is carried out by using heuristic procedures. Determining chance constraint values p_{it} is carried out by using a cyclic coordinate descent method in combination with a two-level simulation model [105].

The problem of monitoring the building system is solved [105] on the basis of the developments outlined in Chapter 7. The only non-essential difference boils down to substituting resource reallocation for target amount rescheduling among the projects. The problem is thus as follows:

At each emergency point t determine local targets V_{it}, $1 \le i \le n$, together with local chance constraints p_{it}, control points t_{ik} and new advancement speeds v_{ij}, $1 \le j \le m$, to minimize the expected total expenses C_t

$$\min_{\{p_{it}, V_{it}\}} C_t \qquad (12.4.1)$$

subject to

$$\Pr\{V^f(D) \ge V\} \ge p \qquad (12.4.2)$$

Problem (12.4.1-12.4.2) may be solved by means of a two-level model.

Both control points t_{ik} and construction speeds v_{ij} are determined at the lower (project) level by implementing the model outlined in §7.2. Target amount reallocation is carried out at the upper (company) level. The general idea of the algorithm is as follows:

At each routine emergency point t_q^{em}, $q = 0,1,...,N_{em}$, decision-making centers on minimizing the future costs from point t_q^{em} until D, including the penalty and the storage costs. The costs representing the past (interval $\left[0, t_q^{em}\right]$) are not relevant for this on-line control problem, and there is no need to remember the past decision [79]. The only relevant information to be stored is t_q^{em} and $V_i^f\left(t_q^{em}\right)$. Thus, decision-making at the system level is carried out only at emergency points t_q^{em} including the moment $t = 0$ the system starts constructing.

Decision-making at the system level at each routine emergency moment $t = t_q^{em}$ centers on determining both new chance constraint values $\{p_{it}\}$ and new target amounts V_{it} for the remaining planning horizon $[t, D]$. Values $\{p_{it}\}$ are obtained via simulation, by a combination of a search algorithm and an on-line one-level control algorithm for several projects. The latter work independently and are controlled separately at inspection points. It is generally assumed that at the beginning of the work all the available resources, i.e., the building teams, are previously allocated among the projects. Those resources remain unchanged within the planning horizon, i.e. no resource reallocation is performed. Thus, the corresponding construction speeds v_{ij} for each project U_i remain unchanged too.

If for a certain project U_i at a routine inspection point t_{ik} it is anticipated that the project cannot meet its target V_{it} on time subject to the previously determined chance constraint p_{it}, an emergency is then called, and decision-making is affected at the system level. The remaining target V_t at $t = t_{ik}$, together with the remaining time $D_{t_{ik}} = D - t_{ik}$, is then updated. New quasi-optimal values $\{p_{it}\}$, $t = t_{ik}$, together with new target amounts $\{V_{it}\}$, are then determined. The newly corrected plan is assigned to all building projects, and the construction process proceeds further, until either the new emergency point or until the moment the target amount is completed. Thus, decision-making at the system level centers on numerous recalculations of the system's plan subject to the chance constraint. This is carried out by using a forecasting simulation model with input values $\{V_{it}, p_{it}\}$, $t = t_{ik}$. The matrix $Z = \{V_{it}, p_{it}\}$ which delivers the minimum of total accumulated costs subject to the chance constraint p, is taken as the *optimal corrected plan*. Afterwards, that corrected plan is passed to the projects, and on-line decision-making is carried out at the project level.

PART III

ORGANIZATION SYSTEMS IN STRATEGIC MANAGEMENT

Chapter 13. Active Organization Systems

§13.1 Description of an active system

13.1.1 *Introduction*

The most important and the most interesting (both in theory and in practice) class of large-scale organization systems are systems incorporating a human or a group of humans. There are most diverse examples of such systems. Among them are sociological, economic, political, administrative, and other systems. A characteristic feature of this class of systems is the presence of subsystems whose objective functions do not coincide in the general case with the overall function of the system. Moreover, subsystems containing a human are active in the sense that for maximizing their objective function they tend to use not only the available physical resources, such as production capabilities, but also information channels. The subsystems receive over these channels information about the activity of other subsystems, in particular subsystems that are controlling with respect to the subsystems under consideration, and they communicate to the controlling subsystems their capabilities, i.e., a description of their models.

An active organization system (*AOS*) is well defined if:

1) its structure is determined, i.e., for any subsystem we know the controlling subsystem and the set of controlled subsystems (the levels of the hierarchy are specified);
2) a model of each subsystem is given, i.e., a method of representation of the set of feasible plans and an objective function that depends on the plan of the given subsystem, the plans of its subordinate subsystems, the control established by the controlling subsystem, and the control established by the given subsystem for the controlled subsystems;
3) the connection is specified between the plans of the subsystems of the lower level of the hierarchy and the plans of the subsystems of the upper level, i.e., to each set of feasible plans of the subsystems subordinate to the subsystem under consideration, we assign a plan of this subsystem. In the latter case we naturally have an accumulation of information.

It can be well-recognized that an *AOS* differs from a non-active OS by comprising man-machine subsystems with non-antagonistic goals (targets), while in many OS those targets prove to be antagonistic (e.g., the distinction of a group of building projects in the projects' portfolio with restricted resources may reduce the quality values of other, less "lucky", building projects). The game each element entering an AOS is playing is always non-antagonistic to other games. An example of an AOS in stochastic project management will be outlined below, in §13.4.

13.1.2 Model of a two-level AOS

The structure of a two-level AOS is formed by a Center (C), by n active elements (AE) subordinate to the Center, and by variables describing the state of the system. In order to account for the "external" connections of the system's elements we introduce the structural element "environment". To the environment we refer also certain "passive" elements of the system (for example, a centralized warehouse). In the deterministic models we shall assume that both the C as well as the AE know the state of the environment. Fig. 13.1. shows an consisting of a Center, a centralized warehouse, and two active elements.

For the i-th AE we specify the state vector y_i (the realization vector in the economic interpretation), the control vector c_i, and the sets of their possible values: $y_i \in Y_i$ and $c_i \in C_i$, $i \in I = \{i/i = 1,2,...,n\}$.

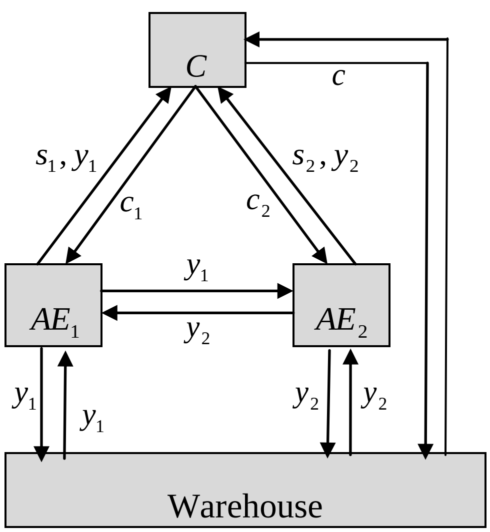

Figure 13.1. The AOS structure

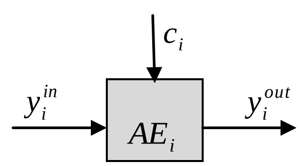

Figure 13.2. The AOS inputs and outputs

Example 1

It can be well-recognized that literature on mathematics and economics presents a large number of models of the "expenditure-output" type, in which the element realization vector y_i is specified by means of an input realization vector y_i^{in} and an output realization vector y_i^{out}: $y_i = (y_i^{in}, y_i^{out})$, while the set of possible realizations of the element is specified in the following way: $y_i^{in} \in Y_i^{in}$ and $y_i^{out} \in Y_i^{out}(y_i^{in})$, i.e., we specify the set of possible input realizations and the set of output realizations as a function of the input realizations (see Fig. 13.2).

For the whole AOS we specify the set of realization vectors $y = \{y_i, i \in I\}$ (realizations of the AOS), the set of control vectors $c = \{c_i, i \in I\}$ (controls of the AOS), as well as sets of possible values of these parameters: $y \in Y$ and $c \in C$. In the general case $Y = Y^g \cap \left(\prod_{i=1}^{n} Y_i \right)$, where Y^g represents global constraints on the realization y of the AOS: $y \in Y^g$. Analogously, $C = C^g \cap \left(\prod_{i=1}^{n} C_i \right)$. The presence of a purpose in the AOS will be associated with the presence of the system's target function $W = \Phi(c, y)$ (for example, the economic utility-income function, the profit, the expenditures, etc.); the C will be reckoned as the administrative organ and it will be described in terms of the description of its actions with respect to the control of the AOS. We shall also assume that the C 's target function coincides with the AOS 's target function. Thus, the model of the AOS can be represented in the form: $W = \Phi(c, y)$ being the target function of the AOS, where $c = \{c_i, i \in I\}$, $y = \{y_i, i \in I\}$:

- $c_i \in C_i$, $i \in I$, $c = \{c_i\} \in C = C^g \cap \left(\prod_{i=1}^{n} C_i \right)$,
- $y_i \in Y_i$, $i \in I$, $y = \{y_i\} \in Y = Y^g \cap \left(\prod_{i=1}^{n} Y_i \right)$.

13.1.3 Description of the Center's actions

The method of organizing the functioning of the AOS 's model will be called its functioning mechanism. Let us describe a number of components of the functioning mechanism of the model of a two-level AOS and the capabilities of the C with respect to forming (changing) them.

Control by means of introducing constraints. One of the properties of "activeness" is that the AE has a freedom of choice of the realization y_i from the set Y_i of possible realizations. In hierarchical systems the C can "judge" the set of possible realizations of each system elements by introducing for it a centrally established set of possible realizations B_i ($i \in I$). The set B_i of the i-th element can depend, firstly, on the controls c_i established by the C and, secondly, on the realizations y_j, $j \neq i$, chosen by other elements of the system (this is frequently due to the presence of "horizontal connections" between the system's elements, e.g., reciprocal deliveries). First of all we consider the case when the AE are independent in the sense that set B_i of possible realizations of each AE depends only on the control c_i and does not depend upon realizations y_j, $j \neq i$, selected by other AE: $B_i(c_i)$, $i \in I$. It can be well-recognized that when forming sets

$B_i(c_i)$ the C must satisfy condition $\forall c_i \in C_i : B_i(c_i) \cap Y_i \neq \emptyset$, $i \in I$. The introduction of sets $B_i(c_i)$ by the C is called control by means of introducing constraints.

Procedures for forming the estimates. In most cases, functioning of hierarchical systems takes place under conditions of incomplete information available to the C on the models of the elements subordinate to it. This situation is reflected in the theory of AOS formally in the following manner. It is assumed that sets $Y_i(r_i)$, $B_i(c_i, r_i)$, and $Y(r)$, where $r = \{r_i, i \in I\}$, are given in a parametric form known to the C. Concerning the values of the vector-valued parameters r_i it is assumed that their dimensions are finite and that C knows only the set Ω_i of possible values of r_i, i.e., $r_i \in \Omega_i$ ($i \in I$), whereas each AE knows precisely the values of its "own" vector-valued parameter r_i. If the knowledge of parameters $r = \{r_i\}$ only within the set $\Omega = \prod_{i=1}^{n} \Omega_i$ is inadequate for an effective control of the system, then C can organize a procedure for forming estimates $s = \{s_i\}$ of the parameters.

Control law in the *AOS*. This is a procedure for which the C determines the AOS 's control $c = \{c_i\}$ on the basis of the information available to it [34-35]. We shall examine the following structure of the control vectors: $c_i : c_i = (\lambda, x_i)$, where x_i stands for the plan vector and vector λ of components is called, as before, the control. The components of the control vector λ can be common for a part or for all AE (for example, costs). The plan x_i represents the desired value' determined by the C, of all or a part of the components of the realization vector $y_i = \{y_{ij}\}$ of the i-th AE. At first we consider the case when the dimensions of vectors y_i and x_i coincide (completely plannable realizations) for all AE ($i \in I$).

A plan x_i of the i-th AE is said to be realizable if there exists a realization $y_i \in Y_i(r_i)$ such that $y_i = x_i$. It can be well-recognized that in an AOS with completely plannable realizations the set $X_i(r_i)$ of realizable plans of each AE coincides with the corresponding set $Y_i(r_i)$ of possible realizations: $X_i(r_i) = Y_i(r_i)$ ($i \in I$). Analogously, for the set $X(r)$ of realizable plans of the whole AOS: $X(r) = Y(r)$.

For the sake of simplicity we will henceforth assume that for each AE the set B_i depends on the plan $x_i : x_i = B_i(x_i, r_i)$ only. The problem statement and the results outlined below generalize without difficulty to the case when sets B_i depend on both the plan x_i as well as the control λ: $B_i(x_i, \lambda, r_i)$. Concerning the sets $B_i(x_i, r_i)$ it is natural to require fulfillment of the following condition: if $x_i \in X_i(r_i)$, then $x_i \in B_i(x_i, r_i)$, i.e., if plan x_i is realizable, then the set $B_i(x_i, r_i)$ contains the realization $y_i = x_i$.

Suppose that under the definition of plan the control C implements a certain procedure for determining the estimates $s=\{s_i\}$ of parameters $r=\{r_i\}$ comprised in the models of the corresponding AE. It can be well-recognized that $s\in\Omega$, but in the general case $s\neq r$. The control law in the AOS can now be defined as the mapping $s\xrightarrow{\pi}\lambda$, $x:\pi(s)=(x(s),\lambda(s))$.

Criterial control. The right of decision making attributed to AE causes to the latter to adopt targets of their own. This circumstance is reflected by the introduction for each AE of a target function $W_i = f_i(\lambda, x_i, y_i)$ (for example, as in the case of the C, the economic utility function). The action of the C regarding forming or changing target functions of an AE is called *criterial control* [32-33].

The phenomenon of individual targets in the AE can lead to a situation where the realization y_i chosen by an AE may not coincide with the corresponding plan x_i. We will assume that the AE is penalized when the plan and the realization do not coincide. Formally this can be reflected by the following constraint on the AE 's target function:

$$f_i(\lambda, x_i, y_i) < f_i(\lambda, y_i, y_i),\ \text{if}\ \ x_i\neq y_i,\ i\in I, \qquad (13.1.1)$$

i.e., for a given realization y_i the value of the target function of the i-th AE is maximal if the realization y_i was planned for. An analogous condition holds for the target function of the whole AOS:

$$\Phi(\lambda, x, y) < \Phi(\lambda, y, y),\ \text{if}\ \ x\neq y. \qquad (13.1.2)$$

In practice the possibility of forming the target functions of the elements is connected with the possibility of determining a payment system, of introducing penalties and encouragements, of organizing competitions, and of making awards depending upon the position occupied, etc., which in the economic interpretation corresponds to creating a motivation system.

Functioning mechanism of an AOS. This is said to be realizable if any set of locally admissible realizations of the AE satisfies global constraints, i.e., $\forall S\in\Omega$, $\forall y_i\in B_i(x_i(s), r_i)$, $i\in I: y=\{y_i\}\in Y^g(r)$. A sufficient condition for the realizability of an AOS 's functioning mechanism is the condition of independence of the system's elements, namely

$$\forall s\in\Omega:\sum_{i=1}^{n}B_i(x_i(s), r_i)\subset Y(r). \qquad (13.1.3)$$

Indeed, in this case any choice of realizations $y_i\in B_i(x_i(s), r_i)$, $i\in I$, yields a realization $y\in Y(r)$ of the system. Let us denote the set of plans x satisfying (13.1.3) by

Z. It can be well-recognized that in a system with independent elements the planning procedure of the C must be such that the plans obtained belong to Z: $x \in Z$. The independence of the system's elements in the sense indicated, as a rule, simplifies the investigation and utilization of the control in the system.

13.1.4 Accounting for the future in AE effectiveness criteria

The target function $W_i = f_i(\lambda, x_i, y_i)$ introduced in *13.1.3* allows us to formalize the presence of a target in the case when the AE attempts optimizing its own utility function only within the functioning period being examined' without accounting for the future consequences of the decisions made "today". This is justified if the decisions made in a given period of functioning do not affect future periods of functioning (more precisely, do not affect the plan x_i, the control λ, and the set $B_i(x_i, r_i)$ of possible realizations in future periods). However, if such an influence does exist, then it is natural to accept that the AE predicts the consequences of the decisions made (another property of "activeness"). A good illustration to the latter statement boils down to the "planning from achievement" principle well known in economics, when the production output of an enterprise within a given period influences the plans for future periods. Under these conditions it may turn out advisable for the enterprises to reduce their work effectiveness "today" so as to ensure advantageous work conditions "tomorrow". The method and the extent of accounting for the future for the various elements are determined mainly by subjective characteristics of the managers. The function reflecting the subjective target of an AE within a given functioning period with due regard to future periods will be called the effectiveness criterion of the AE, allowing for the future functioning periods in contrast to the target function f_i determining the economic effect of the AE only in the "current" period. Thus, the effectiveness criterion of the AE, taking into account the k-th functioning period of N_i future periods, may be determined as

$$W_i^k = f_i\left(\lambda^k, x_i^k, y_i^k\right) + \sum_{q=k+1}^{K+N_i} f_i\left(\lambda^q, x_i^q, y_i^q\right). \tag{13.1.4}$$

Parameter N_i is referred to as the "degree of foresight" of the i-th AE.

Another form of reflecting the future might be presentation of the effectiveness criterion for the AE as a sum of the element's target function in the current period and of the weighted sum of the element's target functions within succeeding periods:

$$W_i^k = f_i\left(\lambda^k, x_i^k, y_i^k\right) + \sum_{q=k+1}^{\infty} \delta_i^{q-1} f_i\left(\lambda^q, x_i^q, y_i^q\right). \tag{13.1.5}$$

From the principal point of view one can admit the case when summarizing in the criterion extends only over N_i succeeding periods. The coefficient δ_i, often referred to in economic research as the "discount coefficient", characterizes the degree of foresight of the element. It is usually assumed that $0 < \delta_i < 1$. The peculiarity of effectiveness criterion (13.1.5) stems from its sliding nature. Indeed, one can well-recognize that $W_i^k = f_i(\lambda^k, x_i^k, y_i^k) + \delta_i \cdot W_i^{k+1}$, i.e., the effectiveness criterion in period k is the sum of the target function in period k and the weighted effectiveness criterion of the next period.

Other methods for accounting for the future may be suggested [34-35]. Let us emphasize only the following important detail. The Center does not know the AE 's effectiveness criterion even if it knows the form of its target function within an individual functioning period. The difficulties in determining the extent of allowing for the future in the effectiveness criterion of the elements concerns not only the C but also the AE themselves, since the prediction of the consequences of the decisions made is a rather complex problem. Furthermore, the extent of allowing for the future in the effectiveness criteria of the AE may change from one functioning period to another. Therefore, a serious requirement on the control law is the independence (or weak dependence) of the behavior of an element (the decision-making principle) from (on) the method of taking the future into account in the element's effectiveness criterion.

Let there be given a model of an AOS and its functioning mechanism. It can be well-recognized that the functioning of such an AOS consists of separate periods. Each period includes three stages: formation of the estimates, planning, and realization of the plan. At the stage of forming the estimates the C determines the estimate $s = \{s_i\}$ of parameters $r = \{r_i\}$. At the planning stage the C determines the control $\lambda(s)$ and the plan $x(s)$ of the AOS by the control law $\pi(s)$ and communicates them to the AE. At the realization stage each AE chooses a realization $y_i \in B_i(x_i(s), r_i)$, $i \in I$, after which the achieved value of the target functions of the elements and of the Center are determined.

Let us enumerate properties of the "activeness" of the organizational subsystems formalized in the description of an active element:

a) the presence of a purpose and the accounting for future consequences of the decisions made. Formally this property is reflected by the fact that the expression for the effectiveness criterion for a given period incorporates target functions of future periods;
b) a definite freedom of action in communicating information and on realization of plans. Indeed, each element can communicate any estimate s_i from set Ω_i and choose any realization y_i from the appropriate set $B_i(x_i(s), r_i)$;
c) the knowledge of the structure and of the functioning mechanism of the system.

The AOS described is in fact a multicriterion system in which both the Center as well as the elements have the right to take independent decisions (the C can form or change the functioning mechanism of the AOS). The situation is therefore of a conflict (game) nature and calls for a game-related theoretic approach.

§13.2 Game concepts in active organization systems

13.2.1 Game methods

A basic feature of most two-level management systems is the human presence on both the upper and the lower levels, and hence the presence (in the general case) of non-coinciding objectives at the Center and at the elements of the lower level. In these systems both C and AE have certain possibilities to influence by their actions the values of their objective functions. For C these possibilities boil down to formation of a certain mechanism of operating the system. Elements can influence the values of their objective functions by communicating to C the information and the choice of their state. As mentioned above, in §13.1, this is clearly a game situation, and it should be investigated by methods of the game theory. However, as noted by various investigators (see, e.g., [32, 36]), there are certain difficulties in directly applying the most developed section of the game theory usually referred to as the "classical theory of games" [32, 176]. Let us note that the "classical theory of games" does not take into account various possibilities of the players depending on the sequence of their moves, as well as the possibility of repetition of the plays of a game; the idea the players have about their game is not presented, neither the information they possess during the game, neither the effect of this information on the possibilities of the players. The analysis of equilibrium situations according to Nash is often unfounded without relating it to the cooperative actions of the players; in cooperative "classical theory" the presence of several types of cooperative actions is not taken into account, i.e., the exchange of information between players and the joint choice of strategies, as well as combining of resources. Consideration of this range of problems became known by the name "games with non-antagonistic interests" [32-38]. Games that have been studied in greater detail are games of two players with non-antagonistic interests. For higher dimensions (three- and n-person games) the possibilities of cooperation between players are examined, as well as the degree of information of the first player with regard to the principles of cooperation and the possibility of its breaking up. The obtained results are a generalization of the solution of a two-person game with a fixed sequence of moves [32-33, 38].

In accordance to this, a game is considered from the viewpoint of one of the players with whom we associate the side that operates in the game (in two-level system the Center is the operating side). Such an approach makes it possible to investigate a game by the methodology of operations research [32, 36, 176]. The description of the game contains descriptions of the efficiency criteria and of the constraints imposed on the players, as well as of the indeterminacies occurring within the game. There exist indeterminate factors with fixed distribution laws, aside indeterminate factors for which only their domain of variation is known. In the latter case they can be grouped into indeterminate factors that are due to insufficient study of certain processes or quantities, indeterminate factors that account for imprecise knowledge of the aim of the operation or of the efficiency criterion, and indeterminate factors that are due to the presence of objects that are acting to a certain extent independently of the operating side and which pursue their own aims.

Therefore, a game can be described in two manners, i.e., we can have an objective description usually not exactly known to the players, and a subjective description corresponding to the information available to each player with regard to the game and the interests and possibilities of the other players [35-36].

The possibilities of the players in a game can b e described as follows. The players may select their own strategies from the set of allowed strategies. It is essential that the strategy of a player can be constructed as a function of the available information about the strategies of other players. The players are also permitted to form coalitions:

a) by exchanging information;
b) by taking joint action in cooperating according to a criterion with certain rules of sharing the common profit [32-33];
c) by combining available resources.

In games with non-antagonistic interests the players choose their strategies by implementing principles of selecting efficient strategies. Most common examples of such strategies are [32-36, 38]:

- optimization after averaging on the basis of random events;
- various modifications of the principle of guaranteed output as a function of the sequence of moves and the information available to the players;
- "absolutely" optimal strategies;
- cooperative principles of selecting an equilibrium; and
- cooperative actions.

The set of solutions of a game is defined as the set of situations that can be selected by the players on the basis of the above principles of selecting efficient strategies adopted by them. In various cases the operation of hierarchical systems is related to the repeatability of the same situations encountered by the system. In a game-theoretical analysis of the operation of hierarchical systems, it is therefore necessary to take into account the repeatability of the plays of a game. In particular, by taking into account the repeatability of a play, it is necessary to include into the set of principles of selecting efficient strategies by the players also the possibility of utilizing information obtained from the previous plays of the game. Thus it may happen that the set of solutions of a particular game can be different in different plays. To overcome this difficulty it is therefore convenient to introduce under such circumstances the concept of [32-36, 38]:

- the set of stable solutions of a game;
- the set of globally stable solutions of a game.

In the theory of games with non-antagonistic interests it is important to specify the advantages of the operating side in a game. With regard to two-level systems this makes it possible to express formally the "priority of the actions of C" considered in papers on hierarchical systems [36]. The right of first move of the operating side and knowledge of the interests and principles of selecting efficient strategies by other players in a game make it possible for the operating side to determine more exact estimates of its criterion of efficiency as compared to the classical maximin (the principle of guaranteed output for the player making the first move). A similar principle has been proposed [36] in formulating the problem of selecting (by C) the control law in an active system. In this case the principle of guaranteed output for the player making the first move presupposes the determination of the optimal guaranteeing strategy of the operating side not on the set of solutions of the game by other players in each play, but on the set of globally stable solutions of the game by other players; the latter has been one of the basic factors in introducing the term "metagame control" [36].

Game models that have been studied in greater detail were those in which it is assumed that the hypothesis of indicator behavior of the elements is satisfied (see, e.g., [141]). Corresponding researches deal with problems of existence of a stable equilibrium, its uniqueness, and global stability. It can be well-recognized that equilibrium situations in the sense of Nash are stable equilibrium situations in the case of an indicator behavior. In such cases it is not assumed that there are coalitions in the actions of the players when Nash equilibrium is reached, this being in contrast to the assumption made in games without repetition of plays. Experimental results obtained in the area of business games [32] support the assumption of an indicator behavior in a number of models.

When the repeatability of the plays of a game is taken into account, we discover yet another basic feature, i.e., the criteria of efficiency of the players can be determined not only by their win in the current play, but also by their wins in future plays of the game (i.e., the players have foresight - see, e.g., [32, 35-36]). The need to account for the foresight occurs, for example, in situations when a decision is taken for a planning period of infinite length [35-36], as well as in cases when the procedure of data compilation uses adaptive or combined methods. Various game models taking into account the foresight of the players have been constructed, accompanied by examining the possibility of ensuring stable coalitions of players when the plays of a game are repeated [35-36].

13.2.2 Game-theoretical formulation of control problems in hierarchical systems

In this Section we will consider the analysis and synthesis of operating mechanisms. In general, researches dealing with the formulation of control problems in two-level systems can be divided into two groups. The first group includes researches in which control problems are formulated and studied within the "framework" of a game-theoretical model (pure game-theoretical formulations) [35-36]. In these studies the problem of search for an optimal "control" strategy of C is formulated as a problem of determining, in a game without repeated plays, the optimal guaranteeing strategy of the player performing the first move, on the set of solutions of the game by other players. To "game-theoretical formulations" there belong also models of collective behavior considered in [35-36]. Problems considered in these papers can be regarded as game-theoretical problems of analyzing equilibrium situations in the case of an indicator behavior of the players in games with repeated plays.

The main attention in problems of the second group is drawn to taking into account the peculiar features of models and mechanisms of operation in two-level systems while utilizing (to a certain extent) game-theoretical concepts. These approaches include, among others, the following studies [32-38]:

- analysis of mechanisms of operation in a decentralized economy environment;
- analysis of equilibrium states in a production network;
- models of collective behavior of automata;
- various results on iterative planning and control;
- developments on information theory of hierarchical systems dealing with various problems of analysis and synthesis of mechanisms of operation without organizing the procedure of data compilation;
- analysis and synthesis of mechanisms of operation in active systems.

Problems considered in the outlined above second group of researches can be divided into problems of, on one hand, analysis, and on the other hand, synthesis of mechanisms of operation. Such a division is quite obvious and reasonable. Analysis involves determination of the properties and conditions of applicability of a certain mechanism of operation. Synthesis suggests determination, within given constraints, of mechanisms of operation that either ensure the validity of certain properties or that are optimal in the sense of a certain criterion. It can be well-recognized that in the final resort problems considered in these papers can be formulated also within the framework of a general "pure-game-theoretical" model of games with non-antagonistic interests. However, precisely by taking into account the specific features of a model of a two-level system and of its mechanism of operation, it is possible to obtain additional constructive results. Although the boundary between these groups of formulations is sometimes imprecise and arbitrary, such a division may nevertheless be useful for additionally characterizing them.

§13.3 Using active systems theory for designing new organization systems

13.3.1 The basis of designing a complex object

Consider the process of designing a complex object which can be regarded as an organization system (OS). From the system approach viewpoint, designing boils down to the model constructed and formulated in a predetermined language, e.g., when dealing with structural design this would be the language of technical drawings, diagrams, etc. For technological design, a formulated model can be described in terms of a process or control program for numerical programmed control equipment.

Application of the term "model" suggests that the process of object (system) design does not provide the whole variety of features and parameters which can exactly and fully describe the regarded object. What is created as a result of the above process is the construction abstracted and approximated with the required (assigned) completeness degree. Such an approach favors the analysis of computer-aided design (CAD) for multilevel modeling. Thus, a number of conditions should be fulfilled in designing complex objects.

The first condition boils down to the presence of a language describing the application domain (designed models). The second condition is the presence of knowledge containing the analysis and synthesis methods for a hierarchical design process and type (base) design elements. In our case these are base elements of an OS used to "assemble" the required design. The third condition requires presence of instruments for experimental checks and estimating the developed model of the OS. It can be well-recognized that the active systems theory most efficiently complies with the first and the second conditions, while instruments of business games and game simulation experiments comply best with the third condition.

The main subject of study in the active systems theory is an OS (mechanism or functioning), and the problem of creating efficient (optimal) mechanisms of functioning becomes the main problem of active systems theory. Significantly, the active systems theory language is rather close to the general language of the application domain, which on the one hand facilitates utilization of experts' (experienced managers') knowledge, and on the other hand assists in adaptation of developed designs for real-life OS.

As for business games and game simulation experiments, taking into account the "human factor" in social and economic systems, one can hardly find a more efficient method of checking the OS design than game simulation experiments (business games) with the participation of people as the system active elements.

13.3.2 Design stages

It is necessary to point out a number of stages and levels in the process of OS design. In the first stage the object position in the external medium is determined, and the system relations and characteristics, serving as the criteria for the created object, are established (determined). In the second design stage the object is decomposed into a complex of simpler elements by structural and functional features.

Proceeding from the decomposition on the basis of requirements and internal object relations determined in the first stage, the object elements criteria are determined. The first level decomposition elements can be decomposed into simpler parts on the second level. The second level decomposition criteria are determined similarly to those of the first level. The total number of decomposition levels depends on the complexity of a particular designed object.

The above two stages may be considered as the pre-design stage. As a matter of fact, the design proper starts with determining the hierarchical set of design levels. This is the development of the object concept, when the main features are established, and the most important criteria are taken into account. A model design level is the next stage, on which the elements, models and dynamics of their interaction are formed. On this level the designed object concept can be checked, and base elements can be chosen from the available set, if the established main criteria do not permit unique preference. In case of a negative result being obtained during model experiments, the designer returns to the previous level to develop a different approach (concept).

The next level is a draft design on which a more detailed description of the models and elements integrated into the object is obtained, with allowance for a more complete set of criteria. In case when the established requirements cannot be complied with, the model design level is revised.

The last level is that of the detailed design. On this level the designer formalizes the full description of the object in terms of the user language, and represents the dynamics of its behavior according to the state of the external medium. Note that, depending on the object complexity, some of the above design stages may coincide.

Generally, as already outlined in §13.2, it can be considered that designing a complex object consists of two main alternating phases: analysis and synthesis. A certain element, synthesized on the lower design level, is analyzed on the higher one and/or included into the designed object, or rejected and returned to the lower level for re-synthesizing. Thus, any complex object or system can be decomposed into some finite elements set definable as the base on. In addition, the base elements set is constantly refilled as the knowledge and social demands develop. Similarly, the design process can be represented in the form of a finite number of procedures. Naturally, they are specific at the particular design level. A set of these procedures is formalized is formalized in the form in the form of structural models, methods, etc. A formalized language for describing the base elements and design procedures provides prerequisites for CAD. The particular implementation of the CAD process depends on the available hardware and design level.

Thus, the enhancement of OS design requires implementing a complex of theoretical (conceptual and methodological), instrumental and technical means providing (in a dialog mode) development of the concept, draft and detailed designs of the OS. An important component of this complex includes service instruments supporting and renewing the libraries of base elements, as well as methods of synthesis and analysis. Our further discussion considers the main elements of the support complex for designing OS comprising the first two design levels, namely, conceptual and draft designs.

13.3.3 The design of organization mechanisms

Let us consider the idea of an organization mechanism (OM) within the scope of the active systems theory. On the general level, an OM can be described as a set of procedures, statements and rules (both formal and informal), regulating people's activities in the organization. Such a qualitative definition classifies an OM among complex systems with determinate behavior. The analysis and synthesis of the mechanism of functioning, as a complex system, requires decomposition into elements. A complex multilevel system can be described as a union of two-level blocks or elementary control loops. Each elementary control loop has its own mechanism. Further on we consider the problems of designing mechanisms for two-level blocks which can be used for synthesizing a complex system [37] (see Fig. 13.3).

The next decomposition stage is performed by the functions of the mechanism elements. Here, we first single out the functions of control and aim definition which are implemented by the control and aim definition mechanisms, respectively. The control mechanism, in its turn, is decomposed into a number of elementary blocks (sub-mechanisms) according to the main control functions. And, as a rule, on the conceptual control level a simplified variant of decomposition (i.e., into three blocks) is used:

- generation of data for decision making;
- planning (decision making);
- using incentives.

In the draft design stage the decomposition is more detailed and, generally, it corresponds to the classification of control functions adopted in economic literature (see Fig. 13.4):

- data generation;
- forecasting and planning;
- account and control;
- analysis and estimation;
- use of incentives.

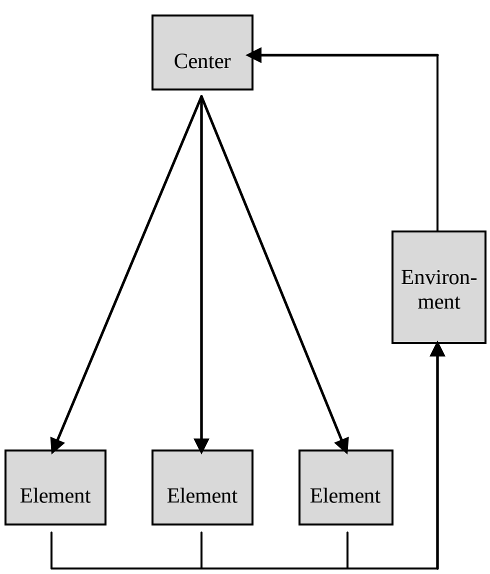

Figure 13.3. Complex system synthesis

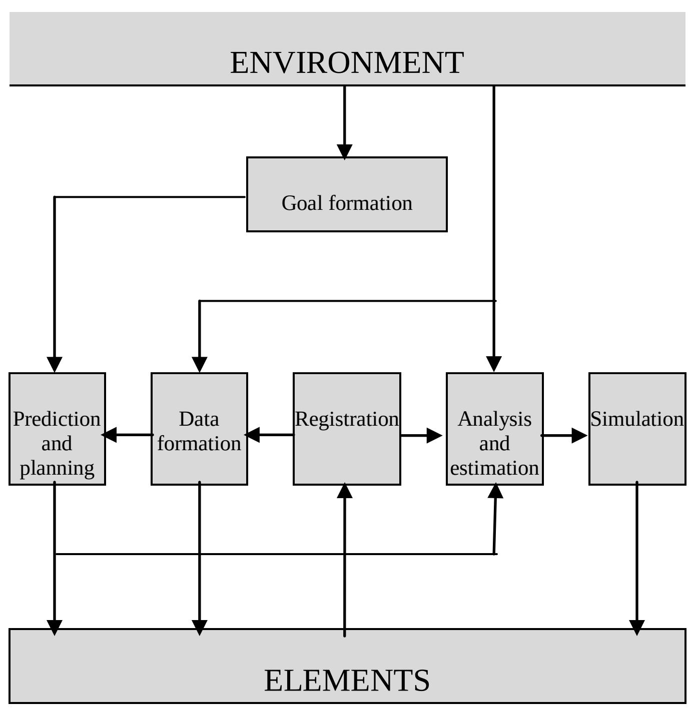

Figure 13.4. Detailed decomposition modules

For the formal description of OMs within the scope of the active systems theory let us introduce the following terms:

$s = \{s_i\}$ - information generated by the Center regarding the system's elements and external medium;

$y = \{y_i\}$ - system state as a set of measurable characteristics significant from the point of view of the considered design;

$x = \{x_i\}$ - plan in terms of the value (desirable for the Center) of all or some parameters of the elements state (or aggregates of these parameters);

$St = \{St_i\}$ - value of the incentive (material or other, but anyway measurable) received by the element.

In the above-introduced language the control mechanism design on the conceptual level can be represented as the set of mappings:

$$x_i = \pi(s); \quad St_i = f_i(x_i, y_i); \quad i = 1,2,...,n.$$

Assume that each element informs the Center about estimates s of the required parameters. Let r denote the true value of these parameters, and $Y_i(r_i)$ the set of possible states of the i-th active element. Next assume that in choosing y from $Y(r)$ the element maximizes the value of incentives. Denote the maximum incentive for plan x by $\varphi(x,s)$. Function $\varphi(x,s)$, estimating the incentive value by information, is called *the element preference function*.

The authors do not seek to further decompose the formal description of control and OMs. Rather, full formal description can be found in [32-34]. Significantly, a detailed formalization has been developed for the following base classes of OMs, making the knowledge base of the considered design support complex:

a) "fair" game mechanisms;
b) coordinated mechanisms;
c) competitive mechanisms;
d) counter-expense mechanisms.

Let us dwell on the procedures of state and activity estimation, the necessary blocks of any OM. Therefore, the challenging problem here is the development of a flexible (universal) subsystem, easily adjustable both for the objectives of the Center and elements indices. This requirement is fulfilled by the AQCEAR system (Automated Qualitative Complex Estimation of Activity Results) and its modified version, AQCES, both being outlined in [37].

As mentioned above, the business games complex and game simulation experiments are the best instrument for the study and analysis of both conceptual and model designs of OMs. A set of simple business games has been developed [37] for studying OMs on the conceptual design level. These games are: “Resource”, “Plan”, “Payment a Team”, etc.

In studying draft design it is expedient to use the complex experiment scheme. According to the scheme, by means of a relatively simple business game it is possible to test the justification of hypothesis regarding the active elements’ (players’) behavior in a complex simulation model of an OM. The justified hypotheses are used to adjust robots’ (artificial players’) behavior in a complex simulation model of an OM. The simulation experiment is carried out on robots, which considerably reduces the experiment’s duration. Methods of planning such complex experiments have been developed on the basis of the experiment planning theory.

Thus, we have touched upon the prerequisites for developing a support complex of OM design. Let us enumerate the formalized modules which should be included in this complex and used during its operation:

1) library of OM’s base blocks;
2) adjustable system of complex estimation and analysis;
3) formalized procedure of carrying out game simulation experiments;
4) formalized analysis and synthesis procedure of the mechanism;
5) means of supporting the OM’s libraries, as well as libraries of analysis and synthesis methods;
6) means of interaction with the user during design synthesis and analysis.

13.3.4 The support complex for organization mechanisms design

The support complex for OMs design is outlined in [37].

The functional structure consists of two macro modules: a macro module of OM synthesis, and that of OM model analysis, both connected to the knowledge base and data base. The knowledge base consists of the set of algorithmic base elements of the OM, which can be complemented according to the established standards. The base elements library is complemented by the support block, which enters the necessary parameters into descriptions and catalogues, thus causing new elements synthesized. The database contains necessary information arrays of normative, reference and on-line data required for the information-reference support of synthesis and analysis.

The macro module of the OM model synthesis opens the available OM base elements in a dialog mode, and provides for linking them into the final model and requesting the data necessary for the experiment. The required data can be generated or obtained from the database by user’s instruction. In this concept, the OM model is formed as a two-level fan-type hierarchical structure [37]. The OM model analysis macro module provides for conducting game simulation experiments with a synthesized model of the mechanism. The user may choose models of automatic devices behavior, and play business games with the participation of human partners.

As an example of the operation of a design support complex for OMs, consider one of its first versions called "Constructor". The Constructor system is supported by the realized program blocks of the knowledge base:

- tax systems (constant, progressive, counter-expense);
- planning procedures;
- estimate generation procedures.

When interacting with the Constructor system, the user has a sequential access to two operation modes: OM model synthesis, and game simulation experiments. The system software is realized on the menu set, from which the necessary mode is chosen.

In the first step, using a dialog with the computer, the designer determines the number of active elements participating in the competition. Then restrictions on the set of possible states of the elements and system are imposed. Those restrictions can be both assigned from the terminal or chosen from the database. The next stage is the determination of the elements and system objective functions. As mentioned above, the user enters them from the available menu. After forming the structure and aims of the OS, the Constructer requests the necessary information for the control mechanism. The procedure is chosen from the class of competitive base mechanisms, and the incentive model is created by determining the tax system. Significantly, the tax scale generation and calculation program, included in the design support complex, provides for performing test calculations and conducting the behavior analysis under various taxation scenario. After determining the control mechanism, the Constructor system requests information regarding the type of active elements (people or robots). Providing answers to the requests accomplishes the OM model synthesis stage.

On the next stage the Constructor system proceeds with simulation experimentation on the model. The experiment in business game form provides for the participation of both human and artificial players. During the game, human players enter the desired prices for execution of the distributed designs, and obtain distribution results in tabular form by the established procedures. The objective functions value is generated. Multi-repeated game experiments provide the necessary data for the estimation hypotheses and the rationality of artificial players' behavior, at the same time enabling verifying the correctness of competitive distribution procedures.

§13.4 Active systems in stochastic project management

It can be well-recognized from §§8.1-8.2 that several projects at the lower level, managed simultaneously by a design office at the upper level, form a truly active system. Indeed, each project's team tries to gain as much as possible even when honoring the management's policy. After the project is realized and finished, the project's team may obtain several kinds of awards, namely,

- the award for high project's utility, and

- the award for delivering the product to the market in time, i.e., before the due date. The award value depends on the product's importance and on the situation on the market. Thus, the due date may undergo drastic changes.

As outlined in §§8.1-8.2, an R&D design office governs various resources (mostly financial ones) by means of distributing (and redistributing, if necessary) the latter among the projects to be realized simultaneously. This fully refers to PERT-COST projects which have been considered in the previous chapters.

Let us consider a particular case (from many other realistic ones) of the projects' competition [30]. Assume that a projects' office at the upper level is in charge of several simultaneously realized PERT-COST type projects $G_i(N,A)$, $1 \le i \le n$, at the lower level. Projects G_i are of different importance and have different priority indices η_i. Assume, further, that at the end of all projects' realization the project management (design office level) receives two kinds of awards: $Aw\ I$ and $Aw\ II$:

- the award for the portfolio's utility is calculated by

  $$Aw\ I = U_s \cdot \gamma_s, \qquad (13.4.1)$$

  where U_s is determined by (8.2.7), and γ_s is pregiven;

- the award for preserving the total office budget is calculated by

  $$Aw\ II = (C_0 - C) \cdot \beta_s. \qquad (13.4.2)$$

Here C is the budget which has been actually spent within the project's realization, and C_0 is the minimal budget for realizing the projects' portfolio determined at moment $t = 0$ by the algorithm outlined in *Section 8.2.5*. Index β_s, similarly to γ_s, is also pregiven. It can be well-recognized that part $Aw\ II$ may become negative.

As to projects G_i at the lower level, they, when finished, also obtain two kinds of awards, namely $Aw_i\ I$ and $Aw_i\ II$, $1 \le i \le n$, as follows:

- award $Aw_i\ I$ depends on the project's utility U_i and is calculated as follows

  $$Aw_i\ I = U_i \cdot \gamma_i,\ 1 \le i \le n, \qquad (13.4.3)$$

  where U_i is determined by (8.2.7), and γ_i is pregiven;

- value $Aw_i\ II$ is the award for delivering the product developed by project G_i (call it henceforth $\mathrm{Pr}od_i$) to the market in order to meet the due date D_i. Value $Aw_i\ II$ is calculated by

  $$Aw_i\ II = L_i + \beta_i \cdot (D_i - F_i), \qquad (13.4.4)$$

where L_i is a constant pregiven value, F_i is the *actual* moment project G_i terminates, and value β_i is pregiven. It can be well-recognized that award $Aw_i\ II$ increases by decreasing F_i, and vice-versa.

Thus, the total award for the design office calculated at moment $t=0$, i.e., at the starting moment for the projects to be realized, is

$$Aw_s = U_s \cdot \gamma_s + (C_0 - C)\cdot \beta_s, \tag{13.4.5}$$

while for each project G_i, $1 \le i \le n$, the total award (at $t=0$ as well) is

$$Aw_i = U_i \cdot \gamma_i + L_i + \beta_i \cdot (D_i - F_i), \tag{13.4.6}$$

Assume, further, that at moment $t=0$ all the information about projects $G_i(N,A)$, $1 \le i \le n$, is available. Thus, for a more or less regular situation at the products' market, we start realizing projects G_i by means of determining the minimal budget C_0 and re-distributing the latter among the projects. Value C_0 enables solving problem (8.2.7-8.2.16) for projects with different priorities together with calculating the forecast utility value for the whole portfolio, as well as local utility values for each project $G_i(N,A)$, $1 \le i \le n$. After calculating values C_i, together with pregiven values D_i and p_i, the projects start functioning.

Assume, in addition, that a certain moment $t>0$ it is recognized that for some or another reason for one of the projects G_ε, $1 \le \varepsilon \le n$, its output product $\mathrm{Pr}od_\varepsilon$ is greatly anticipated on the market. It is announced therefore that for each time unit before the due date D_ε the project, when accomplished, will obtain a very high additional award Aw_ε^*. Under such circumstances project G_ε will surely apply to the upper level in order to decrease the due date D_ε as much as possible, together with increasing the project's index η_ε(also as much as possible!). It goes without saying that the design office has to pay attention to such an application and to undertake decision-making in order to "harmonize" the benefits for the projects entering the portfolio. We suggest the following harmonization procedure:

13.4.1 Procedure for a project portfolio without loosers

The enlarged step-by-step procedure is as follows:

Step 1. For each project G_i, $1 \le i \le n$, calculate both awards $Aw_i I$ and $Aw_i II$ on the basis of the results obtained when implementing algorithm outlined in *Section 8.2.5* at $t=0$. Values $Aw_i I$ are obtained by taking into account calculated values U_i obtained by (8.2.7-8.2.17). Values $Aw_i II$ are obtained by using assumptions $F_i = D_i$, $1 \le i \le n$. After determining values $Aw_i I, II$ both awards are summarized to obtain a forecasting value $Aw_i^o = U_i \gamma_i + L_i$ for each project G_i, $1 \le i \le n$, at $t = 0$.

Step 2. For the project management design office (upper level) calculate value $AwII$ by (13.4.2) on condition that relation $C = C_0$ will hold within the projects' realization.

Step 3. Resolve the initial problem (8.2.7-8.2.16) with new priority indices η_i and new due dates D_i (for project G_ε). Take into account that all projects G_i, $1 \le i \le n$, at moment $t = 0$ are partially realized and, thus, have to be inspected and updated. Calculate by means of inspection the total budget already spent at moment t for all projects. Call it henceforth $\overline{C}$. Determine by means of the algorithm outlined in Section 5 the minimal budget to carry out the remaining updated projects at moment t with new amended indices η_i and D_i. Call that minimal budget C_t^*.

Step 4. Calculate at moment t the newly determined utility values U_{ti} for each updated project G_i, $1 \le i \le n$, taking into account that for G_ε the updated parameters are as follows:

- total value C^* is reallocated among the updated projects according to §§8.1-8.5, and for each project G_i its budget is henceforth called C_{ti}^*, $\sum_{i=1}^{n} C_{ti}^* = C_t^*$;
- due dates D_i remain the same, besides for project G_ε, where D_ε has to be diminished to $D_{\varepsilon t} < D_\varepsilon$;
- indices η_i remain the same, besides for project G_ε, where η_ε has to be increased to $\eta_{\varepsilon t} > \eta_\varepsilon$;

Step 5. Calculate at moment t the corrected award values Aw_{ti} for each project G_i, including for project G_ε the additional award $(D_\varepsilon - D_{\varepsilon t}) \cdot Aw_\varepsilon^*$. Determining Aw_{ti} is carried out in Step 1.

Step 6. Calculate at moment t the newly corrected award for the design office

$$Aw_{ts} = \gamma_s \cdot \sum_{i=1}^{n} U_{ti} + \beta_s \cdot \left(C_0 - \overline{C} - \overline{\overline{C}}_t \right), \qquad (13.4.7)$$

where $\overline{\overline{C}}_t$ denotes the budget actually spent within the period starting from t until the end of the projects' realization.

Step 7. Calculate for each project G_i, $1 \le i \le n$, the corresponding award differences $Aw_{ti} - Aw_i = \Delta_{ti}$. Note that $\Delta_{t\varepsilon}$ will be very high while for some other projects the difference Δ_{ti} may be negative.

Step 8. Calculate the award difference for the design office
$\Delta_{ts} = Aw_{ts} - Aw_s$.

We suggest the following decision-making for the policy with no loosers:

Project G_{ε} *has to transfer from its award value the partial budget to fill up the losses for all projects* G_i, $i \neq \varepsilon$, *with negative values* Δ_{ti}, *as well as for the design office value* Δ_{ts} *in case* $\Delta_{ts} < 0$.

As a matter of fact, many other human trade-off decision-makings may be considered and discussed as well. It can be well-recognized that the developed research fully complies with the methodological grounds outlined in §13.2, namely, with non-antagonistic games the projects are "playing" by means of human personnel behavior. We will consider AOS later on, in Chapter 17, in the course of monitoring hierarchical stochastic project management systems.

Chapter 14. Harmonization Models in Multi-Attribute and Compound Systems

§14.1 Using multi-attribute harmonization models in strategic management

14.1.1 *Introduction*

It can be well-recognized (see, e.g., Chapter 5 and [125-127, 168]) that existing quality techniques in various organization systems (OS) are restricted to market competitive problems only. Those techniques usually center on analyzing the competitive quality of organization systems' output products in order to gain future commercial success. But in that capacity they ignore the quality of the OS functioning, e.g., the quality of designing and creating a new unique product. In Chapters 5, 8-10 we presented several new utility models to estimate the OS's quality in the course of its operation. However, if the company is engaged in designing and creating a new product and, later on, delivering the latter in large quantities to the market, the product's life cycle continues far beyond terminating the design process. Thus, developing quality models of organization systems in their entirety centers on developing new utility models comprising design and production phases as well as the system divestment phase. Nowadays engineering projects where a new system or product are being designed, developed, manufactured and continuously quality tested, may span years, as applicable for the case of a new automobile, or over a decade for a nuclear power plant [168]. New product development takes anywhere from several months to several years. In lengthy processes of this type, decisions made at the outset may have substantial, long-term effects that are usually difficult to forecast. *The trade-off between current objectives and long-term consequences of each decision is a strategic aspect [168] of project management. Thus, the research to be considered refers to strategic management and deals with the most important aspects of that area.* Moreover, special attention is drawn in literature to life cycle costing (see, e.g., [49]) in case a decision having long-term effects deals with *selection of components and parts for a new system or a new product* at the advanced development and detailed design phase.

Thus, an evident conclusion can be drawn that the newly developed utility harmonization models must deal not only with the quality of OS functioning, but with the quality of outcome products as well, especially when subject to severe market competition. In the current Chapter an attempt will be made [14] to enhance utility models in order to cover the whole life cycle of the regarded product. We suggest developing a more generalized utility harmonization model by implementing in the latter the most essential aspects of the MAUT theory [125-127, 150, 168]. The generalized harmonization model should, thus, be applied to all stages of the product life cycle, namely:

1) MAUT multi-attribute models have to undergo certain modifications to be used at the stage of designing the outcome product, on the basis of experts' decision-making in combination with proper ranking and scaling.
2) Analytical and simulation models which have been already incorporated in partial harmonization algorithms (PHM) in safety engineering and project management [8-9, 14, 26], should be applied at the stages of designing and creating the new device.
3) We suggest linking the outlined above MAUT and PHM together in order to optimize the process of designing and creating a new product within its entire life cycle. The suggested optimization algorithm should be of mixed type, i.e., to comprise both analytical calculations and man-computer dialogs at the stage of decision-making on the basis of experts' interviews. *The backbone of the optimization algorithm is that all its elements (including analytical calculations and dialogs with experts) are parts of a generalized search procedure to determine the optimal version of the product to be designed and created.*

It is assumed that the product to be manufactured is composed of several subproducts, e.g., a new automobile comprises an engine, a bonnet, a brake system, etc. Each subproduct, in turn, is a subject of several possible versions. The problem is to determine for each subproduct their optimal versions in order to maximize the product's competitive utility subject to restrictions related to the design process. A two-level optimization algorithm based on the cyclic coordinate search algorithm (CCSA) (see §5.4), is suggested. The internal level is faced with optimizing the product's competitive utility by means of experts' information, while the external level centers on obtaining a routine feasible solution from the point of designing process [14, 26].

14.1.2 The problem's description and definitions

In order to formalize the harmonization problem we will require some new definitions.

Definitions

Call a quantitative parameter entering the project of designing and creating a new product, the *basic project attribute* (BPA) together with its corresponding restriction value. The latter serves as the *worst permissible value that may be implemented into the design project*. Several basic project attributes may be independent as well as dependent parameters. BPA restrictive values are already pregiven by the OS management. However, in the course of carrying out the design project, they may be subject to alterations, e.g., owing to changes of the product's demand on the market.

Call a quantitative parameter entering the output product, i.e., the designed product to be delivered to market, the *basic competitive attribute* (BCA). *BCA values actually form the product's competitive utility in order to gain future commercial success.* Those values are usually calculated by means of expert information.

As outlined above, the system under consideration comprises:

- the phase of the product's designing and creating the pattern example, and
- later on, the second phase related to delivering the product in large quantities to the market.

It can be well-recognized that both BPA and BCA values depend on the set of versions assigned to each subproduct. Note that BPA values are fully determined by the set of versions, i.e., those values can be calculated analytically or by means of simulation. BCA values are calculated through expert information taking into account the set of versions as well. Assume, further, that when benefiting from commercial success, the profit obtained from delivering the product to the market at the second phase, usually exceeds essentially the project's expenses to design the product at the first phase. Thus, *we suggest developing a multi-attribute utility value on the basis of only BCA values. This generalized value has to be maximized in the course of the suggested CCSA algorithm by means of information obtained from experts. As to BPA values, they have to be incorporated in the search procedure in order to satisfy the pregiven restrictions.*

Referring to MAUT models [125-127, 150, 168], we will assume that for each BCA value two opposite estimates have to be pregiven before carrying out the design process:

- the least preferred value having practically very poor chances to win the market competition, and
- the most preferred value which enables the attribute to win the competition.

Note that both opposite estimates for each competitive attribute can be obtained from the expert team on the basis of interview questions. Those estimates play the leading part in the process of questioning experts to obtain the multi-attribute utility values [7-9, 14, 26].

In the model under consideration we will develop a modification of the classical MAUT procedure of both questioning experts and obtaining utility values. This is because existing MAUT expert models cannot be incorporated in a search harmonization procedure.

14.1.3 The suggested expert interview procedure to calculate multi-attribute values

As outlined above, the suggested general idea to maximize the multi-attribute utility value centers on undertaking a search procedure in the multi-dimensional state of possible combinations - possible versions assigned to subproducts. The suggested search procedure is carried out by implementing the cyclic coordinate descent method where each i-th coordinate varies from 1 to r_i, r_i being the number of possible versions which can be assigned to the i-th subproduct.

Let the number of subproducts be q. Thus a routine search point is actually a q-dimensional vector $\vec{D} = \left(d_1, d_2, \ldots, d_q\right)$ with integer numbers. In the course of undertaking the search procedure vector $\vec{D}$ has to satisfy all pregiven BPA restrictions (let them be m), otherwise the routine search point is not considered. If all BPA restrictions are honored, search point $\vec{D}$ has to be passed and later on examined by a team of experts, by means of the following interview procedure. Let the expert team comprise f experts faced with the problem of decision-making on n competitive attributes. It is assumed that (before examining any routine point $\vec{D}$) for each basic competitive attribute BCA two opposite estimates: the least competitive and the most competitive ones - BCA* and BCA** - are already determined.

Each expert E_g, $1 \le g \le f$, after receiving the interview questions, examines and analyzes carefully the input information $\vec{D}$ and *for each* k*-th recurrent competitive attribute* BCA_k gives his personal subjective judgment on:

1. The expert's expected value of BCA_k, which will be henceforth designated as BCA_{gk}; note that
   - estimating BCA_{gk} is carried out always for a concrete set of versions assigned to the subproducts, i.e., for the routine search point $\vec{D}$;
   - value BCA_{gk} has to be always placed between the corresponding pregiven opposite estimates BCA_k^* and BCA_k^{**}.
2. The expert's estimated value (order) of importance of attribute BCA_k to win the competition for the product on the market. Denote henceforth this order of importance by η_{gk}.

After obtaining the answers from all experts we suggest to modify values BCA_{gk}, $1 \le g \le f$, $1 \le k \le n$, to their relative equivalents γ_{gk} as follows:

$$\gamma_{gk} = \frac{BCA_{gk} - BCA_k^*}{BCA_k^{**} - BCA_k^*} . \qquad (14.1.1)$$

Note that relation (14.1.1) does not undergo any changes, both in case relation $BCA_k^{**} > BCA_k^*$ holds, or otherwise. Value γ_{gk} represents, in essence, the relative competitive ability of the routine set of versions $\vec{D}$ due to attribute BCA_k only. Thus, relation

$$\sum_{k=1}^{n} \left(\gamma_{gk} \cdot \eta_{gk}\right) = W_g \qquad (14.1.2)$$

denotes the subjective judgment of the g-th expert about the total value of the product's competitiveness. We suggest calculating the more generalized estimate, which we will henceforth call *the product's competitive utility*

$$U_C = \frac{1}{f}\sum_{g=1}^{f} W_g = \frac{1}{f}\sum_{g=1}^{f}\sum_{k=1}^{n}\left(\gamma_{gk}\cdot\eta_{gk}\right). \qquad (14.1.3)$$

Value U_C calculated by (14.1.3) is just the parameter which has to be maximized in the course of implementing the search algorithm. Note that while using the CCSA algorithm, the number of feasible search points to be examined is less than by implementing other methods. Thus, the number of interview questions to the expert team will be diminished as much as possible.

Note, in conclusion, that according to the MAUT models, pairwise comparisons have to be undertaken by experts, in cases, when dependencies between two or more competitive attributes take place [125-127]. Those techniques may also be used by experts in our models, in the course of determining competitive attributes BCA_k. However, other techniques involving subjective judgments, can be implemented as well [150].

14.1.4 Notation

Let us introduce the following terms:

V - the product to be designed and manufactured;

V_i - the i-th subproduct entering the product, $1\le i\le q$;

q - the number of subproducts;

V_{ij} - the j-th possible version of the i-th subproduct, $1\le j\le r_i$;

r_i - the number of possible versions of subproduct V_i;

$\{d_1, d_2, \ldots, d_q\} = \vec{D}$ - a routine search point (a routine set of versions) for the CCSA algorithm, $1\le d_i\le r_i$, $1\le i\le q$;

BPA_b - the b-th basic project attribute value, $1\le b\le m$;

m - number of BPA values;

BCA_k - the k-th basic competitive attribute value, $1\le k\le n$;

n - number of BCA values;

$BPA_b\left(\vec{D}\right)$ - the b-th basic project attribute value calculated analytically or by means of simulation at the routine search point $\vec{D}$;

Z_b - the worst permissible value for the b-th basic product attribute (pregiven);

Y_b - the best possible value for the b-th basic product attribute, $1\le b\le m$ (pregiven);

BCA_k^* - the worst competitive estimate of the k-th competitive attribute (pregiven by experts);

BCA_k^{**} - the best competitive estimate of the k-th competitive attribute (pregiven by experts);

η_{gk} - priority level (level of importance) of the k-th competitive attribute given by the g-th expert, $1 \le g \le f$;

f - the number of experts entering the team;

BCA_{gk} - the personal subjective judgment of the g-th expert on the expected value of the k-th competitive attribute;

$BPA_{\xi_1, \xi_2, \dots, \xi_{m_1}}$ - basic independent project attributes (pregiven);

$m_1 \le m$ - number of independent basic project attributes (pregiven);

$BPA_{\Theta_1, \Theta_2, \dots, \Theta_{m-m_1}}$ - basic dependent project attributes (pregiven);

$m - m_1$ - number of dependent project attributes;

$BPA_{\Theta_v} = PHM \left\{ D \big/ \overrightarrow{BPA}_{\xi_h}, 1 \le h \le m_1 \right\}$ - the estimate of basic project dependent parameters, $1 \le v \le m - m_1$, obtained by means of implementing partial harmonization models or simulation models on the basis of vector $\vec{D}$;

U_C - the product's multi-attribute competitive utility (to be maximized);

U_P - the project's utility obtained by means of BPA values;

ρ_b - partial utility value for the b-th BPA (pregiven);

ε - the relative accuracy of the harmonization problem's objective (value U_C).

14.1.5 The problem's formulation

Referring to *Sections 14.1.2-14.1.3*, the strategic harmonization problem is as follows: *to determine optimal versions assigned to all subproducts* $d_1^{(opt)}, d_2^{(opt)}, \dots, d_q^{(opt)}$, *to maximize the multi-attribute competitive utility value*

$$\max_{\{\vec{D}\}} U_C , \tag{14.1.4}$$

subject to

$$min\left(Y_b, Z_b\right) \le BPA_b\left(\vec{D}\right) \le max\left(Y_b, Z_b\right), \quad 1 \le b \le m, \tag{14.1.5}$$

where U_C satisfies (14.1.3).

Restriction (14.1.5) means that only feasible solutions $\vec{D}$, i.e., sets of versions which honor pregiven worst permissible constraints Z_b, can participate in the optimization procedure.

We suggest solving the strategic harmonization problem (14.1.3-14.1.5) by means of a two-stage algorithm. At the first stage feasible solutions $\vec{D}$, i.e., combinations of versions assigned to the subproducts, are determined. Those vectors present input information for the second stage, to maximize the multi-attribute competitive utility parameter U_C obtained by means of experts subjective judgments.

Note that in some cases it might be not easy to develop the initial feasible search point $\vec{D}$ at the first stage. We suggest implementing in the algorithm the corresponding subsidiary *Problem AI* which can be formulated as follows:

Determine at least one combination $\vec{D} = \{d_1, d_2, \dots, d_q\}$ *satisfying restriction (14.1.5).*

14.1.6 Subsidiary problem AI

The suggested step-wise algorithm to solve *Problem AI* is as follows:

Step 1. By means of the Monte-Carlo method simulate for each subproduct the index of its version, i.e., simulate integer values

$$d_i = [\alpha_i \cdot r_i] + 1, \quad 1 \le i \le q, \tag{14.1.6}$$

where $\alpha_i = U(0,1)$ is a random value uniformly distributed in $(0,1)$, and $[x]$ is the whole number of x.

Step 2. By means of Monte-Carlo simulate for each i-th subproduct the values of m_1 independent basic project attributes

$$BPA_{\xi_h} = \min\left(Y_{\xi_h}, Z_{\xi_h}\right) + \alpha_h \cdot \left\{\max\left(Y_{\xi_h}, Z_{\xi_h}\right) - \min\left(Y_{\xi_h}, Z_{\xi_h}\right)\right\},\ 1 \le h \le m_1, \tag{14.1.7}$$

where $\alpha_h = U(0,1)$.

Step 3. Using partial harmonization models (see §5.3), determine values of $m - m_1$ dependent basic project attributes $BPA_{\Theta_v} = PHM\left\{\vec{D} \big/ BPA_{\xi_h}, 1 \le h \le m_1\right\}$. In case $m = 3$ there are usually two independent basic attributes (time to accomplish the project and budget assigned to the project) as well as one dependent attribute - reliability for the project to be accomplished on time. In such a case the problem together with the corresponding algorithm is outlined in [9, 11-13]. If m exceeds 3, the problem becomes more complicated.

Step 4. If all values $BPA_{\Theta_v} = PHM\left\{\vec{D} / BPA_{\xi_h}\right\}$, $1 \le v \le m - m_1$, satisfy (14.1.5), search point $\vec{D}$ obtained at Step 1 is a feasible one, thus providing solution to the problem. Otherwise apply the next step.

Step 5. Repeat Steps 2-3 N_1 times, in order to check possible combinations in different m-dimensional subspaces. If for a significantly large N_1 a feasible solution has still not been obtained, apply the next step.

Step 6. Repeat Steps 1-5 N_2 times, where N_2 is a significantly large number. If no feasible solution has been obtained, the initial search point cannot be determined. We have either to alter values (Y_b, Z_b), $1 \le b \le m$, or to select other possible versions of the subproducts.

We have deliberately chosen the so-called undirected Monte-Carlo search method [176] because of its simplicity. If m is not large, using the method for solving *Problem AI* does not cause any particular difficulties.

14.1.7 Cyclic coordinate search method for the problem's solution

To obtain the problem's solution, we suggest implementing the CCSA in the two-level optimization algorithm. The step-wise algorithm is as follows:

Step 1. Solve subsidiary *Problem AI* to obtain a feasible problem's solution, i.e., determine vector $\vec{D}$ which will be used henceforth as the initial search point.

Step 2. Assign to all subproducts' versions entering $\vec{D}$ (obtained at Step 1), the minimal index 1, i.e., $\vec{D} = (1,1,...,1)$. For each subproduct i, $1 \le i \le q$, all other versions can be enumerated in an arbitrary order from 2 to r_i.

Step 3. Transfer the information about the initial search point, i.e., the set of subproducts' versions, to the expert team. After carrying out questioning interviews and receiving the experts' subjective judgments, calculate value U_C by (14.1.3). Coordinates of vector $\vec{D}$ together with value U_C are placed in a special array W. In the course of the optimization process, this array will contain the monotonously increasing utility value U_C together with the corresponding vector of optimized variables $\vec{D}$. Denote henceforth the stored information by U_C^* and $\vec{D}^*$, correspondingly.

Step 4. Start using CCSA with respect to the coordinate variables $d_1, d_2, \dots, d_q$, beginning from the initial search point $\vec{D} = (1,1,...,1)$. The general idea is to increase the first coordinate d_1 by a constant step equal 1, i.e., $d_{1,j} = d_{1,j-1} + 1$, while all other coordinates $d_2, \dots, d_q$ are fixed and remain unchanged in the course of the coordinate optimization. After the first coordinate d_1 is optimized, we fix the index of the latter, and proceed by increasing the second coordinate d_2 by a step equal 1 (coordinates $d_3, \ldots, d_q$ being fixed). Afterwards, when completing optimization of the second coordinate, the latter is fixed as well, and we proceed with the third coordinate d_3, and so forth, until all coordinates are looked through by the partial coordinate increasing procedure. Go to Step 5.

Step 5. Proceed with the optimization process starting again from the first coordinate d_1 with the index obtained in the course of carrying out Step 4. Check two opposite directions: $d_1 - 1 \Rightarrow d_1$ and $d_1 + 1 \Rightarrow d_1$ and choose one of them which results in obtaining a feasible solution as well as in increasing utility value U_C. If such a direction can be chosen, proceed changing d_1 in that direction by a constant step equal 1. The same procedure has to be undertaken with other coordinates. Note that, similar to Step 4, only one coordinate undergoes optimization, while all other coordinates remain fixed and unchanged.

If in the course of carrying out Steps 4-5 a routine feasible search point $\vec{D}$ coincides with the previously obtained and stored in array W feasible point $\vec{D}^*$, the corresponding utility value U_C^* is taken as the quasi-optimal solution of the harmonization problem. Thus, the search process terminates. Go to Step 9.

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Implementing the optimization search process at Steps 4-5 centers on numerous applications to a group of Steps 6-7, which actually examine the routine search point and carry out decision-making as follows:

- either to accept the routine search point as a successful one, i.e., to proceed with the search procedure from that point on, or
- to reject the routine search point and change the optimizing coordinate.

As outlined above, the coordinate optimization centers on examining a routine search point $\vec{D}$ in order to check:

- the search point's feasibility, and
- the increase of the corresponding total utility U_C relatively to the previously obtained maximal value U_C^*.

In order to check the feasibility, apply the next step.

Step 6. To check the routine search point's feasibility, one has to carry out Steps 2-5 of *Problem AI* outlined in *Section 14.1.6*. Note that coordinate values $d_1, d_2, \ldots, d_q$ enter the routine search point $\vec{D}$.

If in the course of carrying out Step 6 a feasible solution has been obtained, go to the next step. Otherwise reject the routine search point and go to Step 8.

Step 7. Undertake questioning interviews of experts and, after obtaining their subjective judgments, calculate value U_C by (14.1.3). If U_C exceeds U_C^* (stored in array W), examine the relative increase of the utility value by calculating

$$\Delta U_C = \frac{1}{U_C^*}\left(U_C - U_C^*\right) . \tag{14.1.8}$$

If relation $\Delta U_C < \varepsilon$ holds, the optimization process terminates. Go to Step 9. Otherwise, when $\Delta U_C \geq \varepsilon$ holds, accept the routine search point as a successful one. Go to Step 4, to continue the search procedure.

If U_C does not exceed U_C^*, the routine search point has to be rejected. Go to the next step.

Step 8. Assume that in the course of the optimization search process, the i-th coordinate, i.e., value d_i, has increased its index by 1, while other values $d_1,\ldots, d_{i-1}, d_{i+1},\ldots, d_q$ were fixed and remained unchanged. If $i \neq q$, decrease value d_i by 1, $d_1 - 1 \Rightarrow d_1$, fix value d_i and start optimizing the next coordinate d_{i+1}. So to Step 4.

If $i = q$, i.e., all coordinates have been partially optimized, the process is then repeated starting with d_i again. Go to Step 5.

Step 9. The optimization search process terminates, and the information stored in array W, i.e., $\vec{D}^*$ with objective U_C^*, is taken as the optimal solution of the harmonization problem.

14.1.8 Case of compound study

It can be well-recognized that in previous sections the competitive utility value U_C has been favored over the project utility value U_P. However, under certain conditions those different utility parameters may be regarded as practically of equal importance. Thus, the problem of maximizing the competitive utility value U_C has to be substituted for maximizing the compound utility

$$\beta_P \cdot U_P + \beta_C \cdot U_C = U_T , \tag{14.1.9}$$

where U_T is the total utility value and β_P and β_C are properly chosen coefficients to present both utility parameters in similar ranking and scaling.

The problem is, thus, to determine the optimal set of versions for each subproduct $\vec{D}$ in order to maximize the total utility U_T

$$\underset{\{\vec{D}\}}{Max}\, U_T \;=\; \underset{\{\vec{D}\}}{Max} \left\{ \beta_P \cdot U_P \;+\; \beta_C \cdot U_C \right\} \tag{14.1.10}$$

subject to (14.1.5), where U_C is calculated by (14.1.3) and U_P satisfies [11-14]

$$U_P \;=\; \sum_{b=1}^{m} \left(\rho_b \cdot BPA_b \right) . \tag{14.1.11}$$

We suggest optimizing harmonization model (14.1.3, 14.1.5, 14.1.9-14.1.11) by using the same search algorithm as being outlined in *14.1.7*. Only minor modifications have to be implemented, namely:

1. Step 1 has to be substituted by the algorithm of solving *Problem AII* to maximize the project's utility value

   $$\underset{\{\vec{D}\}}{Max}\, U_P \;=\; \underset{\{\vec{D}\}}{Max} \left[\sum_{b=1}^{m} \left(\rho_b \cdot BPA_b \right) \right] \tag{14.1.12}$$

   subject to (14.1.5).

   Problem (14.1.5, 14.1.12) together with the corresponding algorithm has been outlined in [11-14] and in Chapter 8, for the three-attribute harmonization model in project management.

2. Step 6 has to be substituted by solving problem (14.1.5, 14.1.11) as well.

3. Competitive utility value U_C^* in array W has to be substituted by the total utility value U_T^*.

4. Relation (14.1.8) has to be substituted by another one, honoring modification (14.1.9).

   All other steps of the algorithm do not undergo any changes.

14.1.9 <u>Example on designing a new passengers vehicle</u>

An example on designing a new passengers vehicle which is widely presented in the literature on project management (see, e.g., [168]), can illustrate implementation of the outlined above harmonization model. The example is subject to restrictions by competitive attributes which have been used within a long period by questioning experts and obtaining from the latter all kinds of subjective decision-making. However, no harmonization models have been suggested and no optimization problems have been solved.

Three basic attributes define usually the R&D project's utility [11, 14]:

BPA_1 - budget assigned to the whole project;

BPA_2 - time to accomplish the project;
BPA_3 - reliability for the project to be accomplished on time on condition of pregiven BPA_1 and BPA_2.

Thus, there are two independent attributes (BPA_1 and BPA_2) and a dependent one (BPA_3). For the case of a PERT-COST project the harmonization model together with the optimization algorithm obtained a detailed solution in §§5.3-5.4, 8.1-8.4.

As to competitive attributes, nine of them have been singled out [168] and are usually examined by experts in case of designing a new vehicle:

BCA_1 - relative fuel economy;
BCA_2 - initial cost;
BCA_3 - life cycle cost per mile;
BCA_4 - maintainability (special scaling);
BCA_5 - safety (special scaling);
BCA_6 - refuel time;
BCA_7 - unrefueled range;
BCA_8 - maximum startup time;
BCA_9 - minimum speed-up time from 0 to 80 mph.

As to the worst and the best competitive attribute estimates, they are as follows (for the last decade - based on best experts' opinion):

1. $BCA_1^* = 20$ mpg equivalent;
   $BCA_1^{**} = 80$ mpg equivalent;
2. $BCA_2^* = \$25{,}000$;
   $BCA_2^{**} = \$\ 5{,}000$;
3. $BCA_3^* = \$1.00/mile$;
   $BCA_3^{**} = \$0.20/mile$;
4. $BCA_4^* = 0$ (special scaling);
   $BCA_4^{**} = 10$ (special scaling);
5. $BCA_5^* = 0$ (special scaling);
   $BCA_5^{**} = 10$ (special scaling);
6. $BCA_6^* = 8$ hours;
   $BCA_6^{**} = 0.17$ hours;
7. $BCA_7^* = 50$ miles;

BCA_7^{**} = 250 miles;

8. BCA_8^{*} = 600 seconds;

   BCA_8^{**} = 5 seconds;

9. BCA_9^{*} = 60 seconds;

   BCA_9^{**} = 5 seconds;

It can be well-recognized that an experienced decision-maker is capable of undertaking subjective judgment for any BCA_k, $1 \le k \le 9$, together with scaling the attribute's level of importance η_k.

14.1.10 Conclusions

1. The problem of maximizing the product's utility by means of considering optimal components for that product, is widely regarded in the literature as an important strategic area in project management. Thus, developing new harmonization models on that subject refers to *strategic harmonization models*. Those models practically cover the entire life cycle of any newly designed and developed product.

2. The backbone of the models under consideration is the generalized search procedure comprising partially harmonization models on the basis of heuristic approaches with decision-making on the competitive ability of the designed product, to be undertaken by a qualified expert team.

3. The suggested search algorithm is based on the cyclic coordinate search method. The latter may either comprise:

   - an optimization procedure in the area of basic competitive attributes only, in order to maximize the total competitive utility subject to restrictions for basic parameters of the designing project, or
   - a search procedure to maximize the total product's utility comprising as summands both the competitive utility and the project's utility values.

   The two outlined above procedures are not of any principal difference; one can be obtained from another by implementing only minor modifications.

4. An emphasis has to be drawn that, in dependence on the novelty of the designed product, the market's demands, etc., other variables to be optimized may be introduced in the harmonization model. However, the basic concepts linked to the necessity of developing a mixed type optimization procedure comprising a combination of heuristic methods and interview dialogs with experts, have to remain unchanged.

§14.2 Harmonization models for designing compound systems

14.2.1 The system's description

The solution of engineering design problems generally requires a compromise between several objectives, including a trade-off among cost and reliability parameters. Those problems become extremely important in cases when an overall compound system is composed of several subsystems. The objective is to use the reliability model to assign reliability to the subsystems so as to achieve a specified reliability goal for the system. The optimization model may be to minimize the total costs of developing the subsystems subject to the condition that the reliability of the system must meet a certain pregiven level (the direct problem) or to maximize the reliability subject to certain cost constraints (the dual problem). However, it can be well-recognized that most of the publications on that area deal with relatively simple system configurations (e.g. for series and parallel systems) where the functional relationship between the subsystems' failures and the top system failures is well known (see, e.g. [56, 115, 123]). In cases when this relationship is complex for other system configurations, e.g., when the linkage between the subsystems is carried out under random disturbances, the number of such publications remains very scanty.

We will consider a complicated system to be designed which is composed of several subsystems. The functional relationship between the subsystems and the system output parameters can be formalized only by means of a simulation model which comprises a variety of random parameters. Subsystems' failures are not independent, and the linkage between subsystems is carried out via various information signals. Each subsystem can be designed and developed independently and is a subject of several alternative measurable versions, including the cost of designing and creating the subsystem and its reliability.

The problems to be considered are as follows: in the system under consideration assign optimal reliability and cost parameters (versions) to all subsystems in order to minimize the total costs of designing and creating, subject to the specified reliability target for the system (the direct problem), and to optimize the subsystems' reliability and cost parameters in order to maximize the system's reliability subject to the restricted total costs (the dual problem).

The solution of both problems is based on a two-level heuristic algorithm. At the upper level a search of optimal subsystems' parameters is undertaken, while the lower level is faced with numerous realization of the simulation model to obtain representative statistics. The output data of the search procedure at the upper level is the input data for the simulation model.

The results obtained are later on considered within the general problem of the designed system standards harmonization. We formulate an optimization problem to assign optimal versions to all subsystems in order to provide harmonization to the system reliability and cost standards [9, 27].

14.2.2 Notation

Let us introduce the following terms:

S - the system to be designed and created;

$S_i \subset S$ - the i-th subsystem entering S, $1 \le i \le n$;

n - the number of subsystems;

S_{ij} - the j-th version of designing subsystem S_i, $1 \le j \le m_i$;

m_i - the number of possible versions of designing and creating the subsystem S_i;

C_{ij} - the average cost of designing and developing S_{ij} (pregiven);

R_{ij} - reliability value of subsystem S_{ij} (pregiven);

SM - simulation model with input subsystems' reliabilities and the outcome system reliability;

$R\{a_i\}$ - system reliability value obtained by means of simulation, $R\{a_i\} = SM\{a_i\}$, where integer value $a_i = j$, $1 \le a_i \le m_i$, is the ordinal number of version j of subsystem S_{ij}, $1 \le i \le n$;

C - the total costs of designing and creating the system, $C = \sum_{i=1}^{n} C_{ia_i}$;

R^* - pregiven specified system reliability;

C^* - pregiven restricted total cost amount to design and create system S;

ΔC - accuracy estimate (pregiven);

α_R - parametrical utility "weight" of the system reliability;

α_C - parametrical utility "weight" of the system total costs.

14.2.3 The problem's formulation

The direct cost-optimization problem [9, 27] is as follows:

Determine the optimal set of integer values a_i, $1 \le i \le n$, which requires the minimal amount of costs

$$\underset{\{a_i\}}{Min} \sum_{i=1}^{n} C_{ia_i} \tag{14.2.1}$$

subject to

$$R\{a_i\} = SM\{a_i\} \ge R^*, \quad 1 \le i \le n, \quad 1 \le a_i \le m_i . \tag{14.2.2}$$

The dual problem is as follows:

Determine the optimal set $\{a_i\}$, $1 \le i \le n$, in order to maximize the system reliability by means of simulation

$$R = \underset{\{a_i\}}{Max}\ R\{a_i\} \tag{14.2.3}$$

subject to

$$\sum_{i=1}^{n} C_{ia_i} \le C^* . \tag{14.2.4}$$

Note that the costs of unifying subsystems $\{S_i\}$ into a complex system S are assumed to be negligibly small in comparison with the total costs of designing and creating all those subsystems.

It can be well-recognized that if the number of subsystems n, as well as the number of alternative options m_i to design subsystems S_i, is high enough, both problems (14.2.114.2.2) and (14.2.3-14.2.4) are NP-complete [66]. Thus, an optimal solution can be obtained only by means of a look-over algorithm that checks the feasibility of each of $\prod_{i=1}^{n} m_i$ combinations $\{a_i\}$. If the number of combinations is high enough and taking into account that each combination requires numerous simulation runs, solving both problems by means of precise classical methods meets unavoidable computational difficulties (see justification in Chapter 5). To avoid this obstacle, we suggest a high-speed two-level approximate heuristic algorithm. At the bottom level a simulation model to realize the functional relationship between reliability values of local subsystems S_i, is implemented. At the upper level a search procedure to determine optimal values $\{a_i\}$, has to be carried out.

Note, in conclusion, that for any subsystem S_i increasing its version number $a_i = j$ results in increasing both costs C_{ij} and the reliability value R_{ij}. Thus, the m_i-th version has the highest reliability R_{im_i}, as well as requires the highest costs C_{im_i}. If for each S_i its highest version has been chosen, it can be well-recognized that the overall system S has the highest possible reliability $R^{**} = SM\{a_{m_i}\}$, $1 \le i \le n$. Thus, if relation $R^{**} < R^*$ holds, problem (14.2.1-14.2.2) has no solution.

We will assume henceforth that both relations

$$R\{a_{m_i}\} = SM\{a_{m_i}\} \ge R^* \tag{14.2.5}$$

and

$$\sum_{i=1}^{n} C_{i1} \le C^* \tag{14.2.6}$$

hold.

14.2.4 Two-level heuristic algorithm for solving the direct cost-optimization problem

As outlined above, the system reliability $R = SM\{a_i\}$ is a complicated non-linear function of values $\{a_i\}$. This enables solution of problem (14.2.1-14.2.2) by using the cyclic coordinate search algorithm ($CCSA$) with optimized variables $\{a_i\}$ [133]. The justification of using $CCSA$ is outlined in Chapter 5. To solve the problem, SM is implemented to obtain representative statistics for calculating $R = SM\{a_i\}$. The expanded step-by-step procedure of $CCSA$ is as follows:

Step 1. Choose an initial search point $\vec{X}^{(0)} = \{m_1, m_2, \dots, m_n\}$. According to (14.2.5), search point $\vec{X}^{(0)}$ is a feasible solution.

Step 2. Start using $CCSA$ which minimizes value $\sum_{i=1}^{n} C_{ia_i}$ with respect to the coordinate variables. Decrease the first coordinate $x_1^{(0)} = m_1$ by a constant step equal 1, i.e., $x_1^{(0)} - 1 \Rightarrow x_1^{(1)}$, while all other coordinates $x_2 = m_2,\ x_3 = m_3,\ \dots,\ x_n = m_n$ are fixed (see Step 1) and remain unchanged. In the course of undertaking the search steps the feasibility of every routine search point $\vec{X}$ is examined by performing numerous simulation runs by means of the SM in order to check relation

$$SM\{\vec{X}\} \ge R^*. \tag{14.2.7}$$

The process of decreasing the first coordinate $\boldsymbol{x_1}$ terminates in two cases:

- if for a certain value $x_1 = j \ge 1$ relation (14.2.7) ceases to hold;
- if for all values $1 \le x_1 \le m_1$ relation (14.2.7) remains true.

For the first case we set $x_1 = j+1$, while in the second case $x_1 = 1$ is fixed.

Step 3. After the first coordinate x_1 is optimized in the course of carrying out Step 2, we proceed with the $CCSA$ by decreasing the second coordinate x_2 by a constant step, i.e., $x_2^{(0)} - 1 \Rightarrow x_2^{(1)}$, while all other coordinates, namely, x_1 (the new optimized value at Step 2), $x_3, \dots, x_n$ are fixed and remain unchanged. After examining the coordinate x_2 by a step-wise decrease via simulation, its newly obtained value is fixed, similarly to x_1, and we proceed with the third coordinate x_3, and so forth, until x_n is reached and checked by the constant step decreasing procedure.

Step 4. After all coordinates $\{x_i\}$ are checked by means of the $CCSA$ (first iteration), the process is then repeated starting with x_1 again. The $CCSA$ terminates after a current iteration does not succeed in bringing any changes to the search point $\vec{X} = (x_1, x_2, \dots, x_n)$. Thus, the n-dimensional search point $\vec{X}$ is then taken as the quasi-optimal solution of the direct problem (14.2.1-14.2.2).

Call henceforth the above algorithm of $CCSA$ to solve the direct problem (14.2.1-14.2.2) - *Algorithm I*. Note that in the course of implementing *Algorithm I* the total costs $C = \sum_{i=1}^{n} C_{ia_i}$ decrease monotonously at each step $\vec{X} = \{a_i\}$.

After obtaining an approximate solution $\vec{X} = \{a_i\}$ we suggest to undertake a corrective random search procedure designated henceforth as *Algorithm II*. The enlarged step-by-step procedure of *Algorithm II* is as follows:

Step 1. Choose an initial search point $\vec{X}^{(0)} = \{a_i\}$ which has been determined in the course of implementing *Algorithm I*. Denote, in addition, the required total costs to design the system with $\{a_i\}$, by

$$C\left(\vec{X}^{(0)}\right) = \sum_{i=1}^{n} C_{ia_i} . \tag{14.2.8}$$

Step 2. Simulate n random independent values p_i, $1 \le i \le n$, uniformly distributed in the interval $[-1, +1]$.

Step 3. Introduce a random step $\vec{X}^{(1)} = \vec{X}^{(0)} + \Delta\vec{X}$ obtained by

$$\vec{X}^{(1)} = \vec{X}^{(0)} + \vec{\beta}, \quad \vec{\beta} = (\beta_1, \beta_2, \dots, \beta_n), \tag{14.2.9}$$

where local steps equal 1 and

$$\beta_i = \begin{cases} +1 & if \quad p_i \ge 0 \\ -1 & if \quad p_i < 0 \end{cases},$$

subject to additional constraints for the i-th coordinate $\vec{X}_i^{(1)}$, $1 \le i \le n$,

$$\vec{X}_i^{(1)} = \begin{cases} m_i & if \quad X_i^{(0)} = m_i \quad and \quad p_i \ge 0 \\ 1 & if \quad X_i^{(0)} = 1 \quad and \quad p_i < 0 . \end{cases} \tag{14.2.10}$$

Step 4. Calculate by means of the SM frequency rate $R\left\{\vec{X}^{(1)}\right\}$ and compare the latter with R^*. If $R\left\{\vec{X}^{(1)}\right\} \ge R^*$ apply the next step. Otherwise go to Step 6.

Step 5. Calculate the total costs to design the system with $\vec{X}^{(1)} = \{a_i + \beta_i\}$. If relation

$$C\left(\vec{X}^{(1)}\right) = \sum_{i=1}^{n} C_{i,a_i+\beta_i} < \sum_{i=1}^{n} C_{ia_i} = C\left(\vec{X}^{(0)}\right) \quad (14.2.11)$$

holds, go to Step 7. Otherwise apply the next step.

Step 6. Set $C\left(\vec{X}^{(1)}\right)$ equal to K, where K is a very large number (take, e.g., $K = 10^{17}$). Go to the next step.

Step 7. Repeat Steps 2-6 Z times, i.e., undertake Z independent steps $\vec{X}^{(0)} + \Delta\vec{X} \Rightarrow \vec{X}^{(1)}$.

Step 8. Determine the *minimal* cost value $C\left(\vec{X}^{(1)}\right)$ from Z values (14.2.11). Denote it by $C^{*(1)}$.

Step 9. If $C^{*(1)} \geq C\left(\vec{X}^{(0)}\right)$ the search process terminates. That means that search point $\vec{X}^{(0)}$ cannot be improved. Go to Step 11. In case $C^{*(1)} < C\left(\vec{X}^{(0)}\right)$ apply the next step.

Step 10. Set $\vec{X}^{(1)} \Rightarrow \vec{X}^{(0)}$, $C^{*(1)} \Rightarrow C\left(\vec{X}^{(0)}\right)$, and return to Step 2.

Step 11. Take $\vec{X}^{(0)}$, together with its corresponding budget value $C\left(\vec{X}^{(0)}\right)$, as the quasi-optimal solution of *Algorithm II*.

Note that since using a search step of pregiven length in the n-dimensional space with a finite number of feasible solutions cannot result in an infinite monotonic convergence, the random search process always terminates.

As outlined above, we suggest using *Algorithm II* on condition that the initial search point $\vec{X}^{(0)}$ is determined by using *Algorithm I*.

14.2.5 The dual cost-optimization problem

The step-by-step algorithm to solve problem (14.2.1-14.2.2) (call it henceforth *Algorithm III*) is based on the bisection method [176] and runs as follows:

Step 1. Calculate reliability values by means of the SM

$$R_{min} = SM\{1, 1, \ldots, 1\}, \quad (14.2.12)$$

$$R_{max} = SM\{m_1, m_2, \ldots, m_n\}. \quad (14.2.13)$$

Step 2. Calculate cost values

$$C_{min} = \sum_{i=1}^{n} C_{i1} , \tag{14.2.14}$$

$$C_{max} = \sum_{i=1}^{n} C_{im_i} . \tag{14.2.15}$$

Note that relation $C_{min} \le C^*$ holds, otherwise problem (14.2.3-14.2.4) has no solution. In case $C^* \ge C_{max}$ there is a trivial solution: $\{a_i\} = \{m_i\}$. Thus, we will assume that a reasonable relation

$$C_{min} \le C^* \le C_{max} \tag{14.2.16}$$

holds.

<u>Step 3</u>. Calculate

$$R' = 0.5 \cdot \left(R_{min} + R_{max} \right). \tag{14.2.17}$$

<u>Step 4</u>. Solve direct cost-optimization problem (14.2.1-14.2.2) with $R' = R^*$. Denote the minimal cost objective value obtained in the course of implementing *Algorithms I-II,* by C'.

<u>Step 5</u>. Compare values C' and C^*. If $\left| C^* - C' \right| < \Delta C$, go to Step 9. Otherwise go to Step 6. Here $\Delta C > 0$ designates the pregiven problem's solution accuracy as outlined in *14.2.2*.

<u>Step 6</u>. Examine relation $C_{min} \le C' < C^*$. In case it holds, go to Step 7. Otherwise, i.e., in case $C^* \le C' \le C_{max}$, go to Step 8.

<u>Step 7</u>. Set $R' \Rightarrow R_{min}$. Go to Step 3.

<u>Step 8</u>. Set $R' \Rightarrow R_{max}$. Go to Step 3.

<u>Step 9</u>. Solution $\{a_i\}$ of the direct problem (14.2.1-14.2.2) obtained at Step 4, is taken as the quasi-optimal solution of problem (14.2.3-14.2.4).

14.2.6 <u>Harmonization models in designing compound engineering systems</u>

As outlined above, in *14.2.1*, engineering design problems generally require a compromise between certain parameters of the system to be designed, e.g., a compromise between cost and quality parameters. If a system to be designed and created is compound in nature and consists of several local subsystems with complex configuration, such a compromise may be realized by means of certain optimization problems [27]. Let us describe two different situations which lead to a "compromise optimization":

Strategy A

A company is faced with designing and creating a new complicated technical system which consists of several subsystems. The latter have to be designed and further on created at the company's design office. Each subsystem may be created in several technical versions, as outlined above. The problem is to determine optimal versions for each subsystem to be designed, in order to:

- meet the system reliability restriction from below;
- meet the system total cost restriction from above;
- optimize a trade-off function between reliability and cost parameters.

Both restrictions can be formalized by relations (14.2.2) and (14.2.4).

Strategy B

A highly complicated compound technical system has to be created (e.g., a new aircraft). The system comprises several subsystems (with complex configuration) which are *already manufactured* by several different companies (and, quite possible, in different countries). Each company manufactures only one version of a certain subsystem while other companies may produce other versions. Thus, each subsystem is available in several alternative versions provided to the international market with pregiven cost and reliability parameters. The compromise optimization problem is similar to that outlined above for *Strategy A*.

It can be well-recognized, however, that both from the point of logical assumptions and considering the solution method, those optimization problems are different. *Strategy A* is based on the assumption that for each subsystem S_i reducing the costs C_{ij} results in reducing its reliability level R_{ij}, and vice versa. This simplifies essentially the solution method.

However, for *Strategy B* the relation between cost and reliability parameters for different competing versions may be entirely different, since certain subsystems may be produced and purchased in different countries and thus affected by their domestic policies in business and standardization.

A detailed description of different strategies (there may be more than two of them), together with developing optimization problems and the corresponding methods of solution, do not lie within the framework of this discussion. However, we will show the nature of the "compromise optimization" by an example of *Strategy A*.

We suggest formalizing the "compromise optimization" problem as follows:

Determine optimal integer values (versions) a_i to maximize a "system priority value" which is composed of local priority functions $\alpha_R(R)$ and $\alpha_C(C)$ (see Notation in *14.2.2*),

$$\underset{\{a_i\}}{Max} \left(\alpha_R \left[R\{a_i\} \right] + \alpha_C \left[C\{a_i\} \right] \right)$$

subject to (14.2.2) and (14.2.4).

It goes without saying that decreasing the total cost C increases the corresponding priority function $\alpha_C(C)$, while decreasing reliability value R decreases value $\alpha_R(R)$.

Thus, we suggest introducing the concept of harmonization by means of a compromise, trade-off optimization. Finally, we obtain:

$$\underset{\{a_i\}}{Max} \left(\alpha_R \left[R\{a_i\} \right] + \alpha_C \left[C\{a_i\} \right] \right) \qquad (14.2.18)$$

subject to

$$R\{a_i\} \geq R^*, \qquad (14.2.19)$$

$$C\{a_i\} \leq C^*. \qquad (14.2.20)$$

This is a complicated stochastic optimization problem since value $R\{a_i\}$ is calculated through a simulation model and can be determined in frequency terms only. As to functions α_R and α_C, we suggest to assume they are deterministic.

14.2.7 Harmonization model's algorithm

The enlarged step-wise procedure of the suggested problem's solution is as follows:

Step 1. Solve cost-optimization problem (14.2.1-14.2.2) by means of *Algorithms I-II*. Denote the quasi-optimal solution as $a_1^*, a_2^*, \dots, a_n^*$.

Step 2. Solve cost-optimization problem (14.2.3-14.2.4) by means of *Algorithm III*. Denote the quasi-optimal solution by $a_1^{**}, a_2^{**}, \dots, a_n^{**}$.

Step 3. Calculate $C' = \sum_{i=1}^{n} C_{ia_i^{**}}$.

Step 4. If relation $C' > C^*$ holds, problem (14.2.18-14.2.20) has no solution. Otherwise apply the next step.

Step 5. Determine three n-dimensional areas:

- area I which comprises n-dimensional points $\vec{X} = \{a_i\}$ between $\vec{X}^{(1)} = \{1, 1, \dots, 1\}$ and $\vec{X}^{(2)} = \{a_i^*\}$;
- area II which comprises n-dimensional points $\vec{X} = \{a_i\}$ between

$\overrightarrow{X}^{(2)} = \{a_i^*\}$ and $\{a_i^{**}\} = \overrightarrow{X}^{(3)}$;

- area III which comprises n-dimensional points $\overrightarrow{X} = \{a_i\}$ between $\overrightarrow{X}^{(3)} = \{a_i^{**}\}$ and $\{a_{m_i}\} = \overrightarrow{X}^{(4)}$.

Step 6. Note that solution $\{a_i^*\}$ of problem (14.2.1-14.2.2), as well as solution $\{a_i^{**}\}$, are approximate ones. However, it can be well-recognized that:

- an overwhelming majority of n-dimensional points $\overrightarrow{X}$ entering area I does not meet reliability level R^*;
- an overwhelming majority of n-dimensional points $\overrightarrow{X}$ entering area III does not meet total cost restriction C^*.

Both assertions can be easily checked by simulating points $\overrightarrow{X}$ by means of the Monte-Carlo method in areas I and III with coordinates $X_i^{(1)}$ and $X_i^{(3)}$ as follows:

$$X_i^{(1)} = \left[a_i^* \cdot \beta_i \right] + 1, \quad 1 \le i \le n, \quad \beta_i \in U(0,1),$$

$$X_i^{(3)} = \left[a_i^{**} + \left(m_i - a_i^{**} \right) \cdot \alpha_i \right] + 1, \quad 1 \le i \le n, \quad \alpha_i \in U(0,1),$$

where $[x]$ denotes the whole part of x and α_i, β_i are random values uniformly distributed in $[0,1]$.

Later on, by means of the SM, the outlined above assertions can be easily verified. Practically speaking, points $\overrightarrow{X}$ in areas I and III do not meet restrictions (14.2.19) and (14.2.20).

Step 7. A Monte-Carlo sub-algorithm (call it henceforth *Algorithm IV*) is suggested to solve problem (14.2.18-14.2.20) for area II. The sub-steps of *Algorithm IV* are as follows:

Step 7.1. Simulate by means of the Monte-Carlo method points $\overrightarrow{X}$ in area II with co-ordinates $X_i^{(2)}$,

$$X_i^{(2)} = \left[\left(a_i^{**} - a_i^* \right) \cdot \beta_i + a_i^* \right] + 1, \quad 1 \le i \le n, \quad \beta_i \in U(0,1).$$

Step 7.2. Check by means of SM and $C' = \sum_{i=1}^{n} C_{ia_i^*}$ restrictions (14.2.19-14.2.20). If at least one restriction does not hold apply sub-Step 7.1. Otherwise go to the next sub-step.

Step 7.3. Calculate for point $X_i^{(2)}$, by means of the SM and $C = \sum_{i=1}^{n} C_{ia_i}$, system priority value $\alpha_R \left(SM \left\{ a_{ia_i} \right\} \right) + \alpha_C \left(\sum_{i=1}^{n} C_{ia_i} \right)$.

Step 7.4. Undertake a random search outlined in *Algorithm II*, by substituting maximization for minimization. Take the local optimum obtained in the course of the random search, as a local solution.

Step 7.5. Check the number of local solutions generated in the course of implementing the optimum trial random search method. If the number of such solutions exceeds N, go to the next sub-step. Otherwise apply sub-Step 7.1.

Step 7.6. Choose the maximum of local solutions obtained at sub-Steps 7.1-7.5. The result should be taken as the approximate solution of the trade-off problem (14.2.18-14.2.20).

The above global random search method is highly recommended in [9, 67, 176] and can be considered as an effective one for solving harmonization problems of type (14.2.18-14.2.20).

As for harmonization problems related to *Strategy B*, using the global random search method is less effective. This is because optimization methods for *Strategy B* may deal with a lot of isolated n-dimensional points in both areas I and II (see *Algorithm IV*). It normally causes much computational troubles to detect those points.

Note, in conclusion, that in the harmonization model (14.2.18-14.2.20) partial harmonization models do not exist, since all basic factors (C and R) are independent. Similarly to model (10.1.9-10.1.11), both basic factors are set by means of restrictions (14.2.19-14.2.20) and should be pregiven beforehand.

Chapter 15. Strategic Hierarchical Harmonization Model for Complex Holding Corporations

§15.1 The problem's formulation

15.1.1 Introduction

In the three recent decades extensive research has been undertaken in developing various quality assessment techniques for marketing competitive problems. The authors of the multi-attribute utility theory [125-127, 150, 168] dealt mostly with marketability of output products in order to gain future commercial success.

In papers [9, 11-15, 23] utility concepts have been used to estimate various system's quality parameters, especially in project management, in the course of the project's realization. New harmonization models comprising analytical, heuristic and simulation algorithms, have been developed. In Chapter 14 a new harmonization model has been suggested to optimize the process of designing, creating and marketing a new product within its entire life cycle. The developed optimization algorithm is of a mixed type, in order to comprise both analytical calculations and man-computer dialogs at the stage of decision-making on the basis of experts' information. Thus, the results obtained in this area are restricted as yet to developing competitive quality models for *output products* to be delivered in large quantities to the market.

The Chapter under consideration presents an essential extension of the recently outlined results [15]. We suggest to develop a generalized utility harmonization model by implementing in the latter highly complicated hierarchical organization systems, e.g., complex holding corporations. The latter are usually involved in financial management of designing and creating simultaneously various new outcome products as well as providing services. A holding corporation usually comprises several subsidiary corporations or companies, which are actually the direct designers and producers of the new products and services.

Given the budget of the holding corporation, the problem is to maximize the corporation's competitive utility on the markets by undertaking optimal budget allocation among subsidiary corporations. The latter, in turn, have to reallocate independently the assigned budget among the projects entering the subsidiary corporation. At the project level a man - computer dialog has to be carried out in order to obtain subjective judgment from the experts on the competitive ability of the product's (or service's) attributes to gain commercial success on the market. Thus, the general harmonization model of the hierarchical holding corporation requires by itself a proper hierarchical structure together with the multi-level optimization algorithm.

A three-level search algorithm of the problem's solution, i.e., of determining the optimal holding corporation's budget structure to maximize the corporation's marketability, is suggested. The higher and the intermediate levels are faced with optimizing the budget allocation's structure by means of the newly developed couple reverse cyclic coordinate search method (CRCCSM). The lower level centers on optimizing the product's or service's competitive utility by means of experts' information.

The problem of developing the general harmonization model for a hierarchical holding corporation is essentially more complicated, than a similar model for an individual project. However, the gain of implementing the model for real holding corporations may be tremendous.

15.1.2 The problem's description

A holding corporation S (e.g., Boeing, General Electric, Pepsico, etc.), comprising several subsidiary corporations S_i (e.g., Pizza Hut, Pepsi, Kentucky Fried Chicken - all entering Pepsico), is considered. For each subsidiary corporation S_i its corresponding index of importance (priority index) η_i is pregiven. Each subsidiary corporation comprises, in turn, several projects Q_{ij}, $1 \le j \le n_i$, in order either to design and create new products V_{ij} or to develop new service systems. All the subsidiary corporations are functioning simultaneously within a common planning horizon with the due date D. For each project Q_{ij} its basic project's attribute BPA_{ijb}, $1 \le b \le m_{ij}$, together with the corresponding restriction values BPA^*_{ijb} and BPA^{**}_{ijb}, are pregiven. Here BPA^*_{ijb} denotes the worst acceptable value of BPA_{ijb}, while BPA^{**}_{ijb} stands for the best possible value. A quantitative parameter entering the outcome product which actually forms the product's competitive utility in order to gain future commercial success on the market, is called the basic competitive attribute (BCA). For each routine k-th competitive attribute of product V_{ij}, BCA_{ijk}, two opposite estimates have to be predetermined:

- the least preferred value BCA^*_{ijk} having practically very poor chances to win the market competition, and
- the most preferred value BCA^{**}_{ijk} which enables the attribute to win the competition.

Note that both opposite estimates for each competitive attribute can be obtained from the expert team on the basis of interview questions. Those estimates play the leading part in the process of questioning experts to obtain the multi-attribute utility values [125-127, 140, 150].

15.1.3 The expert interview procedure

In Chapter 14 we have outlined a modification of the classical MAUT procedure of both questioning experts and determining competitive utility values. In the present chapter we have implemented additional modifications, namely:

- for each project Q_{ij} its priority index η_{ij} (within the set of n_i projects entering the subsidiary corporation S_i), is assigned. Determining η_{ij} has to be carried out in advance either by the subsidiary corporation management, or by means of expert information;
- the experts' subjective judgment on competitive attributes $\left\{ BCA_{ijk} \right\}$, $1 \le k \le d_{ij}$, is based on analyzing the basic project attribute values BPA_{ijb}, $1 \le b \le m_{ij}$;
- for important basic project attributes (e.g., in safety engineering, environmental protection, etc.) which actually may have a strong influence on the future public opinion and thus the desired commercial success, experts have to forecast the corresponding restriction estimates, especially values BPA^{*}_{ijb}. Those estimates are usually determined in cost values (see §15.2);
- similarly to §14.1, the g-th expert's estimated index of importance of the k-th attribute BCA_{ijk} to win the competition for product V_{ij} on the market, namely parameter η_{ijgk}, has to be introduced;
- denote the g-th expert's personal subjective judgment to forecast the expected value BCA_{ijk}, by BCA_{ijgk};
- similarly to §14.1, after obtaining answers from all experts entering the expert team, we suggest to modify values BCA_{ijgk}, $1 \le k \le d_{ij}$, $1 \le g \le f$, to their relative equivalents γ_{ijgk} as follows:

  $$\gamma_{ijgk} = \frac{BCA_{ijgk} - BCA^{*}_{ijgk}}{BCA^{**}_{ijgk} - BCA^{*}_{ijgk}};$$
- the product's V_{ij} competitive utility, or it's marketability M_{ij}, is calculated by means of the experts' averaged conclusion

  $$M_{ij} = \frac{1}{f} \cdot \sum_{g=1}^{f} \sum_{k=1}^{m_{ij}} \left(\gamma_{ijgk} \cdot \eta_{ijgk} \right);$$
- we suggest to introduce the concept of the subsidiary corporation's S_i competitive utility, or marketability, by

  $$M_i = \sum_{j=1}^{n_i} \left(\eta_{ij} \cdot M_{ij} \right).$$

  Later on we will replace terms M_i and M_{ij} by more correct designations, namely, $M_i(C_i)$ and $M_{ij}(C_{ij})$. The latter denote the dependence of the competitive utility on the assigned budget values C_i and C_{ij}, correspondingly;
- the level of marketability of the holding corporation by use of expert information is calculated by means of

  $$M = \sum_{i=1}^{n} \left(\eta_i \cdot M_i \right).$$

It is the value which has to be maximized in the course of solving the general harmonization problem.

15.1.4 Notation

Let us introduce the following terms:

Holding Corporation Level

S - holding corporation;

C - holding corporation's budget (pregiven);

S_i - the i-th subsidiary corporation, $1 \le i \le n$;

n - number of subsidiary corporations;

C_i - budget assigned to the i-th subsidiary corporation (to be determined and optimized);

$C_{i\ min}$ - the minimal possible budget to be assigned to S_i (to be calculated beforehand);

$C_{i\ max}$ - the maximal budget to be assigned to S_i (to be calculated beforehand). In case $C_i > C_{i\ max}$ value $\delta C_i = C_i - C_{i\ max}$ will be redundant;

η_i - index of importance (priority index) of the i-th subsidiary corporation (pre-given);

M - holding corporation's marketability (to be maximized);

$M_i(C_i)$ - the i-th subsidiary corporation's marketability on the basis of budget C_i assigned to the corporation (a random value obtained by expert information);

D - the due date (pregiven);

ΔC - the search step length at the upper level (pregiven);

ε - the relative accuracy of the search procedure at the upper level (pregiven);

$\vec{X} = \{C_1, ..., C_n\}$ - a routine search point for the Couple Reverse Cyclic Coordinate Search Method (CRCCSM) algorithm, in order to reallocate C among n subsidiary corporations.

Subsidiary Corporation Level *(S_i Corporation)*

Q_{ij} - the j-th project entering the S_i-th corporation, $1 \le j \le n_i$;

V_{ij} - the product (services) to be designed and created by realizing Q_{ij};

n_i - number of projects (products, services) entering S_i (pregiven);

C_{ij} - budget assigned to Q_{ij} (to be determined and optimized);

$C_{ij\ min}$ - the minimal possible budget to be assigned to project Q_{ij} (to be calculated beforehand);

$C_{ij\ max}$ - the maximal budget to be assigned to Q_{ij} (to be calculated beforehand). In case $C_{ij} > C_{ij\ max}$ value $\delta C_{ij} = C_{ij} - C_{ij\ max}$ will be redundant;

η_{ij} - priority index of project Q_{ij} (relatively to other projects entering S_i);

M_{ij} - product's V_{ij} marketability;

$M_{ij}(C_{ij})$ - product's V_{ij} marketability determined (by expert information) on the basis of the assigned budget C_{ij};

ΔC_i - the search step length at the intermediate level (in the course of reallocating budget C_i among n_i projects);

ε_i - the relative accuracy of the search procedure with step length ΔC_i (pregiven);

$\vec{X}_i = \{C_{i1},...,C_{in_i}\}$ - a routine search point for the CRCCSM algorithm, in order to reallocate C_i among n_i projects.

<u>Project Level</u> *(Project Q_{ij})*

V_{ij} - the product to be designed and manufactured;

BPA_{ijb} - the b-th basic project attribute value, $1 \le b \le m_{ij}$;

m_{ij} - number of BPA values in product V_{ij};

BCA_{ijk} - the k-th basic competitive attribute value, $1 \le k \le d_{ij}$;

d_{ij} - number of BCA values in product V_{ij};

BPA^{*}_{ijb} - the worst permissible (acceptable) value for the b-th basic project attribute value BPA_{ijb} (pregiven);

BPA^{**}_{ijb} - the best possible value for BPA_{ijb} (pregiven);

BCA^{*}_{ijk} - the worst competitive estimate for the k-th competitive attribute BCA_{ijk} (pregiven by experts);

BCA^{**}_{ijk} - the best competitive estimate for BCA_{ijk} (pregiven by experts);

η_{ijgk} - priority index (level of importance) of the k-th competitive attribute of product V_{ij}, given by the g-th expert, $1 \le g \le f$;

f - the number of experts entering the decision-making team;

BCA_{ijgk} - the personal subjective judgment of the g-th expert on the expected value of the k-th competitive attribute BCA_{ijk};

M_{ij} - the product's V_{ij} marketability, determined by means of expert information;

M^{*}_{ij} - the minimal acceptable marketability (competitive utility) of product V_{ij},

$1 \le i \le n,\ 1 \le j \le n_i$ (pregiven);

ΔC_{ij} - the search step length at the project level, in the course of determining value $C_{ij\ min}$.

15.1.5 *The problem's formulation*

The strategic hierarchical harmonization problem is based on multistage budget reallocation and is as follows: determine budget values $\{C_i\}$ assigned to subsidiary corporations $\{S_i\}$, as well as budget values $\{C_{ij}\}$ assigned to individual projects $\{Q_{ij}\}$, in order to maximize the holding corporation's marketability

$$\underset{\{C_i\},\{C_{ij}\}}{Max}\ M \tag{15.1.1}$$

subject to

$$\sum_{i=1}^{n} C_i = C, \tag{15.1.2}$$

$$\sum_{j=1}^{n_i} C_{ij} = C_i, \tag{15.1.3}$$

$$M_{ij}\left(C_{ij}\right) \ge M_{ij}^{*} \tag{15.1.4}$$

$$Min\left\{BPA_{ijb}^{*}, BPA_{ijb}^{**}\right\} \le BPA_{ijb} \le Max\left\{BPA_{ijb}^{*}, BPA_{ijb}^{**}\right\}, \quad 1 \le i \le n,\ 1 \le j \le n_i,\ 1 \le b \le m_{ij}, \tag{15.1.5}$$

where (see Chapter 14)

$$M_{ij} = \frac{1}{f} \cdot \sum_{g=1}^{f} \sum_{k=1}^{m_{ij}} \left(\gamma_{ijgk} \cdot \eta_{ijgk}\right), \tag{15.1.6}$$

$$\gamma_{ijgk} = \frac{BCA_{ijgk} - BCA_{ijgk}^{*}}{BCA_{ijgk}^{**} - BCA_{ijgk}^{*}}, \tag{15.1.7}$$

and

$$M = \sum_{i=1}^{n} \left\{ \eta_i \left[\sum_{j=1}^{n_i} \eta_{ij} \cdot M_{ij}\left(C_{ij}\right) \right] \right\}. \tag{15.1.8}$$

We suggest solving the strategic hierarchical harmonization problem (15.1.1-15.1.8) by means of a three-stage algorithm. At the first stage a heuristic budget reallocation procedure implementing the CRCCSM, is carried out, namely, the problem

$$\underset{\{C_i\}}{Max} \sum_{i=1}^{n} \eta_i \cdot M_i(C_i) \tag{15.1.9}$$

subject to

$$\sum_{i=1}^{n} C_i = C, \tag{15.1.10}$$

$$C_{i\ min} \le C_i \le C_{i\ max}, \tag{15.1.11}$$

has to be solved. Here values $C_{i\min}$ and $C_{i\max}$ are not pregiven and have to be determined beforehand. Quasi-optimal values $\{C_i\}$ serve as input parameters for another, similar optimal budget reallocation problem at the intermediate hierarchical level

$$\underset{\{C_{ij}\}}{Max} \left\{ \sum_{j=1}^{n_i} \left[\eta_{ij} \cdot M_{ij}(C_{ij}) \right] \right\} \tag{15.1.12}$$

subject to

$$\sum_{j=1}^{n_i} C_{ij} = C_i, \tag{15.1.13}$$

$$C_{ij\ min} \le C_{ij} \le C_{ij\ max}, \quad 1 \le i \le n, \tag{15.1.14}$$

where values $C_{ij\min}$ and $C_{ij\max}$ also have to be determined beforehand. Note that evident relations

$$C_{i\ min} = \sum_{j=1}^{n_i} C_{ij\ min}, \tag{15.1.15}$$

$$C_{i\ max} = \sum_{j=1}^{n_i} C_{ij\ max}, \quad 1 \le i \le n, \tag{15.1.16}$$

hold.

At the lower level each random value $M_{ij}(C_{ij})$ has to be calculated by means of experts' judgments, on the basis of multiple interviews, and taking into account restrictions (15.1.5-15.1.6).

Note that other, non-essential modifications of problems (15.1.1-15.1.16), may be suggested, but the basic principles of the solution remain the same. In order to solve problem (15.1.1-15.1.8) we have to implement a new coordinate search algorithm which will be outlined below. To determine constraints $\left\{ C_{ij\min}, C_{ij\max} \right\}$, we will use auxiliary Problem A [14] which will be outlined below.

§15.2 Auxiliary problems and methods

15.2.1 Auxiliary problem A

It has been outlined above that calculating values $C_{ij \min(\max)}$ and $C_{i \min(\max)}$ has to be carried out beforehand of solving problem (15.1.1-15.1.8), as well as solving problems (15.1.9-15.1.11) – (15.1.12-15.1.14). The main difficulties arise in determining estimates $\left\{ C_{ij \min} \right\}$, since estimates (15.1.15-15.1.16) and

$$C_{ij\ max} = \sum_{b=1}^{m_{ij}} BPA_{ijb}^{**}, \quad 1 \le i \le n, \tag{15.2.1}$$

are evident.

To determine values $C_{ij \min}$ we will use the outlined above multi-attribute harmonization problem in strategic project management (see §14.1) to be denoted henceforth as *Problem A*. Note that the latter can be applied to individual projects only. To simplify the problem's solution we will, thus, omit unnecessary indices i and j. The problem is to maximize the total project's competitive utility (marketability) which results in maximizing the global marketability [12-15, 22]

$$M = \beta_P \cdot U_P + \beta_C \cdot U_C , \tag{15.2.2}$$

where U_P stands for the project's utility, U_C is the product's competitive multi-attribute utility, while β_P and β_C are properly chosen coefficients to present both utility parameters in similar scaling and ranking. Note that in modern project management most BPA estimates can be presented in cost values, as well as their corresponding lower and upper bounds BPA^* and BPA^{**}. Namely, to honor the company's good name, the following project's restrictions BPA^*, i.e., restrictions (15.1.5), have to be incorporated:

a) the budget for designing the product;
b) the budget for creating the product's pattern example;
c) the budget to undertake proper safety engineering actions;
d) the budget to implement proper actions to prevent jeopardizing environmental safety;
e) the budget to carry out advertisement programs;
f) the budget to realize various customer service and organizational procedures supporting the product's marketability, etc.

Thus, a conclusion can be drawn that for any product V the summarized constraint

$$C^* = \sum_{b=1}^{m} BPA_b^* , \tag{15.2.3}$$

where m is the total amount of basic project's attributes, can be regarded as the lower bound of the minimal budget to be assigned to the project in order to design and create a new product, i.e., relation

$$C^* \leq C_{min} \tag{15.2.4}$$

holds. *Here $C_{\min}$ denotes the minimal budget which enables honoring basic project attributes' together with the project's marketability restriction*

$$C_{min} = Min\left[C: \; M(C) \geq M^* \right] \tag{15.2.5}$$

subject to (15.1.5). Here $M(C)$ stands for the product's marketability on the basis of budget C assigned to the project for designing and creating the product.

Auxiliary *Problem A* as well as the corresponding *Model A* (see §14.1) enables determining $M(C)$, while M comprises both the utility of designing and creating the product's pattern example as well as its competitive utility to gain future commercial success.

The heuristic algorithm to solve *Problem A* is outlined in Chapter 14. A two-level optimization procedure based on the cyclic coordinate search algorithm (CCSA) [9-15, 133], is suggested. The internal level is faced with optimizing the product's competitive utility by making use of experts' information, while the external level centers on obtaining a routine feasible solution from the point of designing process.

15.2.2 Solving the problem of determining $\left\{ C_{ij\,\min} \right\}$

For the problem under consideration, in order to determine budget values $\left\{ C_{ij\,\min} \right\}$, $1 \leq i \leq n$, $1 \leq j \leq n_i$, we suggest to use auxiliary *Problem A* as follows. First, for each project Q_{ij} separately, values C_{ij}^* satisfying

$$C_{ij}^* = \sum_{b=1}^{m} BPA_{ijb}^* , \tag{15.2.6}$$

have to be calculated. Afterwards, for each project Q_{ij} separately, start increasing value C_{ij}^* by cost step ΔC_{ij}. After determining the routine accumulated value

$$C_{ij}^* + \Delta C_{ij} \Rightarrow C_{ij}^* , \tag{15.2.7}$$

solve *Problem A* to determine the marketability value $M_{ij}\left(C_{ij}^*\right)$. The process terminates after relation

$$M_{ij}\left(C_{ij}^*\right) < M_{ij}^* \qquad (15.2.8)$$

ceases to hold. The minimal value C_{ij}^* satisfying (15.1.4), is taken as the problem's solution, i.e., as the minimal budget value $C_{ij\ min}$ which may be assigned by S_i to project Q_{ij}. Other restrictive values $\left\{C_{ij\ max}\right\}$, $\left\{C_{i\ min}\right\}$ and $\left\{C_{i\ max}\right\}$ can be determined by using (15.1.15-15.1.16, 15.2.1).

15.2.3 *Couple Reverse Cycling Coordinate Search Method (CRCCSM)*

It can be well-recognized that both budget reallocations problems (15.1.9-15.1.11) and (15.1.12-15.1.14) deal with random values, since all marketability estimates for each element of the hierarchical system "holding corporation – subsidiary corporation – project (product)" - are partially determined by means of experts' subjective judgments. Moreover, the objectives at each hierarchical level are in fact non-linear ones. This makes impossible to solve those problems via mathematical programming models. Thus, only heuristic approaches enable an approximate satisfactory solution.

In our previous publications we have used the cyclic coordinate search algorithm (CCSA) [9-15, 133] in order to optimize a complicated non-linear multi-dimensional objective. However, it is impossible to use CCSA for solving budget reallocation problems (15.1.9-15.1.11) and (15.1.12-15.1.14), since undertaking a search for a certain coordinate results in introducing reverse amendments for other coordinates [5, 15]. Due to dependability restrictions (15.1.11) and (15.1.14), the CCSA requires essential modifications.

We have developed a modified version of the CCSA - the Couple Reverse Cyclic Coordinate Search Method (CRCCSM), which enables carrying out an optimization search procedure for a three-level hierarchical system under consideration. The general approach of using CRCCSM for multi-dimensional systems under random disturbances is as follows.

Given:

- variables (coordinates) $X_1, X_2, \dots, X_n$ to be optimized;
- non-linear multi-dimensional objective $F\left(X_1, X_2, \dots, X_n\right)$ with complicated logical links and subject to random disturbances (e.g., comprising subjective experts' judgments);
- linear restrictions

$$\sum_{i=1}^{n} X_i = C, \tag{15.2.9}$$

$$X_{i\,min} \le X_i \le X_{i\,max}, \quad 1 \le i \le n, \tag{15.2.10}$$

where C, $X_{i\,min}$ and $X_{i\,max}$ are constant pregiven values;

- non-linear restrictions under random disturbances

$$F_q(X_1, X_2, \dots, X_n) \ge W_q, \quad 1 \le q \le r, \tag{15.2.11}$$

- search step length ΔX common for all coordinates X_i;
- relative accuracy $\varepsilon > 0$ of the search procedure.

The problem is to determine quasi-optimal values $X_1^*, X_2^*, \dots, X_n^*$ *to maximize the objective* $F(\vec{X})$ *subject to all restrictions.*

The enlarged step-wise procedure of the problem's solution by implementing the CRCCSM is as follows:

Step 1. By any mean establish a feasible problem's solution, i.e., determine values $X_1, X_2, \dots, X_n$ satisfying relations (15.2.9-15.2.11). The techniques to be used depend usually on the structure of objective F. However, reasonable approaches for carrying out the step are outlined in various publications (see, e.g., [9]). Values X_i, $1 \le i \le n$, obtained in the course of Step 1, are taken as the initial search point $\vec{X}^0$. Denote vector $\vec{X}^0$'s coordinates by X_{0i}, $1 \le i \le n$.

Step 2. For initial point $\vec{X}^0$ calculate it's corresponding objective value $F(\vec{X}^0)$. Note that since $\vec{X}^0$ presents a *feasible solution*, restrictions (15.2.9-15.2.11) will be honored.

Store vector $\vec{X}^0$ in a special array A together with the corresponding objective value. In the course of the optimization process array A will contain the problem's approximate solution.

Step 3. First, coordinate X_1 has to be optimized, by advancing with the constant search step ΔX in two opposite directions:

$X_{01} + \Delta X \Rightarrow X_{11}$ and $X_{01} - \Delta X \Rightarrow X_{11}$. If both directions can be taken, go to Step 11.

If, for a certain reason, only one direction can be chosen, e.g., $X_{01} + \Delta X \Rightarrow X_{11}$ or $X_{01} - \Delta X \Rightarrow X_{11}$, apply the next step. If no changes can be introduced for the first coordinate, go to Step 7. Note that cases of less than two possible directions usually take place when value X_1 is on the border, i.e., relations $X_{01} = X_{1\,min}$ or $X_{01} = X_{1\,max}$ hold.

Step 4. Assume that only direction $X_{01} + \Delta X \Rightarrow X_{11}$ can be chosen. Calculate X_{11} and choose a reverse direction for the second coordinate to decrease the latter, i.e., determine $X_{02} - \Delta X \Rightarrow X_{12}$. If such a reverse direction cannot be undertaken (usually when $X_{02} = X_{2\,min}$ holds), try to implement a reverse direction for the third coordinate X_3, and so forth, until either:

- for a certain coordinate X_i, $2 \le i \le n$, such a reverse direction can be chosen and value $X_{0i} - \Delta X \Rightarrow X_{1i}$ is calculated. Go to Step 5.
- or for any coordinate X_i, $2 \le i \le n$, no reverse direction can be chosen. Go to Step 7.

If, in the course of carrying out Step 4, direction $X_{01} - \Delta X \Rightarrow X_{11}$ is chosen, substitute the reverse direction by *increasing* the second coordinate. In all other aspects the step remains unchanged.

Step 5. Calculate objective F for the new vector $\{X_{11}, \ldots , X_{1i}, \ldots , X_{0n}\} = \vec{X}^1$ where, besides coordinates X_1 and X_i, all other coordinates remain unchanged. If either:

- at least one restriction (15.2.9-15.2.11) is not honored, or
- objective value F did not increase,

- go to the next step. Otherwise apply Step 8.

Step 6. Apply Step 4 and try to determine a new coordinate X_i with a possible reverse direction. If such a coordinate X_i, $2 \le i \le n$, can be determined, go to Step 5. Otherwise apply the next step.

Step 7. Applying this step means that the first coordinate X_1 cannot be optimized. Substitute coordinate X_1 by the next coordinate X_2 and apply Step 3. While carrying out anew Steps 3-7 the index of the coordinate to be optimized has to be increased by one, i.e., the optimized coordinate changes from X_1 to X_2, later on from X_2 to X_3, and so forth, until all coordinates $X_1, X_2, \ldots , X_n$ are examined. Then go to Step 13.

Step 8. Check relation

$$\frac{F\left(\vec{X}_1\right) - F\left(\vec{X}_0\right)}{F\left(\vec{X}_0\right)} < \varepsilon \ . \qquad (15.2.12)$$

If the relation holds, go to Step 10. Otherwise apply the next step.

Step 9. Take search point $\vec{X}^1$ for the initial one, i.e., transform $\vec{X}^1 \Rightarrow \vec{X}^0$. Store the newly obtained vector $\vec{X}$ together with its objective value F. Go to Step 3. Note that in the course of carrying out Step 9 the index of the optimized coordinate has not to be changed for the next one in Steps 3-7.

Step 10. The optimization process terminates, and vector $\vec{X}^{1}$ together with its corresponding objective value is taken as the quasi-optimal problem's solution.

Step 11. Check both new search points (vectors)

$$\vec{X}^{1} = \{ X_{01} + \Delta X, X_{02}, \ldots, X_{0n} \} \qquad (15.2.13)$$

and

$$\vec{X}^{2} = \{ X_{01} - \Delta X, X_{02}, \ldots, X_{0n} \}, \qquad (15.2.14)$$

i.e., check restrictions (15.2.9-15.2.11) together with calculating objective values $F\left(\vec{X}^{1}\right)$ and $F\left(\vec{X}^{2}\right)$. If at least one restriction (15.2.9-15.2.11) proves not to be satisfied, or objectives $F\left(\vec{X}^{1}\right)$ and $F\left(\vec{X}^{2}\right)$ did not increase for both search points $\vec{X}^{1}$ and $\vec{X}^{2}$, go to Step 7. If for one of the search points all the restrictions are honored and the objective value has increased (let it be $\vec{X}^{1}$), go to Step 8. If for both search points under examination all the restrictions are honored and both objectives have increased their values, choose the search point with the highest increase. For the sake of definition, let it be $\vec{X}^{1}$. Apply the next step.

Step 12. If relation (15.2.12) holds go to Step 10. Otherwise take search point $\vec{X}^{1}$ as the initial one and go to Step 4, i.e., proceed changing the optimized coordinate *in the same direction* as it has been chosen by determining point $\vec{X}^{1}$. Similarly to Step 9, the index of the optimized coordinate has not to be altered in the course of repeating Steps 4-7.

Step 13. After all coordinates $X_{1}, X_{2}, \ldots, X_{n}$ have been examined by means of the CRCCSM algorithm, proceed the optimization process further in cyclic succession, i.e., after coordinate X_{n} return optimizing X_{1}, etc.

The optimization process terminates if in the course of examining all coordinates $X_{1}, X_{2}, \ldots, X_{n}$ the latter do not undergo any changes. In case like that vector $\vec{X}$ entering array A, together with the objective value, are taken as the problem's quasi-optimal solution.

§15.3 Three-level heuristic algorithm for the hierarchical harmonization problem

The general idea of the three-level algorithm to solve the strategic harmonization problem for complex holding corporations is presented on Fig. 15.1. The step-wise procedure of the algorithm is based on the results outlined in §15.2 and comprises the following subalgorithms:

Subalgorithm 1. Using auxiliary *Problem A* (see §15.2), outlined in details in §14.1, determine for each project Q_{ij} independently value $C_{ij\,\min}$ in order to cover the minimal acceptable marketability M_{ij}^{*}. The general idea of *Subalgorithm 1* is outlined in §15.2. The input values are $\left\{ BPA_{ijb} \right\}$ which are forwarded to the expert team to undertake a multi-stage mutual man-computer dialog.

Subalgorithm 2. This block centers on determining a feasible initial search point for solving problem (15.1.9-15.1.11) at the highest hierarchical level by means of the CRCCSM. Note that input values for *Subalgorithm 2* are $\left\{ C_{i\,\min} \right\}$ and $\left\{ C_{i\,\max} \right\}$ obtained from relations (15.1.15-15.1.16, 15.2.1) on the basis of the input parameters of *Problem A*, as well as of priority indices η_i, $1 \le i \le n$. The substeps of *Subalgorithm 2* are as follows:

2.1. Reorder all priority indices η_i in descending order.

2.2. Reorder all corresponding ordinal numbers of subsidiary holdings S_i.

2.3. Assign to all S_i their minimal budget values $C_{i\ min}$, $C_{i\ min} \Rightarrow C_i$, $1 \le i \le n$.

2.4. Set $\alpha = 1$.

2.5. Calculate $W_\alpha = Min\left[\Delta C = C - \sum_{i=1}^{n} C_{i\ min} ,\ C_{\alpha\ max} - C_{\alpha\ min} \right]$.

2.6. Determine for the subsidiary holding S_α its feasible budget $W_\alpha + C_{\alpha\ min} = C_\alpha$.

2.7. Update $\Delta C - W_\alpha \Rightarrow \Delta C$. If $\Delta C = 0$ go to Substep 2.10. Otherwise apply the next substep.

2.8. Set $\alpha + 1 \Rightarrow 1$.

2.9. If $\alpha \le n$ go to Substep 2.5. Otherwise apply the next substep.

2.10. Vector $\vec{C_i}$ serves as a feasible solution of problem (15.1.9-15.1.11) and as the initial search point X.

It is assumed that a reasonable relation $\sum_{i=1}^{n} C_{i\,\min} \le C$ holds, otherwise the problem has no solution. Another relation $\sum_{i=1}^{n} C_{i\,\max} > C$ is evident, otherwise values $C_{i\,\max}$, $1 \le i \le n$, are the sought optimum variables at the higher hierarchical level.

Subalgorithm 3 centers on implementing the couple reverse cyclic coordinate search method for problem (15.1.9-15.1.11), with initial feasible search point X. The CRCCSM is outlined in §15.2. Steps 2, 5 and 11 result in applying *Subalgorithm 4* at the intermediate hierarchical level (subsidiary corporation level). The search step length equals ΔC.

Subalgorithm 4. The input values for the subalgorithm are values $\{C_i\}$, $1 \le i \le n$. For each corporation S_i independently, the couple reverse cyclic coordinate search method has to be carried out, in order to redistribute budget value C_i among n_i projects Q_{ij}, $1 \le j \le n_i$, i.e., to solve budget allocation problem (15.1.12-15.1.14). Thus, instead of the n -dimensional CRCCSM of *Subalgorithm 3*, a n_i-dimensional search procedure has to be implemented. Determining the initial search point is carried out similarly to the analytical approach outlined in *Subalgorithm 2*. Steps 2, 5 and 11 of the CRCCSM procedure (see §15.2) are realized by applying the following *Subalgorithm 5* at the project level. For each subsidiary corporation S_i the search step length equals ΔC_i. Thus, values C_{ij} serve as input values for that subalgorithm.

Subalgorithm 5 is carried out by calculating the product's competitive utility and, later on, by determining the product's marketability M_{ij}, on the basis of experts' information. Establishing value M_{ij} is facilitated via (15.1.6-15.1.7), on the basis of the averaged experts' subjective judgments [13, 150, 168]. The calculated value $M_{ij}(C_{ij})$ is provided to *Subalgorithm 4*, where for each subsidiary corporation marketability values $M_i(C_i)$ have to be maximized according to (15.1.12). Thus, a multiple information exchange among *Subalgorithms 4* and *5* takes place until, for all projects Q_{ij}, $1 \le i \le n$, $1 \le j \le n_i$, conditional sub-optimal values $M_{ij}^{(opt)}$ and $M_i^{(opt)}$ are determined, together with the corresponding values C_{ij} and C_i. Only afterwards values $M_i^{(opt)}$ are provided to *Subalgorithm 3* (see Steps 2, 5 and 11 of CRCCSM in §15.2) in order to calculate objective F - the marketability value of the holding corporation at the routine search point of problem's (15.1.9-15.1.11) solution.

In the course of carrying out the CRCCSM by *Subalgorithm 3* by means of multiple applications to *Subalgorithm 4*, conditional sub-optimal values $M_i^{(opt)}$, $1 \le i \le n$, undergo numerous changes, until the search process at the higher level terminates. Finally obtained values M, M_i, M_{ij}, C_i and C_{ij}, $1 \le i \le n$, $1 \le j \le n_i$, are taken as the quasi-optimal solution.

Thus, a double optimization cycle to solve the hierarchical harmonization problem has to be implemented.

§15.4 Conclusions

The following conclusions can be drawn from the Chapter:

1. The harmonization model for a large hierarchical holding corporation refers to a very important area of strategic management. The desired future of any complex holding corporation is to gain commercial success on international and domestic markets. Since nowadays the budget of such corporations often exceeds billions of dollars, even small quality refinements may result in tremendous additional profits.

2. The suggested harmonization model for a holding corporation comprises local harmonization models for subsidiary corporations. The latter enter the general holding corporation and usually form a hierarchical "tree". Thus, the harmonization model itself turns out to be a hierarchical model, with a governing sub-model at the higher level. The intermediate level comprises harmonization models for subsidiary corporations. At the lower level harmonization models for individual projects, to design and create new products and/or new service systems, have to be incorporated.

3. The harmonization model for a holding corporation actually refers to multi-dimensional budget reallocation models under random disturbances. This is an extremely complicated non-linear model which cannot obtain an analytic solution. Only heuristic, approximate approaches may be implemented.

4. Since one of the most simple and efficient methods to optimize non-linear multi-dimensional models - the cyclic coordinate search algorithm (CCSA, see [133]) - cannot be applied to hierarchical budget reallocation models, we developed an efficient modified version of the CCSA - the couple reverse CCSA (CRCCSM) which has been applied to the optimization process for the general harmonization model of the holding corporation.

5. The developed harmonization model, thus, comprises three levels together with an auxiliary expert's team at the fourth level. The first two levels comprise a double cycle optimization model by means of CRCCSM. The project level calculates the marketability of all individual projects and service systems independently, while the experts' level provides experts' subjective judgments on basic competitive attributes for the future commercial success of new products and service systems.

6. The suggested harmonization model is an essential extension of our previous publications on harmonization models for estimating the project's utility outlined in [9]. As a matter of fact, the results obtained in the past are not more but several structural elements in the newly developed hierarchical tree.

7. If necessary, due to specific market requirements, etc., additional project's and competitive attributes may be incorporated in the harmonization model.

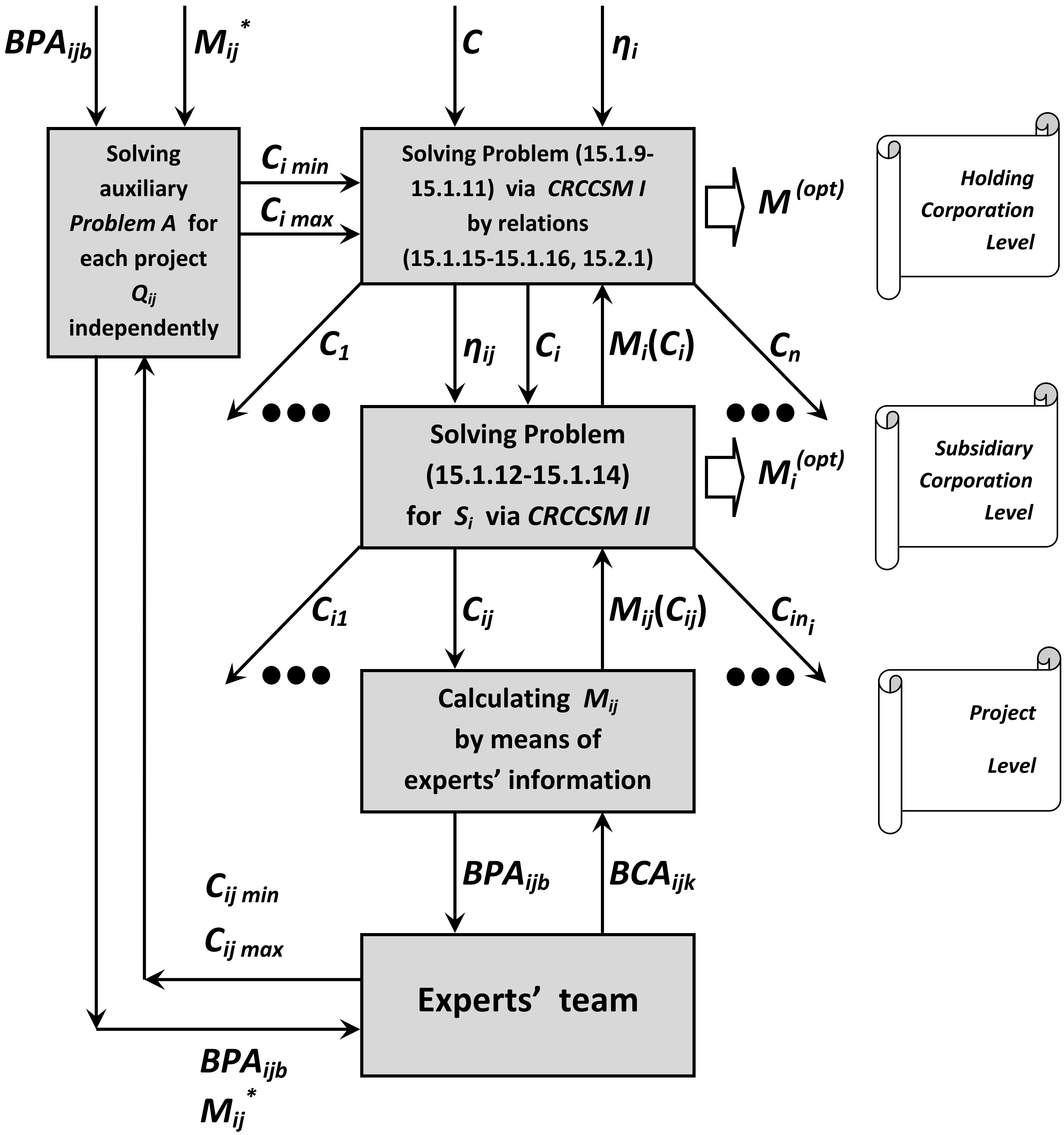

Figure 15.1. *The general idea of the three-level heuristic harmonization algorithm*

Chapter 16. Optimization Models in Strategic Marketability

§16.1 The problem

16.1.1 Introduction

In Chapters 8, 14-15 we have outlined various harmonization models to determine system quality parameters in project management. Those models deal mostly with marketability estimates of outcome products in order to gain future commercial success. Those models present an essential extension of the recently developed multi-attribute utility theory [125-127, 150].

In this Chapter we will solve the problem of optimizing the individual project's utility, i.e., maximizing the marketability estimate of the new product to be obtained in the course of the project's realization. The process of designing and creating the product is described in the form of a stochastic PERT-COST network model.

Each activity (i, j) entering network model $G(N, A)$ has a random duration $t(i, j)$ depending parametrically on the budget value $c(i, j)$ assigned to that activity. Note that each value $c(i, j)$ has two opposite restrictive values $c(i, j)_{\min} \le c(i, j) \le c(i, j)_{\max}$. In case $c(i, j) < c(i, j)_{\min}$ activity (i, j) cannot be realized, in case $c(i, j) > c(i, j)_{\max}$ cost value $\Delta c_{ij} = c(i, j) - c(i, j)_{\min}$ is redundant.

The product's marketability can be calculated by means of experts' subjective judgment in the form of a function of basic competitive attributes BCA_k, $1 \le k \le n$, [14, 150, 168], which actually determine the future commercial success. It can be well-recognized that values BCA_k depend on the total budget C to be invested in the project's realization to design and to create the new product. Note that besides obtaining proper attribute values, honoring both the pregiven due date of completing the project, as well as the reliability value of meeting the due date on time, has also to be taken into account. Thus, the problem is as follows:

Given:

- the stochastic network graph of PERT-COST type $G(N, A)$ comprising activities of random duration,
- the due date D (pregiven),
- chance constraint $R = Pr\{T(G) \le D\}$ of accomplishing the project on time (pregiven),

- the problem is to determine the minimal project total budget $C_{\min}$ to be distributed among the project's activities (i, j), in order to obtain the highest possible new product's marketability value M. Besides determining total value $C_{\min}$, values $c(i, j)$ assigned to project's activities (i, j) have to be determined as well.

A dual cost-optimization problem can be formulated as follows: given the restricted project's budget C together with pregiven due date and reliability values D and R, determine the maximal marketability value M (here value M may not be the highest possible one). Note that solving the direct cost-optimization problem usually enables the solution of the dual one.

Both problems are solved by means of a heuristic algorithm which comprises three different procedures:

1. A lookover search procedure of determining the optimal value of C.
2. A trade-off "time - cost" model to maximize the project's reliability R with preset value C.
3. A heuristic corrective procedure of redistributing a pregiven budget value C among the project's activities in order to maximize the product's marketability subject to the pregiven chance constraint.

A newly developed important conception of the attribute's sensitivity relative to the corresponding investment is introduced. The suggested compound rate, i.e., the so-called cost-marketability, is imbedded in the developed budget reallocation algorithm [19, 22].

16.1.2 The problem's description and definitions

The newly developed product to be delivered to the market possesses n basic competitive attributes BCA_k, $1 \le k \le n$, together with its corresponding restriction values (see Chapter 14 and [14, 26]. The latter serve as the worst permissible values that may be implemented into the design process of the new product. Several basic competitive attributes may be independent as well as dependent parameters. BCA values form the product's competitive utility in order to gain future commercial success. Those values are usually calculated by means of expert information.

We will assume [14, 150] that for each BCA_k value two opposite estimates have to be established before carrying out the design process:

- the worst competitive estimate BCA_k^* having practically very poor chances to win the market competition, and
- the best competitive estimate BCA_k^{**} which enables the attribute to win the competition.

Note that both opposite estimates for each competitive attribute can be obtained from the expert team on the basis of interview questions.

As an example of competitive attributes, nine of them have been singled out and are usually examined by experts in case of designing a new automotive vehicle (see *Section 14.1.9*).

As to the worst and the best competitive attribute estimates, they are presented in *14.1.9* as well.

As outlined above, the process of designing and creating the new product can be formalized and described by means of a PERT-COST stochastic network model $G(N,A)$ with activities (i,j) of random duration. In order to formalize the problem to be considered we will require additional definitions.

Call the k*-th array* $AR_k \subset G(N,A)$, $1 \le k \le n$, a unification of activities $(i,j) \subset G(N,A)$ which are related to attribute BCA_k and have to be carried out in order to create a product with that attribute. Denote henceforth those activities $\{(i_k, j_k)\}$. It goes without saying that for certain different arrays AR_{k_1} and AR_{k_2}, $1 \le k_1, k_2 \le n$, their intersection may not be empty, i.e., two different arrays may comprise similar activities. An array is usually presented not in the form of a graph, but rather in the form of a list of activities.

As an example of an activity array for a new vehicle [14, 168], array AR_6 comprises activities $\{i_6, j_6\}$ to be processed in order to raise the unrefueled vehicle's range as much as possible.

Call the k*-th sub-budget* $C_k < C$ the partial budget assigned to carry out activities entering array AR_k,

$$C_k = \sum_{\{(i_k, j_k)\}} c(i_k, j_k), \qquad (16.1.1)$$

where $c(i_k, j_k)$ represents the budget assigned to activity (i_k, j_k).

Call the *minimal budget value* C_k^* the minimal budget required to obtain the new product with the least preferred attribute BCA_k^*. Call the *maximal budget value* C_k^{**} the assigned budget which results in the best attribute's value BCA_k^{**}.

Call the *priority level* (level of importance) η_k *of the* k*-th competitive attribute* BCA_k a positive value $0 < \eta_k < 1$, $\sum_k \eta_k = 1$, determined by experts and denoting the contribution of a BCA_k unit in the marketability estimate M. Thus, relation

$$M = \sum_{k=1}^{n} (\eta_k \cdot BCA_k) \qquad (16.1.2)$$

holds.

Call the *cost-sensitiveness of each attribute* BCA_k, $1 \le k \le n$, the ratio

$$\gamma_k = \frac{\eta_k \cdot \left(BCA_k^{**} - BCA_k^{*}\right)}{C_k^{**} - C_k^{*}}, \tag{16.1.3}$$

which determines the contribution of each cost unit of sub-budget C_k invested in the project, to the entire product's marketability M. Relation $\gamma_{k_1} > \gamma_{k_2}$ signifies that attribute BCA_{k_1} has a higher cost-sensitivity than attribute BCA_{k_2}.

16.1.3 Notation

Let us introduce the following terms:

$G(N,A)$ - the PERT-COST network model to formalize the process of designing and creating a new product;

(i,j) - activity with random duration $t(i,j)$ entering $G(N,A)$;

$c(i,j)$ - budget assigned to activity (i,j);

$c(i,j)_{min}$ - the lower possible value of budget $c(i,j)$ (pregiven);

$c(i,j)_{max}$ - the upper bound of value $c(i,j)$ (pregiven);

BCA_k - the k-th basic competitive attribute value of the newly developed product, $1 \le k \le n$;

n - number of BCA values;

BCA_k^{*} - the worst competitive estimate of the k-th competitive attribute (pregiven by experts);

BCA_k^{**} - the best competitive estimate of the k-th competitive attribute (pregiven by experts);

η_k - priority level (level of importance) of the k-th competitive attribute (pregiven by experts);

D - due date of project $G(N,A)$ (pregiven);

R - reliability (chance constraint) value for accomplishing the project on time (pregiven);

C - total budget invested in the project (to be determined);

$T\left[G \middle| C = \sum_{(i,j)} c(i,j)\right]$ - random project's duration on condition that budget C has been invested and later on redistributed among activities (i,j);

AR_k - a set of activities entering $G(N,A)$ and related to the attribute BCA_k (to be determined beforehand);

$\{(i_k, j_k)\}$ - activities entering AR_k;

m_k - number of activities entering AR_k;

C_k - budget assigned to carry out activities entering AR_k; note that usually $\sum_{k=1}^{n} C_k > C$ holds, since sets $\{AR_k\}$ have common activities;

C_k^* - budget required to provide attribute BCA_k with the worst value BCA_k^* (given by experts);

C_k^{**} - budget required to provide the best competitive value BCA_k^{**} (given by experts);

γ_k - cost-sensitivity value for attribute BCA_k calculated by (16.1.3) (to be determined beforehand);

M - the product's marketability calculated by (16.1.2);

M^* - the minimal possible marketability to be obtained by investing $\{C_k^*\}$, $1 \le k \le n$, to each array $\{AR_k\}$. Value M^* is always obtained by assigning $c(i,j)_{min}$ to each activity $(i,j) \subset G(N,A)$;

M^{**} - the maximal possible product's marketability. It can be well-recognized that M^{**} can be obtained by assigning $c(i,j)_{max}$ to each (i,j). However, in most practical cases M^{**} can be reached by investing budget $C < \sum_{(i,j)} c(i,j)_{max}$;

ΔC - the cost unit to be transferred from activity to activity;

δC - the cost step, i.e., the total budget's step, for solving the general problem.

16.1.4 The problem's formulation

The problem is as follows:

Determine the minimal budget value $C = \sum_{(i,j)} c(i,j)$ which

- results in the product's marketability being equal M^{**}, and
- satisfies chance constraint

$$Pr\left\{ T\left[G \middle| C = \sum_{(i,j)} c(i,j) \right] \le D \right\} \ge R. \tag{16.1.4}$$

Since any product's marketability value M is, practically speaking, an implicit function $M\{C\}$ of the project's budget C, the optimization problem can be formalized as follows:

Determine optimal budget value C^{opt} together with assigned activity budgets $\{c(i,j)^{opt}\}$ which results in the maximal product's marketability M^{**},

$$C^{opt} = Min\left\{ C : M(C) = M^{**} \right\} \qquad (16.1.5)$$

subject to (16.1.4).

It can be well-recognized that problem (16.1.4-16.1.5) is a multi-parametrical non-linear stochastic optimization problem, which allows only approximated heuristic solutions. The main difficulty of solving problem (16.1.4-16.1.5) is to formalize the implicit function $M\{C\}$ which reflects the experts' subjective judgments. Assume that for any k-th attribute, $1 \le k \le n$, its contribution $\eta_k \cdot BCA_k$ can be determined by

$$\eta_k \cdot BCA_k = BCA_k^* + \gamma_k \cdot \left(C_k - C_k^*\right), \qquad (16.1.6)$$

where γ_k stands for the cost-sensitivity of attribute BCA_k to be calculated beforehand by (16.1.3), and C_k represents the partial budget to carry out activities entering array AR_k. Thus, we assume that cost-sensitiveness remains constant for each k-th attribute throughout interval $\left[BCA_k^*, BCA_k^{**} \right]$.

The above assumption enables heuristic solution of problem (16.1.4-16.1.5) by using two subsidiary procedures.

§16.2 Subsidiary procedures

16.2.1 Reliability optimization (Procedure I)

In [9, 12, 70, 109] a trade-off cost-reliability procedure is considered. The problem is as follows:

Given:

a) a PERT-COST type network model $G(N,A)$ comprising activities $(i,j) \subset G(N,A)$ with random time durations $t(i,j)$ depending parametrically on the local budgets assigned to those activities $c(i,j)$;
b) upper and lower budget levels $c(i,j)_{min}$ and $c(i,j)_{max}$ for each activity (i,j);
c) the project's due date D;
d) probability density function $p\left[t(i,j), c(i,j) \right]$ for each activity duration $t(i,j)$ for all $(i,j) \subset G(N,A)$;
e) the total project's budget C,

- determine local budgets $c(i,j)$ to all activities in order to maximize the project's reliability

$$\underset{\{c(i,j)\}}{Max}\ Pr\left\{ T\left[G \middle| C=\sum_{(i,j)} c(i,j) \right] \le D \right\} \tag{16.2.1}$$

subject to

$$c(i,j)_{min} \le c(i,j) \le c(i,j)_{max}, \tag{16.2.2}$$

$$\sum_{(i,j)} c(i,j) = C. \tag{16.2.3}$$

The problem's solution algorithm is outlined in details in Chapter 8 and is based on extensive simulation of the p.d.f.'s $p\left[t(i,j),c(i,j)\right]$ together with various heuristic procedures. Most of them are based on examining special activity values $\mu(i,j)\cdot P\left\{(i,j)\middle| c(i,j)\subset L_{cr}\right\} = \lambda\left\{(i,j)\middle| c(i,j)\right\}$, where $\mu(i,j)$ stands for the average value of $t(i,j)| c(i,j)$ and $P\left\{(i,j)\middle| c(i,j)\subset L_{cr}\right\}$ denotes the probability of activity (i,j) to be on the critical path in the course of the project's realization, on condition that each activity (i,j) obtains local budget $c(i,j)$, subject to (16.2.2) and (16.2.3). A unit cost transfer procedure from activities (i,j) with small $\lambda(i,j)$ values to activities with larger $\lambda(i,j)$ values, is arranged in order to diminish the project's critical path as much as possible.

Call henceforth problem (16.2.1-16.2.3) *Problem I*. Note, that a dual problem can be formulated as follows:

Given the chance constraint (reliability value) R, determine the *minimal* total project's budget C, together with local assigned cost budgets $c(i,j)$, subject to (16.2.2-16.2.3). Thus, the problem satisfies

$$Min\ C\left(C: Pr\left\{ T\left[G \middle| C=\sum_{(i,j)} c(i,j) \right] \le D \right\} \ge R \right) \tag{16.2.4}$$

subject to (16.2.2) and (16.2.3).

Call henceforth problem (16.2.2-16.2.3) *Problem II*. Both problems - the direct and the dual one - are outlined in Chapter 8 together with their detailed solutions.

Note, in conclusion, that both *Problems I* and *II* comprise a standard problem as follows:

Given:

- the project's budget C;

- assigned cost values $c(i,j)$ satisfying (16.2.2) and (16.2.3);
- the project's due date D,

  - determine the probability of meeting the project's target on time, i.e.,

$$Pr\left\{ T\left[G \middle| C = \sum_{(i,j)} c(i,j) \right] \le D \right\}. \tag{16.2.5}$$

Value (16.2.5) can be determined by simulating the project's realization in order to obtain representative statistics. Call henceforth problem (16.2.5) *Problem III*.

16.2.2 *Subsidiary corrective Procedure II to increase product's marketability and corporate sustainability*

The procedure outlined below presents a heuristic solution of the problem as follows:

Given:

- the PERT-COST type network model with activities of random duration $t(i,j)$ depending parametrically on the budget values assigned to that activities (p.d.f. given);
- total project's budget C;
- assigned budget values $c(i,j)$ satisfying (16.2.2) and (16.2.3);
- project's due date D;
- project's reliability R satisfying (16.2.5);
- arrays $\{AR_k\}$, $1 \le k \le n$, which actually define n BCA_k; $\{AR_k\}$ are given in the form of activity sets;
- values BCA_k^*, BCA_k^{**}, η_k, $C_k^* = \sum_{(i,j) \subset AR_k} c(i,j)_{min}$, $C_k^{**} = \sum_{(i,j) \subset AR_k} c(i,j)_{max}$ and γ_k, $1 \le k \le n$, obtained by (16.1.3);
- marketability value M calculated by (16.1.2), (16.1.3) and (16.1.6), where $C_k = \sum_{(i,j) \subset AR_k} c(i,j)$; it goes without saying that reasonable relations $C_k^* \le C_k \le C_k^{**}$, $1 \le k \le n$, hold;

  - the problem is to determine new assigned values $\bar{c}(i,j)$ to maximize value (16.1.2) subject to (16.2.2), (16.2.3) and

$$Pr\left\{ T\left[G \middle| C, \bar{c}(i,j) \right] \le D \right\} \ge R . \tag{16.2.6}$$

Thus, the objective to be maximized is as follows:

$$\underset{\{\bar{c}(i,j)\}}{Max} \sum_{k=1}^{n} \eta_k \left[BCA_k^* + \gamma_k \left(\overline{C}_k - C_k^* \right) \right] \tag{16.2.7}$$

subject to

$$\sum_{\{(i,j)\}} \bar{c}(i,j) = C\,, \tag{16.2.8}$$

$$c(i,j)_{min} \le \bar{c}(i,j) \le c(i,j)_{max}\,, \tag{16.2.9}$$

$$Pr\left\{ T\left[G \middle| C\,, \bar{c}(i,j) \right] \le D \right\} \ge R\,, \tag{16.2.10}$$

$$C_k^* \le \overline{C}_k = \sum_{(i,j)} \bar{c}(i,j) \le C_k^{**}. \tag{16.2.11}$$

The step-wise heuristic procedure of solving problem (16.2.7-16.2.11) is as follows:

<u>Step 1</u>. Reorder the cost-sensitivity values γ_k of attributes BCA_k, $1 \le k \le n$, in descending order. Thus, denote henceforth BCA_1 the competitive attribute with the highest cost-sensitivity, BCA_2 - with lower cost-sensitivity, etc. Attribute BCA_n obtains, thus, the least cost-sensitivity. It goes without saying that the corresponding arrays $\{AR_k\}$, $1 \le k \le n$, receive from now the same relative order numbers as well.

<u>Step 2</u>. For each array AR_k reschedule the corresponding entering activities $\left(i_{k_\xi}\,, j_{k_\xi} \right)$, $1 \le \xi \le m_k$, in descending order of the product

$$\mu\left(i_{k_\xi}\,, j_{k_\xi} \right) \cdot p\left(i_{k_\xi}\,, j_{k_\xi} \right) = \upsilon\left(i_{k_\xi}\,, j_{k_\xi} \right), \quad 1 \le \xi \le m_k\,, \tag{16.2.12}$$

where $\mu\left(i_{k_\xi}\,, j_{k_\xi} \right)$ stands for the average value of the activity's duration and $p\left(i_{k_\xi}\,, j_{k_\xi} \right)$ represents the probability of the activity to be on the critical path in the course of the project's realization. The regarded probabilities can be easily calculated by solving *Problem III*, with pregiven C and $c(i,j)$, in the course of maximizing reliability value (16.2.5).

All activities (i,j) in arrays AR_k with $p(i,j) = 0$ have to be rescheduled at the end of the list (for each array) in descending order of their average values $\mu(i,j)$.

Note that Steps 1-2 have to be carried out beforehand and do not actually participate in solving problem (16.2.7-16.2.11).

Step 3. Set $v=1$ and $w=n$.

Step 4. Determine activity entering AR_v *with the highest order* for which the assigned budget is less than the corresponding maximal possible budget value. Let this activity be $\left(i_{v_{\rho_1}}, j_{v_{\omega_1}}\right)$. Apply the next step. If such an activity does not exist, go to Step 12.

Step 5. Determine activity entering AR_w *with the lowest order* for which the assigned budget exceeds the corresponding minimal possible budget value. Let the activity be $\left(i_{w_{\rho 2}}, j_{w_{\omega 2}}\right)$. Apply the next step. If such an activity does not exist, go to Step 10.

Step 6. If value $C_w - \Delta C$, i.e., $\sum_{\xi=1}^{m_w} c\left(i_{w_\xi}, j_{w_\xi}\right) - \Delta C$, becomes less than C_w^*, go to Step 10. Otherwise apply the next step.

Step 7. If value $C_v + \Delta C$, i.e., $\sum_{\xi=1}^{m_v} c\left(i_{v_\xi}, j_{v_\xi}\right) + \Delta C$, exceeds C_v^{**}, go to Step 12. Otherwise apply the next step.

Step 8. Calculate value $Pr\left\{T\left[G\middle|C, c(i,j)\right] \le D\right\}$ by solving *Problem III* (see *16.2.1*) with new values $c\left(i_{w_{\rho_2}}, j_{w_{\omega_2}}\right) - \Delta C \Rightarrow c\left(i_{w_{\rho_2}}, j_{w_{\omega_2}}\right)$ and $c\left(i_{v_{\rho_1}}, j_{v_{\omega_1}}\right) + \Delta C \Rightarrow c\left(i_{v_{\rho_1}}, j_{v_{\omega_1}}\right)$. If the new reliability value becomes not less than the pregiven target R, apply the next step. Otherwise go to Step 10.

Step 9. Transfer of the cost unit value ΔC from array AR_w to array AR_v is carried out. Thus, array AR_v obtains activity $\left(i_{v_{\rho_1}}, j_{v_{\omega_1}}\right)$ with a new increased budget, while array AR_w comprises activity $\left(i_{w_{\rho_2}}, j_{w_{\omega_2}}\right)$ with the new diminished budget. New values C_v and C_w are calculated. Go to Step 4.

Step 10. Applying this step means that no cost transfers can be implemented from any activity entering array AR_w. Thus, counter w works, $w-1 \Rightarrow w$.

Step 11. If w becomes equal v, *Procedure II* is accomplished. Go to Step 13. If w

exceeds v, go to Step 4.

Step 12. Applying this step means that value C_v of array AR_v cannot be increased. Counter v works, $v + 1 \Rightarrow v$.

Step 13. Calculate the new increased marketability value M for new local assigned budgets $\bar{c}(i, j)$ entering the project. Note that some of them have not been altered in the course of carrying out *Procedure II* while other activities undergo changes by means of cost transfers as described above.

It can be well-recognized that the general idea of *Procedure II* is to facilitate transfer of cost amounts from arrays of attributes with smaller cost-sensitivity rates to arrays representing more sensitive attributes.

§16.3 Enlarged heuristic procedure

In strategic management a company is usually faced with the necessity both of gaining maximal profit from distributing finished products on the market, as well as developing new products for the future. A certain part of the profit obtained has usually to be invested in creating newly developed products to be later on delivered in large quantities to markets. Thus, determining the budget to be invested in R&D projects in order to maximize the long-term profit becomes one of the main problems in strategic management. This problem is connected with raising the new product's marketability as much as possible. In this course, problem (16.1.4-16.1.5) outlined above becomes especially significant from both theoretical and applied points of view.

Thus, given

- the R&D project to develop a new product;
- the project's due date D;
- the project's reliability R;
- the forecasted maximal value of the project's marketability M^{**},

\- problem (16.1.4-16.1.5) boils down to determining the minimal budget C which results in the highest marketability rate M^{**} while honoring chance constraint R.

The enlarged procedure of solving problem (16.1.4-16.1.5) is as follows:

Stage 1. Solve *Problem II* (see *16.2.1*) to obtain the minimal budget C honoring chance constraint R. The output values to be obtained are values C and $\{c(i, j)\}$ honoring restrictions (16.2.2) and (16.2.3).

Stage 2. Carry out *Procedure II* (see *16.2.2*) to improve the product's marketability as much as possible by means of cost transfers from attribute to attribute. Denote by $\overline{M}$ the obtained marketability value.

Stage 3. Compare values $\overline{M}$ and M^{**}. If those values are equal, problem (16.1.4-16.1.5) is solved, and values $\overline{C}$ and $\overline{c}(i,j)$ obtained at Stage 2, are taken as the optimal ones. Otherwise, in case $\overline{M} < M^{**}$, apply the next step.

Stage 4. Increase C by δC. Solve *Problem I* (see *16.2.1*). The problem's output values are taken as the input for carrying out *Procedure II*. Go to Stage 2.

The process terminates when value $\overline{M}$ becomes equal M^{**} at Stage 3. It can be well-recognized that the number of iterations is finite since value C increases with a constant step δC and cannot exceed $C^{**} = \sum_{\{(i,j)\}} c(i,j)_{max}$.

§16.4 Additional application areas

The discussed problem (16.1.4-16.1.5) can be linked with many other essential problems related to strategic management. Note that the problem has to be solved on preliminary stages of the project's realization, to determine the total R&D project's budget. However, modern holding corporations deal simultaneously both with delivering products to the market and with researching and developing new products for the future. Thus, in our opinion, the following problem can be suggested.

A holding company is faced with developing a new product in order to deliver it later on to the market for high-demand sells. After developing the new product and determining the cost of manufacturing the product's unit (e.g., a vehicle, a consumer product, etc.), the unit price has to be decided upon. Existing approaches on that subject [179] enable optimizing the net profit of delivering the already created new product to the market *with a fixed, preset marketability*. However, as it has been shown in the Chapter, the product's marketability turns out to be a variable function which depends actually on the project's budget assigned for developing the new product. Thus, the project's budget has an essential influence both on the product's marketability and on the forecasted purchased quantities of that product. We suggest undertaking future research to consider a combined problem where the project's expenses, the manufacturing expenses of producing large quantities of the product, the selling price of the product and the future purchased quantities of the product have to be unified in one general model. The problem's objective is to maximize the net profit on a long-term period comprising both the stage of designing the product, as well as delivering the latter to the market. On our opinion, the optimized variable should be the project's budget C which influences all other model components. The most complicated part of the research is to formalize the linkage between the product's marketability, the level of market's demand for that product, and its selling price. However, it is possible to obtain a heuristic solution of such a generalized problem, probably with the help of expert judgment.

Another important area is based on considering multi-period planning horizons for designing and creating new products. Problem (16.1.4-16.1.5) suggests a single, i.e., indivisible period. Subdividing the latter into several periods (e.g. 3÷5 years to design and to create a new product) and determining capital investments for each such period separately, may raise essentially the net profit to be obtained within the product’s life cycle.

§16.5 Marketability model in project management

Assume that at a certain pregiven moment (due date) a newly designed product has to be developed in order to be later on manufactured and delivered in large quantities to the market. Assume, further on, that in order to design and to develop a new product with marketability value M a certain budget value $C_{R\&D}$ has to be invested. Thus, value M can be regarded as an implicit function of value $C_{R\&D}$. Assume that the total product's marketing period T comprises m subperiods (e.g., years) T_i. Denote the decreasing product's marketability for the i-th subperiod by M_i, $1 \le i \le m$, where $M_i > M_j$ for each $i < j$.

A reasonable assumption can be implemented: if M is an implicit function of $C_{R\&D}$, then values of the descending row $\{M_i\}$ are implicit functions of $C_{R\&D}$ as well. Thus, denote $M_i = F_i(C_{R\&D})$, $1 \le i \le m$, where $M_1 \equiv M$.

For each sub-period T_i relation holds:

$$Q_i = N_i - \ell_i S_i, \quad 0 < S < \frac{N}{\ell}, \tag{16.5.1}$$

where N_i represents the potential market's capacity for the considered product, Q_i stands for the quantity of the product's purchased units within subperiod T_i, S_i denotes the product's selling price in that period and ℓ_i is the product's price-sensitivity [179] depending on marketability value M_i. Assume that the product's life cycle terminates at the end of the m-th subperiod. A standard optimization problem for any sub-period T_i (to simplify the notation we will omit index i) can be suggested [179]. Given:

- the cost of manufacturing a single item C_{man};
- values N and ℓ (pregiven by means of experts' information);
- other overhead marketing expenses K (non-unit related expenses),

- determine the selling price S in order to obtain the maximal revenue π for the considered subperiod

$$\begin{aligned} \underset{S}{Max}\, \pi &= \underset{S}{Max}\left[Q \cdot S - C_{man} \cdot Q - K\right] = \underset{S}{Max}\left[(N - \ell S)S - C_{man}(N - \ell S) - K\right] = \\ &= \underset{S}{Max}\left[S(C_{man} \cdot \ell + N) - S^2 \cdot \ell - K\right] \end{aligned} \tag{16.5.2}$$

subject to evident restrictions

$$0 < S < \frac{N}{\ell}, \; S > C_{man}. \qquad (16.5.3)$$

Problem (16.5.2-16.5.3) can be easily solved by examining the square equation (16.5.2). We will suggest an essentially more generalized marketing problem applicable to the entire product's life cycle, including the period of designing and creating the product and the marketing period comprising m subperiods. The problem's formulation and its solution are based on the assumption that for each marketing sub-period T_i values N_i and ℓ_i are implicit functions of M_i, $1 \le i \le m$. Values M_i, in turn, satisfy $F_i(C_{R\&D})$, where F_i can be approximately formalized by means of expert information. Thus, values N_i, ℓ_i, $1 \le i \le m$, for each period T_i, as well as values C_{man} and K for the entire marketing period, are functions of $C_{R\&D}$. The problem is as follows: *at the stage preceding the new product's design*, i.e., before the project of designing the new product actually starts, determine optimal value $C_{R\&D}$, to maximize the company's profit π within the entire product's life cycle:

$$\underset{C_{R\&D}}{Max}\, \pi = \underset{C_{R\&D}}{Max} \left[\sum_{i=1}^{m} (N_i - \ell_i S_i) S_i - C_{man} \sum_{i=1}^{m} (N_i - \ell_i S_i) - \sum_{i=1}^{m} K_i \right] \qquad (16.5.4)$$

subject to

$$0 < S_i < \frac{N_i}{\ell_i}, \; S_i > C_{man}, \; 1 \le i \le m. \qquad (16.5.5)$$

Here values N_i, ℓ_i, $1 \le i \le m$, C_{man} and K are all implicit functions of $C_{R\&D}$, while values S_i are obtained by solving problem (16.5.2-16.5.3) for each subperiod T_i independently. The problem's solution results in undertaking a one-dimensional search for the optimized value $C_{R\&D}$ in order to maximize the marketability parameter M as much as possible. For each search value $C_{R\&D}$ objective (16.5.4) is calculated to obtain the maximal life cycle profit π. Note that the optimal value $C_{R\&D}^{(opt)}$ for problem (16.5.4-16.5.5) may not coincide with value $C_{R\&D}$ providing the maximal marketability $M = M_1$. This may happen, e.g., in case when just a minor improvement of parameter M requires an essential increase of C_{man}, which, in turn, leads to diminishing the overall life cycle profit of the product.

The solution of the subsidiary problem to maximize the marketability value M for a preset $C_{R\&D}$ is outlined in §§16.1-16.3.

§16.6 Conclusions

The following conclusions can be drawn from the Chapter:

1. The problem of maximizing the product's marketability by means of optimizing the

project’s capital investments, is an important area in strategic management. The recently developed models cover different periods of the entire life cycle of any newly developed product. This duscussion presents a newly developed model to forecast the product’s marketability before the R&D project of designing the product actually starts.

2. The backbone of the newly developed model is a heuristic procedure to reschedule local budgets assigned to project’s activities in order to maximize the future product’s marketability with pregiven project’s budget and honoring appropriate chance constraint restrictions.

3. The optimization procedure is based on the suggested cost-sensitivity values for each product’s attribute entering the compound cost-marketability rate.

4. The suggested model is structured from standard optimization blocks based on extensive simulation in combination with heuristic procedures.

5. In Chapters 8, 14-15 we have formulated the basic concepts underlying the necessity of developing a mixed type optimization model covering the entire product’s life cycle. Those basic concepts not only remain unchanged in the newly developed model but are strengthened in the course of undergoing greater detailization and covering new levels in strategic project management.

6. The results obtained in §§16.1-16.3 can be used in the problem of optimal budget reallocation within the project’s life cycle subject to the products selling price. This problem, in turn, is an important milestone on the way of solving the overall problem of maximizing the company’s profit.

PART IV

HIERARCHICAL ON-LINE CONTROL MODELS OF ORGANIZATION SYSTEMS

Chapter 17. Hierarchical On-Line Control Model for Stochastic Project Management

§17.1 The system's description

Until now we have examined only two- and three-level cases of various OS. Now we present an essentially more complicated system, for which some results outlined above, in previous Chapters, meet together. We will also present a hierarchical on-line control system in combination with the active systems theory outlined in Chapter 13.

The outlined below hierarchical model combines together two resource reallocation models at the upper level, the on-line control model at the medium level and a resource supportability model at the lower level.

Several activity-on-arc network projects (graphs) with independent activities of random durations are considered. Each activity duration follows a beta probability density function while the cost-duration function is based on the assumption that each activity duration is close to be inversely proportional to the budget assigned to that activity.

A hierarchical control model is suggested which at any control point determines [70, 104]:

- optimal budget values assigned from the company to each project,
- optimal budget reallocation among the project's activities,
- optimal control points to inspect each project,
- optimal resource delivery schedule for project activities; the corresponding resources are hired and maintained on the account of the budget assigned to these activities,

in order to

- minimize the total number of control points for all projects, and
- maximize the probability of meeting the deadline of the slowest project.

The model is based on a stochastic optimization problem with two conflicting objectives and a variable number of constraints. The problem cannot be solved in the general case and allows only heuristic solutions. The general control model is modified to the hierarchical on-line control model, which comprises three optimization problems. *Problem I*, at the company level, enables optimal budget reassignment among the projects. The problem's solution, i.e., the budget assigned to each project, serves as the initial data for *Problem II* (at the project level), where budget is reallocated among the project's activities to maximize the probability of meeting the project's deadline. The solution of *Problem II* serves, in turn, as the initial data for *Problem III*, which carries out on-line control, i.e., determines optimal control points to inspect the progress of the project. This is done by determining the planned trajectories that must be repeatedly corrected in the course of the project's realization. At the lower level the problem boils down to determine the delivery schedule for different types of renewable resources, namely, for

- rare and costly resources which have to be delivered from outside for a relatively small group of project activities;
- non-restricted resources, which are always at the disposal of the project management;
- restricted renewable resources which are feeded-in at random moments when the resources are available and at least one project activity has to be supported with resources in order to start processing (see Chapters 8 and 12).

Thus, practically speaking, the hierarchical control model comprises four levels.

If, at any control point, it turns out that a project deviates from the planned trajectory, an error signal is generated, and decision-making is based on solving *Problem II* to reassign the remaining budget among the remaining project's activities to maximize the probability to meet the deadline. If the problem's solution enables the project's deadline to be met, subject to the chance constraint, a corrected planned trajectory is determined, and *Problem III* is resolved to determine the next control point. Otherwise an emergency signal is generated, and decision-making is carried out at the company level. *Problem I is* resolved under emergency conditions to reassign the remaining budget among the non-accomplished projects. Thus, in the course of controlling a group of projects, the latter are first optimized on line from "top-to-bottom". In the case of an emergency, the generated "bottom-top" signals are converted into control actions to enable the projects' due dates to be met on time. This general idea, which has been outlined in many of our publications (see, e.g., [67-70, 78, 81, 102, 104, 109, etc.]), is recommended to be implemented in any on-line control hierarchical OS.

§17.2 Notation

Let us introduce the following terms:

I. The company level

C - the total company budget assigned at $t=0$ for all project's realization;

n - number of projects;

$G_k(N,A)$ - the k-th stochastic network project (graph) of PERT-COST type, $1 \le k \le n$;

C_k - budget assigned to the k-th project at moment $t=0$;

C_{kt} - budget assigned to the k-th project at moment $t>0$;

$C_k(t)$ - the remaining project's budget at moment $t \geq 0$ (observed via inspection), $C_k(0)=C_k$.

D_k - the due date for the k-th project;

P_k^* - the pregiven minimal possible probability for the k-th project to meet it's deadline on time;

G_{kt} - the remaining part of graph $G_k(N,A)$ at moment $t \geq 0$; $G_{k0}=G_k$;

$T_{kt}\{C_k(t)\}$ - the random duration of G_{kt} on condition that at $t \geq 0$ the remaining budget is $C_k(t)$;

$P_{kt}=P\{C_k(t)\}$ - the probability of meeting the project's G_{kt} deadline on time on condition that at moment $t \geq 0$ the remaining budget is $C_k(t)$, i.e., $P_{kt}=P\{t+T_{kt}\{C_k(t)\} \leq D_k\}$;

$C(t)$ - the remaining company's budget at moment $t>0$.

II. The project level

For simplicity we will omit index k:

$(i,j) \in G(N,A)$ - activity entering the project;

t_{ij} - random duration of (i,j);

$c(i,j)$ - budget assigned to activity (i,j);

$c(i,j)_{\min}$ - the minimal budget with which activity (i,j) can be operated (pregiven);

$c(i,j)_{\max}$ - the maximal budget to operate (i,j) (pregiven);

$t_{ij}\{c(i,j)\}$ - the random duration of activity (i,j) on condition that budget c_{ij} is assigned to (i,j), $c(i,j)_{\min} \leq c(i,j) \leq c(i,j)_{\max}$;

S_{ij} - the moment (i,j) actually starts;

T_{ij}^r - the resource delivery moment for activity (i,j) (a random value, which is determined in the course of the project's realization);

$F_{ij}=S_{ij}+t_{ij}$ - the moment activity (i,j) is accomplished;

V - target amount for the project of PERT-COST type; let $V=C$, where C is the budget assigned to the project;

V^t - the actual realized part of target amount V at moment $t>0$; for PERT-COST projects $V^t=C^t$, where C^t is the budget actually realized at moment t;

$C(t)=C-C^t$ - the remaining (non-realized) budget at moment $t>0$ (observed via inspection);

$T_r(x,t,D)$ - the control trajectory for the project determined at moment $t \geq 0$; this is a straight line connecting points (t, C^t) and $(D, 0)$;

N - number of control (inspection) points in the course of controlling the project;

P^* - the pregiven minimal confidence probability of meeting the deadline on time;

t_g - the g-th control point, $g = 0,1,\ldots,N$; $t_0 = 0$, $t_N = D$;

Δ - the minimal time span between two consecutive control points (pregiven for each project);

§17.3 Optimization *Problem I* at the company level

At moment $t = 0$ the problem is as follows: determine values C_k assigned for each project $G_k(N,A)$, $1 \leq k \leq n$, to maximize

$$J = \underset{C_k}{Max}\,\underset{k}{Min}\,\{P\{C_k\}\} \tag{17.3.1}$$

subject to

$$\sum_{k=1}^{n} C_k = C \tag{17.3.2}$$

and

$$P\{C_k\} \geq P_k^*. \tag{17.3.3}$$

Problem (17.3.1-17.3.3) is a very complicated problem which does not obtain a precise solution. Its heuristic solution is outlined in [70, 77, 109].

The corresponding dual problem for the case of one project centers on determining the minimal budget C with pregiven due date D and minimal confidence probability P^*. One has to determine

$$Min\, C \tag{17.3.4}$$

subject to

$$\Pr\{C\} \geq P^*. \tag{17.3.5}$$

At moment $t > 0$ problem (17.3.1-17.3.3) is as follows: determine the newly corrected values C_{kt}^* to maximize

$$\underset{\{C_{kt}\}}{Max}\,\underset{k}{Min}\,\{P\{C_{kt}\}\} \tag{17.3.6}$$

subject to

$$\sum_{k=1}^{n} C_{kt}^{*} = C(t), \tag{17.3.7}$$

$$P\{C_{kt}^{*}\} \ge P_k^{*}, \quad 1 \le k \le n, \tag{17.3.8}$$

where $C(t)$ is the remaining company budget which has to be redistributed among the projects.

§17.4 Optimization *Problem II* at the project level

For Problem II the input parameters are either C_k or C_{kt}, which, for the sake of simplicity, are designated by C or C_t. Thus, in §§17.4-17.6 index k is omitted.

The problem is as follows: to redistribute C among the project's activities in order to obtain the maximal $P\{C\}$, i.e., to determine values $c(i, j)$

$$\underset{c(i,j)}{Max}\left[P\{C\}\right] = \underset{c(i,j)}{Max}\left[\Pr\{T\{C\} \le D\}\right] \tag{17.4.1}$$

subject to

$$P\{C\} \ge P^{*}, \tag{17.4.2}$$

$$\sum_{(i,j)} c(i, j) = C, \tag{17.4.3}$$

$$(i, j) \in G(N, A) \tag{17.4.4}$$

and

$$c(i, j)_{\min} \le c(i, j) \le c(i, j)_{\max}. \tag{17.4.5}$$

Problem (17.4.1-17.4.5) is solved (see *Section 8.1.4*) by using a combination of heuristic procedures and simulation modeling. Note that problem (17.4.1-17.4.5) is in fact a simplified version of problem (17.3.4-17.3.5).

If $t > 0$, the problem can be modified to a more complicated version

$$\underset{c(i,j)}{Max}\left[\Pr\{t + T\{C(t)\} \le D\}\right], \tag{17.4.6}$$

subject to (17.4.2-17.4.5).

After determining values $\{c(i, j)\}$ control points t_g have to be determined.

§17.5 On-line control *Problem III*

The problem (see [70, 81,109]) is to determine control points t_g, $g = 0,1,...,N$, which deliver the minimum of the number of those points

$$\underset{\{t_g\}}{Min}\, N \tag{17.5.1}$$

subject to

$$\Pr\{C_D = 0\} \ge P^*, \tag{17.5.2}$$

$$t_{g+1} - t_g \ge \Delta, \tag{17.5.3}$$

$$t_1 = 0,\ \ t_N = D. \tag{17.5.4}$$

Problem (17.5.1-17.5.4) is a very complicated problem of non-linear stochastic programming. The problem can be solved by substituting it for another one, i.e., to maximize the time span between two consecutive control points. The problem is to determine values $\{t_g\}$ in order to maximize

$$Max\left\{t_{g+1} - t_g\right\}, \tag{17.5.5}$$

subject to

$$t_{g+1} - t_g \ge \Delta, \tag{17.5.6}$$

$$\Pr\left\{C_t < T_r\left(t, t_g, D\right)\right\} \ge P^*. \tag{17.5.7}$$

In (17.5.7) trajectory $T_r\left(t, t_g, D\right)$ is a straight line connecting two points $\left(t_g, C^{t_g}\right)$ and $(D,0)$. The trajectory line is as follows:

$$T_r\left(t, t_g, D\right) = \frac{t \cdot C^{t_g}}{D - t_g} + \frac{D \cdot C^{t_g}}{D - t_g} = C^{t_g}\ \frac{t + D}{D - t_g}. \tag{17.5.8}$$

Problem (17.5.5-17.5.7) has been solved in [70, 81, 109] by a combination of statistical sequential analysis and simulation.

If $C^{t_{g+1}} \le T_r\left(t_g, t_{g+1}, \Delta\right)$ holds, that means that the project does not deviate from its target and there is no need in any additional control actions. In case $C^{t_{g+1}} > T_r\left(t_g, t_{g+1}, \Delta\right)$ one has to resolve problem (17.4.1-17.4.5) for the remaining part of the budget $C\left(t_{g+1}\right) = C - C^{t_{g+1}}$ and the remaining project $G_{t_{g+1}}(N, A)$. The problem results in maximizing the probability of meeting the target on time by rescheduling the budget among the remaining activities. If in the course of solving problem (17.4.1-17.4.5) we obtain $P\{C(t_{g+1})\} \geq P^*$, that means that a new trajectory has to be developed. Thus, a new control point t_{g+2} is obtained, and the project's realization proceeds. If relation $P\{C(t_{g+1})\} < P^*$ holds, that means that the project is unable to meet its target on time and needs help from the company.

§17.6 Resource delivery *Problem IV*

If there is no essential deviation from the target, a resource delivery schedule $T_{ij}^{(r)}$, $(i,j) \in G_{t_{g+1}}(N,A)$, $t_g < T_{ij}^{(r)} \leq t_{g+1}$, has to be determined (see Chapter 12). That means, that $T_{ij}^{(r)}$ enables processing activities (i,j) which are operated between two adjacent control points t_g and t_{g+1}. Resources have to be hired and delivered from the budget $c(i,j)$ assigned to the corresponding activity (i,j). Note that $c(i,j)$ is determined on the basis of optimization problem (17.4.1-17.4.5) which has been solved at moment $t \leq t_g$.

To determine resource delivery schedule $T_{ij}^{(r)}$ we suggest a heuristic procedure as follows:

Step 1. Simulate the duration of all remaining activities $t_{ij}[c(i,j)]$, $(i,j) \in G_{t_{g+1}}(N,A)$.

Step 2. For all nodes $i \in G_{t_{g+1}}$, besides the sink ones, determine on the basis of simulated values (at Step 1) the earliest moments of realizing event (node) i, $T^{ear}(i)$. Value $T^{ear}(i)$ is the value of the longest path connecting the prospect's source node and node i. The algorithm to determine $T^{ear}(i)$ is outlined in many books on network planning (see, e.g. [67]).

Step 3. Realize steps $1 \rightarrow 2$ M times, where M is large enough to obtain the representative statistics, in order to obtain parameters of random value $T^{ear}(i)$.

Step 4. For each random value $T^{ear}(i)$ on the basis of the statistics obtained determine its lower and upper values $\overline{T}^{ear}(i)$ and $\overline{\overline{T}}^{ear}(i)$.

Step 5. Assume that the density function of random value $T^{ear}(i)$ satisfies the beta-distribution [9, 67-70] with p.d.f.

$$f_t\left[T^{ear}(i)\right] = \frac{12}{\left[\overline{\overline{T}}^{ear}(i) - \overline{T}^{ear}(i)\right]^4}\left[t - \overline{T}^{ear}(i)\right] \cdot \left[\overline{\overline{T}}^{ear}(i) - t\right]^2, \tag{17.6.1}$$

where $\overline{\overline{T}}^{ear}(i)$ and $\overline{T}^{ear}(i)$ are obtained (for each i) at Step 4.

Note that for p.d.f. (17.6.1) the average value of $T^{ear}(i)$ is as follows [67, 70]:

$$E\left\{T^{ear}(i)\right\} = 0.6\,\overline{T}^{ear}(i) + 0.4\,\overline{\overline{T}}^{ear}(i).$$

Step 6. Single out all nodes (events) $i \in G_{t_{g+1}}(N, A)$ satisfying

$$\mathrm{t_g} \le \mathrm{E}\left\{\mathrm{T}^{\mathrm{ear}}(\mathrm{i})\right\} \le \mathrm{t_{g+1}}. \tag{17.6.2}$$

Step 7. For all activities (i, j) satisfying (17.6.2), determine $T_{ij}^{(r)}$ by

$$T_{ij}^{(r)} = E\left\{T^{ear}(i)\right\} = 0.6\ \overline{T}^{ear}(i) + 0.4\ \overline{\overline{T}}^{ear}(i). \tag{17.6.3}$$

Thus, (17.6.3) can be used as an approximate estimation for resource delivery schedule $T_{ij}^{(r)}$.

Fig. 17.1 presents the interconnections between the elements of the multilevel on-line control model. In order to review the model as a whole (see §17.1) index k has been restored.

§17.7 Human factors revisited

17.7.1 The problem's description

In §13.4 we have outlined several problems connected with human behavior in OS. Now we will present another problem in that area referring to multilevel stochastic project management.

Assume that the project management system S comprising several simultaneously realized stochastic network PERT-COST projects at the lower level and the company personnel at the upper level, receives at the end of the planning period an award W_S, depending on the portfolio's utility U_S determined by (8.2.7) or (8.2.30). Usually a certain pregiven part of W_S remains at the company's disposal, while the other part (call henceforth those two parts W_{SI} and W_{SII}, correspondingly) is dispersed among the projects. If n projects $G_k(N, A)$, $1 \le k \le n$, with utilities U_k participate in this division, each project $G_k(N, A)$ receives an award estimated as

$$W_{IIk} = W_{SII} \cdot \frac{U_k}{\sum_{k=1}^{n} U_k}, \quad 1 \le k \le n. \tag{17.7.1}$$

Here the calculation is carried out for each project $G_k(N, A)$ by means of (8.1.10), with values C_k, D_k and R_k being *preplanned and predetermined*.

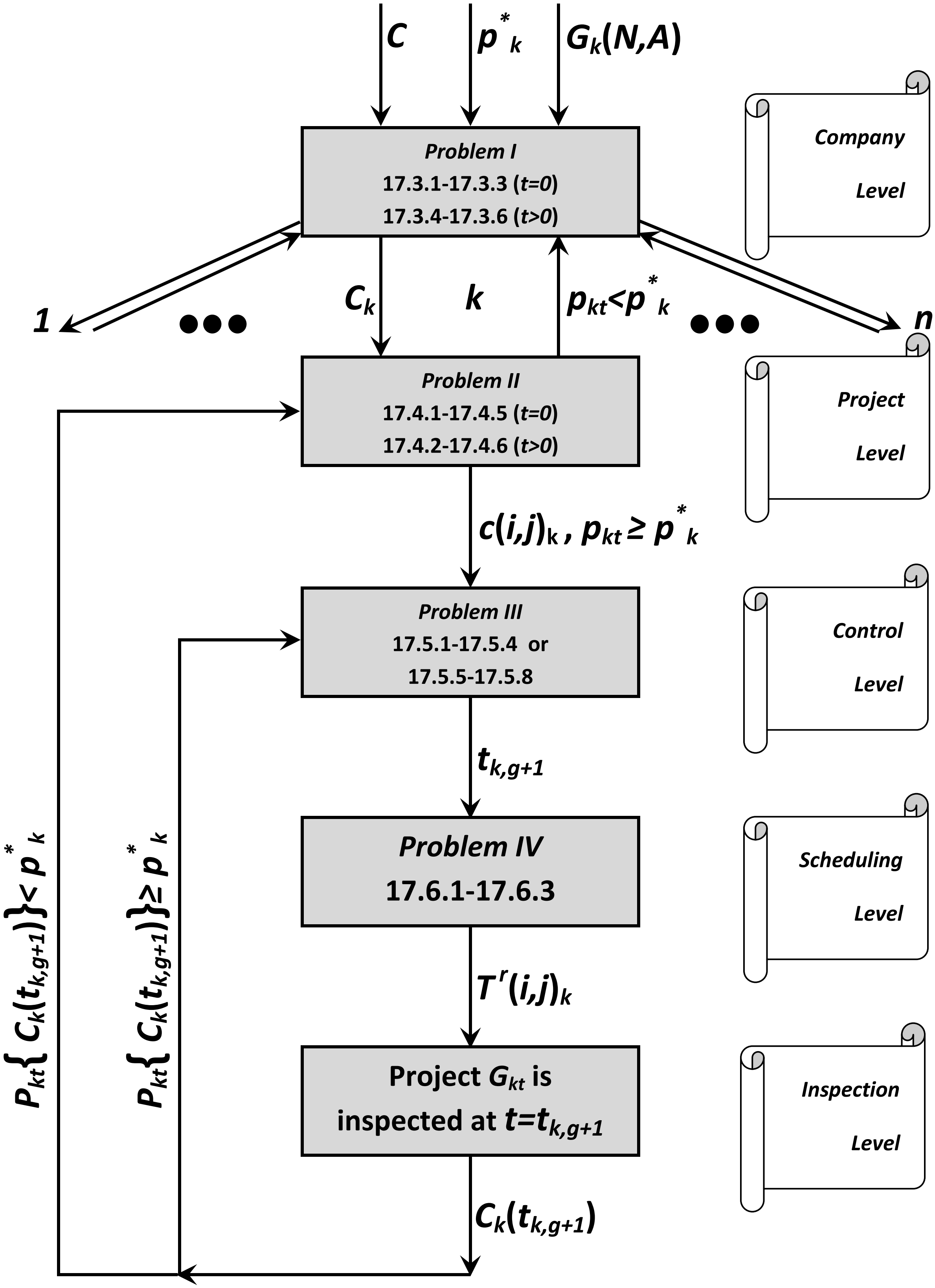

Figure 17.1. Hierarchical on-line control model (*emergency at* $t = t_{k,g+1}$)

Note that in accordance with the theory of active organization systems (see Chapter 13), all executors at the medium level of an OS, i.e., practically all executors responsible for operating project's activities $(i, j)_k$, $1 \le k \le n$, are playing their active game similar to that outlined in §13.2. They deliberately overestimate their activity durations by prolonging the right "tail" of the beta p.d.f. (2.1.3), i.e., parameter b. This results in overestimating the due date D_k and enables the project's personnel both comfortable (non-intense) working conditions accompanied by guaranteed awards. However if the project's personnel would be informed *beforehand* that in case of executing the project within a *shorter preplanned time* their award share would be significantly higher, than for a longer preplanned D_k (and this is one of the psychological backbones of the theory of active systems!), the players might become interested in changing their game strategy. Namely, we suggest that each activity executor be allowed to determine several (2-3) estimates b:

- the basic one (b) which practically guarantees a quiet life for the executor;
- the intermediate estimate $b^* < b$ which actually enables to operate the activity with p.d.f. duration (2.1.3), $b = b^*$, but by means of a higher labor productivity, and
- the minimal value $b^{**} < b^* < b$, which requires from the executor to work in the most exhausting manner, i.e., with the utmost labor productivity.

Honoring the active systems theory applied to our case, we suggest to each activity executor to proclaim beforehand the three estimates for the right bound of the corresponding p.d.f. (2.1.3) for that activity. Such an action would result (see Chapter 8) in diminishing the due date D_k for each project and, later on, in increasing the corresponding utility function for that project as well, i.e., in changing the system's utility U_S to U_S^* and U_S^{**}, correspondingly. Since $U_S^{**} > U_S^* > U_S$ holds, the corresponding awards W_S would increase as well, namely

$$\begin{cases} W_S^* = W_S \cdot \dfrac{U_S^*}{U_S} \\ W_S^{**} = W_S \cdot \dfrac{U_S^{**}}{U_S}. \end{cases} \tag{17.7.2}$$

Thus, each project $G_k(N, A)$ would be granted an essentially higher award, but only on condition that the project executers decided to *declare* beforehand the appropriate improvement of the former due date D_k (to be changed to D_k^* or D_k^{**}) with subsequently implementing this amended due date as a *plan parameter*.

According to the active systems theory, only the project's personnel and nobody else can undertake such an alternative decision: either working harder and earning more or just leaving things as they stand now. It goes without saying that an individual project's decision does not influence other projects. Note that in the course of undertaking such a game the pregiven reliability values R_k for each project remain the same, independently of the due dates' changes.

Let us formulate in appropriate terms the problem under consideration. Two cases will be examined:

A. Case of a single project (a two-level on-line control model), and
B. Case of several projects (a three-level on-line control model).

17.7.2 A two-level model

In the case of one project subordinated to the company level, we will require a few additional terms to be added to the Notation "The project level" outlined in §17.2, namely:

$b_{ij}^* < b_{ij}$ - the modified right bound of the beta-distribution (2.1.3) for each activity by means of intensifying the labor productivity of operating (i, j) as compared to the existing one (received from executors of all activities $(i, j) \subset G(N, A)$);

$b_{ij}^{**} < b_{ij}^*$ - practically the minimal right bound of (2.1.3) which can be achieved by executing (i, j) with the utmost labor productivity (received from all executors as well).

$T\left\{G(N, A)/c(i, j), b_{ij}\right\}$ - the project's random completion time on condition that the project budget value C is reallocated among activities (i, j) and regular or "curtailed" values b_{ij} are used in calculating T.

Note (see Chapters 2 and 5) that b_{ij} actually depends on budget values $c(i, j)$ assigned to activity (i, j) which are unknown beforehand. Therefore we suggest expressing b_{ij}^* and b_{ij}^{**} in relative terms, e.g., $b_{ij}^* = 0.95 b_{ij}$, $b_{ij}^{**} = 0.90 b_{ij}$. When interacting with project's decision makers, one have to bear in mind that consecutive diminishing the right "tail" b_{ij} will sooner or later lead to the equilibrium value b_{ij}^{***} where even working with the utmost intensity may result in failing to meet the project's target on time. Each executor must be aware of such a situation to refrain himself from getting too close to the regarded equilibrium point by estimating b_{ij}. Note that the concept of equilibrium plays an essential role in the theory of active systems.

The problem's formulation is as follows: minimize the p^*-th quantile of $T\left\{G(N, A)/c(i, j), b_{ij}\right\}$

$$Min\ W_{p^*}\left\{T\{G(N,A)\}/c(i,j),b_{ij}\right\} \qquad (17.7.3)$$

subject to

$$b_{ij} > b_{ij}^{***}, \qquad (17.7.4)$$

$$\sum_{\{i,j\}} c(i,j) = C \qquad (17.7.5)$$

and (17.4.5).

Thus, $\{c(i,j)\}$ have to be reallocated among all (i,j) optimally, together with an optimal combination of $\{b_{ij}\}$ for all activities $(i,j)\subset G(N,A)$.

To solve problem (17.4.5, 17.7.3-17.7.5), one has to check all combinations $\{b_{ij}^{**}, b_{ij}^{*}, b_{ij}\}$ submitted by the executors, with subsequently choosing the optimal set $\{b_{ij}\}$ delivering minimum to (17.7.3) by solving optimization problem

$$Min\left\{W_{p^*}\left\{T\left\{G,N,A\right\}_{c(i,j)}\right\}\right\} \qquad (17.7.6)$$

subject to (17.4.5, 17.7.5) with fixed $\{b_{ij}\}$.

After determining the optimal sequence $\{b_{ij}\}$ the latter has to be sent to the executors in order to confirm the possibility of realizing the process of the project. The optimal value (17.7.3) has to be considered as the new preplanned due date D (lower than the former value). If for some activities (i,j) there are objections from the corresponding executors, restriction (17.7.2) has to be corrected and optimization problem (17.4.5, 17.7.1-17.7.3) resolved, until no contradictions take place any more.

We recommend solving problem (17.4.5, 17.7.3-17.7.5) in two stages. At the first stage a cyclic coordinate search algorithm CCSA [133] is implemented in the space of $\{b_{ij}\}$, $(i,j)\subset G(N,A)$, while for a fixed set of $\{b_{ij}\}$ problem (17.4.1-17.4.5) has to be solved at the second stage. In order to simplify the problem's solution one has to apply for *all* activities $(i,j)\subset G(N,A)$ one and the same relations connecting b_{ij}^{*}, b_{ij}^{**} and b_{ij}, e.g., $b_{ij}^{*} = 0.95 b_{ij}$, $b_{ij}^{**} = 0.90 b_{ij}$. Thus, problem (17.7.1-17.7.5) has to be solved only for three combinations.

17.7.3 Case of several projects (a three-level on-line model)

Analyzing such a case results in an essentially more complicated solution. We suggest for projects $G_k(N,A)$ participating in a game similar to that outlined above, as it was stated for the case of one single project, to question all executors responsible for operating activities $(i, j)_k$, $1 \le k \le n$, regarding their psychological possibilities of shortening the corresponding right "tails" b_{ijk}. We suggest to obtain from each executor several (2-3) relative estimates, i.e., b_{ijk}, $0.95b_{ijk}$, $0.90b_{ijk}$, which, on their opinion, may be achieved by the appropriate intensifying effort. Thus, if the k-th project incorporates n_k activities, then at the first level a cyclic coordinate descent algorithm has to be implemented in the $N = \sum_{k=1}^{n} n_k$ -dimensions space in order to obtain for each combination $\{b_{ijk}\}$ one of those three values under examination, namely:

- the existing estimate b_{ijk};
- the slightly curtailed $0.95b_{ijk} = b^{*}_{ijk}$;
- the essentially curtailed $0.90b_{ijk} = b^{**}_{ijk}$, $(i, j)_k \subset G_k(N, A)$, $1 \le k \le n$.

Each coordinate $\{b_{ijk}\}$ serves as the output value of the CCSA algorithm [133] and the input value for the algorithm outlined above, in §§17.1-17.5. Thus, the problem is to determine optimal values $\{c(i, j)_k\}$ and $\{b_{ijk}\}$ in order to maximize the utility of the projects' portfolio by using the model outlined in §8.2. After determining the routine feasible combination of estimates $\{b_{ijk}\}$ by applying CCSA (see Chapters 5 and 8), we solve

$$Max \sum_{k=1}^{n} \rho_k U_k \tag{17.7.7}$$

on the basis of intermediate models outlined in §8.2. An optimal combination of $\{b_{ijk}\}$ and $\{c(i, j)_k\}$ has to be determined in order to solve problem (17.7.7) by obtaining a preliminary optimal solution:

To minimize for all n projects G_k, $1 \le k \le n$, values

$$D_k = W_{p_k^*}\left\{T\{G_k(N, A)\}/c(i, j)_k, b_{ijk}\right\} \tag{17.7.8}$$

subject to

$$b_{ijk} > b^{***}_{ijk} \quad \text{(obtained from all executors)}, \tag{17.7.9}$$

$$\sum_{k=1}^{n} \sum_{\{i,j\}_k} c(i, j)_k = C \tag{17.7.10}$$

and (17.4.5).

Note that after reallocating cost resources among the projects values $W_{p_k^*}$ are obtained and later on minimized independently. Moreover, after determining $\{b_{ijk}\}$ by means of CCSA we may further improve them by solving problem (17.7.8-17.7.10). After solving that problem for the group of n projects the optimal portfolio utility's value (8.2.5) has to be maximized. Since values $\sum_{\{i,j\}_k} c(i,j)_k = C_k$ and p_k^* are practically not changed essentially in the course of "curtailing" parameters b_{ijk} (this affects only the due dates $D_k = W_{p_k^*}\left\{T\{G_k(N,A)\}/c(i,j)_k, b_{ijk}\right\}$, $1 \le k \le n$), increasing the utility of the projects' portfolio can be undertaken only by shortening the activities' "tails".

17.7.4 *<u>Conclusions</u>*

The problem outlined in *17.7.2-17.7.3* is a complicated one since considering psychological interactions and human behavior changes on-line control problems essentially. However, solving that problem results in improving the hierarchical project management system's utility.

What is, in essence, the similarity and the difference between the theory presented in §13.4, and the model outlined in §17.7? Both models refer to stochastic project management. In §13.4 the Center (company) is an active player, while in §17.7 players are system's active elements (projects). Being different in nature, they are similar in results: the OS in both cases raises its utility *without any antagonistic losses among all reciprocating elements entering the OS*.

Many other cases may be examined but it can be well-recognized that, being actually psychological, an active systems' model in conjunction with classical planning and control models, refines the effectiveness of an OS and improves its utility estimates.

§18.1 Introduction

This approach to the interaction problems between different levels in hierarchical control systems is based on the conception of emergency, introduced by Golenko-Ginzburg and Sinuany-Stern [79]. By using the idea that hierarchical levels can interact only in special situations, so-called emergency points, one can decompose a general and complex multilevel problem of optimal production control into a sequence of one-level problems.

In the current Chapter we present applications of the hierarchical control approach to some large and complex industrial systems. They consider three levels: company, section and production unit, where each level is faced with stochastic optimization problems. Each unit produces a given target amount by a given due date (common to all units) and has several possible speeds, which are subject to disturbances. At the unit level, at each control point, decision-making centers on determining both the next control point and the speed to proceed with up to that point. The section level faces problems of either reallocation resources among the section's units or reassigning the remaining target amounts among the units so that the faster one will help the slower one. The company level is faced with similar problems, i.e., reallocating resources or reassigning target amounts among the sections. Two different optimization cases are considered:

- case with a conflicting two-criteria objective, namely, to maximize the probability of completing the production on the due date and to minimize the number of control points, but the first criterion is dominant;
- the objective is to maximize the expected net profit.

Simulation results demonstrate the high probability of target achievement by this approach [124].

§18.2 Notation

Let us introduce the following terms:

V_{kc} - production plan of the c-th product for unit k, $1 \le c \le b$, $1 \le k \le n$;

T_{kc} - planning horizon for production plan V_{kc};

R_{kd} - the d-th type of resource capacity allocated to unit k, $1 \le d \le f$, $1 \le k \le n$;

n - number of production units;

b - number of different resources;

f - number of different products;

$v_{jkc}\left(R_{k1},...,R_{kf}\right)$ - the j-th speed of unit k to manufacture product c, $1 \le j \le m$, $1 \le c \le b$, $1 \le k \le n$. Speeds v_{jkc} are subject to disturbances and represent random values;

m - number of possible speeds common to all units;

$\bar{v}_{jkc}(R_{k1},...,R_{kf})$ - the average of speed $v_{jkc}(R_{k1},...,R_{kf})$; speeds are sorted in ascending order of their average values;

$v_{pkc}(R_{k1},...,R_{kf})$ - the planned speed of unit k to manufacture product c, $p \in \{1,m\}$; in non-critical cases the planned speed is recommended;

$V_{kc}(t)$ - the actual output of product c manufactured by unit k, observed at moment t;

t_{kci} - the i-th inspection moment (control point) of unit k when producing product c, $1 \le i \le N_{kc}$, $0 \le t_{kci} \le T_{kc}$; note that for every new product value i starts from 1;

N_{kc} - number of control points of unit k to produce product c (a random variable determined by the control model);

Δ_{kc} - minimal value of closeness of inspection moment t_{kci} to the planning horizon T_{kc}, $1 \le c \le b$, $1 \le k \le n$;

d_{kc} - the minimal given time span between two consecutive control points t_{kci} and $t_{kc,i+1}$ (in order to force convergence);

W_c - production plan of the factory for product c, $1 \le c \le b$ (pregiven);

T - the due date (pregiven);

R_d - total resources of type d at the factory's disposal (to be determined);

R_{kd}^{min} - lower bound of value R_{kd} (pregiven);

R_{kd}^{max} - upper bound of value R_{kd} (pregiven);

R_d^h - total resources of type d allocated to section h, $1 \le h \le u$;

u - number of sections;

W_c^h - production plan for product c assigned to section h;

$X_{kc}(t)$ - the future amount of product c to be manufactured by unit k during interval of length t (a random variable determined by the control model);

T_g - the g-th overall emergency moment, i.e., the g-th target amount and resource reallocation moment to be implemented at the factory level, $g = 0,1,...$, $T_0 = 0$;

$\{k^h\}$ - set of units entering section h;

C_d - the cost of the d-th resource unit's renting and utilization (per time unit);

C_{pj} - the average processing cost per time unit when speed j is implemented by all factory units; note that C_{pj} is determined in addition to resource utilization values C_d;

C_{ins} - the average cost of performing a single inspection of a production unit;

C_{pen} - the penalty cost imposed on the factory for not accomplishing the total production program at the given due date T;

C_{em} - the cost of reallocating resources and target amounts at the section level in case of emergency;

$C_{ov.em}$ - the cost of implementing control actions at the factory level in case of an overall emergency; note that all cost values $C_d \div C_{ov.em}$ are pregiven;

E - the total factory expenses accumulated at the due date T.

§18.3 Optimal planning model at the factory level

At moment $t=0$, the problem is to determine values R_d, $1 \le d \le f$, to be put at the factory's disposal, as well as to determine optimal target amounts $V_{kc} \ge 0$ and resource capacities $R_{kd} \ge 0$, $1 \le k \le n$, $1 \le c \le b$, $1 \le d \le f$, for each production unit k. The problem is as follows:

$$F = \underset{\{R_d\}}{Min} \left[\sum_{d=1}^{f} [C_d R_d] \cdot T \right] \tag{18.3.1}$$

subject to

$$J = \underset{\{V_{kc}, R_{kd}\}}{Max} \; \underset{k}{Min} \left[\prod_{c=1}^{b} Pr\{V_{kc}(t) \ge V_{kc}\} \right] = 1 \tag{18.3.2}$$

and

$$\sum_{k=1}^{n} V_{kc} = W_c, \quad \sum_{k=1}^{n} R_{kd} = R_d, \quad R_{kd}^{min} \le R_{kd} \le R_{kd}^{max}. \tag{18.3.3}$$

Thus, at moment $t=0$, the factory management has to minimize the total budget for resource $\{R_d\}$ renting and utilization, on condition that those resources are sufficient to accomplish the factory production program on time when operating with the intermediate, planned speed v_{pkc} for all products and all production units. Resources $\{R_d\}$ have to be reallocated among all production units entering the factory, irrespective of their sectional subordination. Thus, values V_{kc} and R_{kd} have to be determined for each production unit.

Problem (18.3.1-18.3.3) is a complicated stochastic optimization problem. We suggest replacing the probabilistic value J by a deterministic value, namely,

$$\underset{\{V_{kc},R_{kd}\}}{Min}\ I = \underset{\{V_{kc},R_{kd}\}}{Min}\ \underset{k}{Max}\left\{\sum_{c=1}^{b}\frac{V_{kc}}{\bar{v}_{pkc}\left(R_{k1},...,R_{kf}\right)}\right\} \le T. \tag{18.3.4}$$

Solving problem (18.3.4) means that the factory obtains the minimal budget $C = F^{opt} \cdot T$ to rent resources and utilize them within the planning horizon to accomplish the target on time. The problem is a non-linear programming problem of general type; we solve it by using a modified coordinate descent method which is essentially faster than the recently developed alternatives (see, e.g., [78]). An approximate method for solving problem (18.3.1-18.3.3) will be outlined in §18.7. Note that problem (18.3.4) has to be solved only *once*, at $t = 0$. Later on, at $t > 0$, other optimization problems have to be solved at the factory level.

After solving problem (18.3.1-18.3.4) each section h, $1 \le h \le u$, obtains resources R_d^h as well as target amounts W_c^h, by summarizing values R_{kd} and V_{kc} for each unit entering the corresponding section. Control is then delegated to the unit level.

§18.4 Control model at the unit level

At the unit level, all units first work independently and are controlled separately. The decisions are taken on-line, in real time. For each unit k, $1 \le k \le n$, and for each product c manufactured by that unit, the problem is to determine at each routine control point t_{kci} both the index of the new speed j and the next control point $t_{kc,i+1}$. The problem thus is both

- to minimize the number of control points

$$\underset{\{t_{kci},j\}}{Min}\ N_{kc} \tag{18.4.1}$$

- and to maximize the probability of meeting the target amount V_{kc} on time, i.e., not later than at the planning horizon T_{kc}

$$\underset{\{t_{kci},j\}}{Max}\ Pr\left\{V_{kc}\left(T_{kc}\right) \ge V_{kc}\right\}, \tag{18.4.2}$$

subject to

$$V_{kc}\left(t_{kc1}\right) = 0; \tag{18.4.3}$$

$$t_{k,c+1,1} = \begin{cases} T_{kc} & if \quad V_{kc}\left(t_{kci}\right) < V_{kc},\ \forall i: t_{kci} < T_{kc} \\ t_{kci} < T_{kc} & if \quad V_{kc}\left(t_{kci}\right) \ge V_{kc},\ c \ge 1 \end{cases}; \tag{18.4.4}$$

$$t_{kc,i+1} - t_{kci} \ge d_{kc} \quad \forall i: 1 \le i \le N_{kc}; \tag{18.4.5}$$

$$T_{kc} - t_{kc} \geq \Delta_{kc} \quad \forall i : 1 \leq i \leq N_{kc}; \tag{18.4.6}$$

$$\bar{v}_{j-1,kc} \cdot (T_{kc} - t_{kc}) < V_{kc} - V_{kc}(t_{kci}) \leq \bar{v}_{jkc} \cdot (T_{kc} - t_{kci}), \quad 1 \leq k \leq n, \quad 1 \leq c \leq b. \tag{18.4.7}$$

Restriction (18.4.3) means that for each production unit k, the starting time of manufacturing product c has to be chosen as the first control point t_{kc1}. Decision-making at that control point centers on determining both the next control point t_{kc2} and the index of the speed, j, to be implemented from the beginning up to the second control point t_{kc2}.

Restriction (18.4.4) means that for the next product $(c+1)$ its first control point $t_{k,c+1,1}$ coincides with the planning horizon T_{kc} for the c-th product, $c \geq 1$, only if that product has to be inspected within the whole planning horizon. Otherwise, if a control point $t_{kci} < T_{kc}$ it has been observed that the c-th product has met its target, then the next $(c+1)$-st product starts manufacturing from that point on.

Restriction (18.4.5) ensures that the time span between two consecutive control points is restricted from below (in order to force convergence). Restriction (18.4.6) enables the inspection moment to be close to the due date. Restriction (18.4.7) means that at all control points t_{kci}, index j denotes the minimal speed that on average guarantees completion of V_{kc} by the due date T_{kc}. Thus, the on-line control model at the unit level prohibits implementing unnecessarily high and exhausting speeds.

The stochastic optimization problem (18.4.1-18.4.7) cannot be solved analytically in the general case; it allows only a heuristic solution. The heuristic algorithm outlined in [78, 102] determines at each control point t_{kci} for unit k engaged in manufacturing product c, the following parameters:

- speed v_{jkc} to be implemented until the next control point $t_{kc,i+1}$. The index of the speed j is the minimal value satisfying (18.4.7);
- the next control point $t_{kc,i+1}$ which is determined by using risk-averse decision-making. Assume that, due to most unfavorable circumstances, the unit will produce product c according to the minimal rate until the next control point. This point has to be determined so that, by applying the average *maximal speed* $\bar{v}_{in.kc}$ from that point on, there will still be enough time to meet target V_{kc} on time.

§18.5 Control model at the section level

If at any moment t it is anticipated that the unit would not be able to meet its target on time, a *local emergency* is declared, and the section level solves the optimization reallocation problem as follows: for section h under emergency, reallocate target amounts W_c^h and resources R_d^h among units k subordinated to that section, i.e., to determine optimal target amounts $V_{kc} \geq 0$ and resource capacities $R_{kd} \geq 0$, $k \in \{k^h\}$,

$1 \le c \le b$, $1 \le d \le f$, to maximize the probability of the slowest unit to meet its deadline on time, namely

$$J = \underset{\{V_{kc}, R_{kd}\}}{Max} \; \underset{k}{Min} \left[\prod_{c=1}^{b} Pr\left\{ V_{kc}(t) + X_{kc}(T-t) \ge V_{kc} \right\} \right] \tag{18.5.1}$$

subject to

$$\sum_{k \in \{k^h\}} V_{kc} = W_c^h - \sum_{k \in \{k^h\}} V_{kc}(t), \quad 1 \le c \le b, \tag{18.5.2}$$

$$\sum_{k \in \{k^h\}} R_{kd} = R_d^h, \quad 1 \le d \le f, \tag{18.5.3}$$

$$R_{kd}^{min} \le R_{kd} \le R_{kd}^{max}, \quad 1 \le d \le f, \quad 1 \le k \le n. \tag{18.5.4}$$

Here values $X_{kc}(T-t)$ are being determined by solving problem (18.3.1-18.3.4, 18.4.1) at the unit level with new target amounts V_{kc} and new resource capacities R_{kd}.

It can be well-recognized that in (18.5.1),

$$\prod_{c=1}^{b} Pr\left\{ V_{kc}(t) + X_{kc}(T-t) \ge V_{kc} \right\} \tag{18.5.5}$$

stands actually for the probability of unit k to accomplish all the products on time, subject to new target amounts V_{kc}. Thus, objective (18.5.1) maximizes the probability of the slowest unit k to meet its due date T on time. Restriction (18.5.2) means that for each product c, $1 \le c \le b$, the sum of target amounts V_{kc} determined by solving problem (18.5.1-18.5.4), has to be equal to the non-accomplished part of the section's target for that product at moment t. Restrictions (18.5.3-18.5.4) require that in the course of manufacturing, all available non-consumable resources at the section's disposal should be reallocated among the units while satisfying applicable boundary limits $R_{kd}^{\min}$ and $R_{kd}^{\max}$.

Reallocation problem (18.5.1-18.5.4) is a complicated stochastic optimization problem. A detailed description of the problem's solution is outlined in [77]. The solution is based on replacing objective (18.5.1) by

$$I = \underset{\{V_{kc}, R_{kd}\}}{Min} \; \underset{k}{Max} \left\{ \sum_{c=1}^{b} \frac{V_{kc}}{\bar{v}_{mkc}\left(R_{ka}, \ldots, R_{kf}\right)} \right\} \le T, \tag{18.5.6}$$

where *only units k subordinated to section h are taken into consideration*. A newly developed approximate algorithm to solve problem (18.5.2-18.5.4, 18.5.6) is presented in §18.7.

Note that the principal difference between objectives (18.3.4) and (18.5.6) boils down to the fact that (18.3.4) is based on *planned speeds* v_{pkc}, while objective (18.5.6) is, in essence, the average time required by the unit to meet its deadline on time, given that *only maximal speeds* v_{mkc} will be actually used throughout.

If solving reallocation problem (18.5.2-18.5.4, 18.5.6) after a recent emergency call at the unit level at moment t, results in obtaining $J > T - t$, section h is unable to accomplish all its products on time, even when introducing the highest production speeds with the utmost intensity. In such a case, an *overall emergency* is declared, and decision-making has to be carried out at the factory level.

The structure of the three-level control model is presented in Fig. 18.1.

§18.6 Control model at the factory level

The control model at the factory level operates in cases of overall emergencies declared.

To reassign, at $t > 0$, the remaining budget among sections, control actions have to be undertaken as follows:

A. The factory budget C has to be updated at moment $t = T_g$, i.e., within the interval $[T_{g-1}, T_g]$ between two routine adjacent overall emergencies

$$C - \sum_{d=1}^{f} \left[\left(T_g - T_{g-1} \right) C_d \cdot R_d \right] \Rightarrow C . \qquad (18.6.1)$$

B. An optimization problem, which is a dual one to the direct problem (1-4), has to be solved as follows:

determine new values R_d, $1 \le d \le f$, as well as values V_{kc} and R_{kd}, $1 \le k \le n$, $1 \le c \le b$, *for each production unit* k *entering the factory*, to minimize (18.5.6) subject to

$$\sum_{d=1}^{f} \left[C_d \cdot R_d \cdot (T - t) \right] \le C \qquad (18.6.2)$$

and

$$\sum_{k=1}^{n} V_{kc} = W_c - \sum_{k=1}^{n} V_{kc}(t), \quad \sum_{k=1}^{n} R_{kd} = R_d , \quad R_{kd}^{min} \le R_{kd} \le R_{kd}^{max} . \qquad (18.6.3)$$

Solving problem (18.5.6, 18.6.1-18.6.3) means that the factory has to rent and utilize resources within the remaining limited budget and to reallocate these resources among the units to meet the target on time. Note that, due to an overall emergency, only maximal production speeds will be used throughout.

After determining new resource values R_d, new resource capacities R_{kd} and target amounts V_{kc} are passed to the units, and the manufacturing process proceeds.

Fig. 18.1 presents interconnections between elements at all hierarchical levels.

§18.7 Approximate method for solving reallocation problems

It can be well-recognized that problems (18.3.1-18.3.4), (18.5.1-18.5.4) and (18.5.6, 18.6.1-18.6.3) are in fact modifications of a general production control reallocation problem. In [77], a "section → unit" reallocation problem is solved by applying the cyclic coordinate descent method [133]. However, the latter cannot be regarded as a high-speed method and may only be applied to control models of small or medium size. Otherwise, i.e., for large-scale hierarchical production systems, a more efficient algorithm has to be developed.

Let us briefly recall the main properties of the coordinate descent method. To solve the general production reallocation problem, one has to determine either of the optimal set of pairs $\{V_{kc}, R_{kd}\}$ or $\{T_{kc}, R_{kd}\}$. Note that knowing the first set results in determining the second one, and vice-versa. Thus, $(nb+nf)$ variables have to be determined, including $(b+f)$ dependent variables, namely, V_{nc}, $1 \le c \le b$, and R_{nd}, $1 \le d \le f$. The latter satisfy constraints

$$R_{nd} = R_d - \sum_{k=1}^{n-1} R_{kd}, \quad V_{nc} = V_c - \sum_{k=1}^{n-1} V_{kc}.$$

The cyclic coordinate descent method minimizes objective (18.3.4) cyclically with respect to the independent coordinate variables.

In order to speed up the approximate method for solving production control reallocation problems, we use two different criteria:

a) the *union criterion*

$$J_1 = \underset{1 \le k \le n}{Min} \left\{ T \Big/ \sum_{c=1}^{b} \left[V_{kc} \Big/ \sum_{d=1}^{f} \left(h_{kcd} \cdot R_{kd} \right) \right] \right\}, \tag{18.7.1}$$

and

b) the *production criterion*

$$J_2 = \underset{1 \le c \le b}{Min} \left\{ \sum_{k=1}^{n} \left[T_{kc} \cdot \sum_{d=1}^{f} \left(h_{kcd} \cdot R_{kd} \right) \right] \Big/ W_c \right\}, \tag{18.7.2}$$

where $\sum_{d=1}^{f} \left(h_{kcd} \cdot R_{kd} \right) = \bar{v}_{mkc}$ represents the average maximal speed of unit k to produce product c with resources R_{kd} (linear relationship holds).

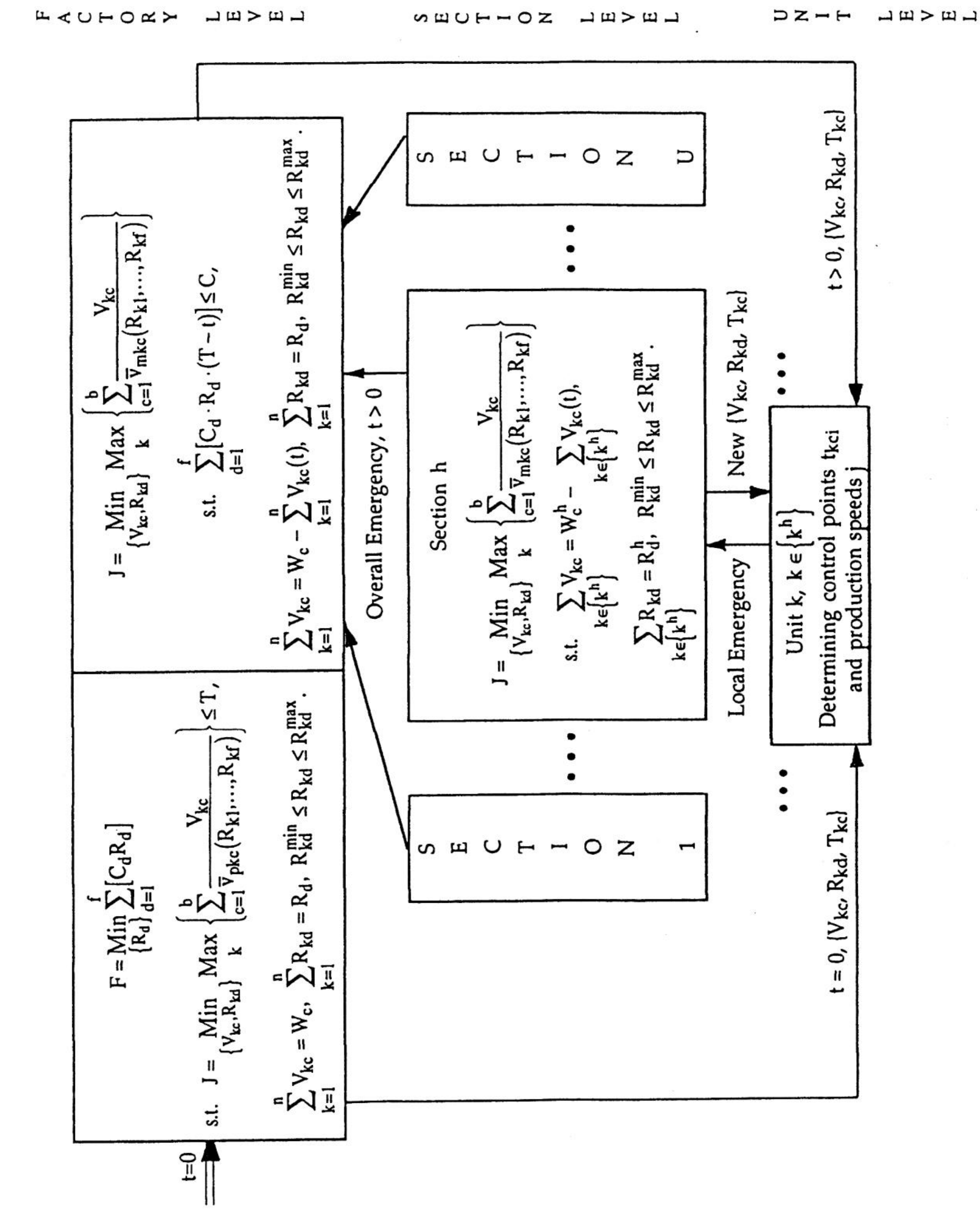

Figure 18.1. *<u>The structure of the three-level control model</u>*

Two corresponding production control reallocation problems have been formulated [78, 124]:

1. Reallocation *Problem 1* (*RP1*):

   Determine optimal values R_{kd} and V_{kc}, $1 \le c \le b$, $1 \le k \le n$, to maximize

   $$\underset{\{V_{kc}, R_{kd}\}}{Max}\; J_1$$

   subject to (18.3.3), where J_1 satisfies (18.7.1), and

2. Reallocation *Problem 2* (*RP2*):

   Determine optimal values R_{kd} and T_{kc}, $1 \le c \le b$, $1 \le k \le n$, to maximize

   $$\underset{\{T_{kc}, R_{kd}\}}{Max}\; J_2$$

   subject to

$$\sum_{c=1}^{b} T_{kc} = T\,,\;\; 1 \le k \le n\,, \quad \sum_{k=1}^{n} R_{kd} = R_d\,, \quad R_{kd}^{min} \le R_{kd} \le R_{kd}^{max}\,,\;\; 1 \le d \le f\,. \qquad (18.7.3)$$

Solving *Problems RP1* and *RP2* results in obtaining optimal production plans $\{V_{kc}, R_{kd}\}$ and $\{T_{kc}, R_{kd}\}$, respectively. It has been proven [124] that:

Theorem 1

For any fixed set $\{R_{kd}\}$ of resource capacities, solving *Problems RP1* and *RP2* results in obtaining equal objectives J_1 and J_2, i.e., $\underset{\{V_{kc}\}}{Max} J_1 = \underset{\{T_{kc}\}}{Max} J_2$ holds.

Theorem 2

Solving optimization *Problems RP1* and *RP2* results in obtaining identical optimal production plans $\{V_{kc}, R_{kd}\}$ and $\{T_{kc}, R_{kd}\}$, with equal objectives J_1 and J_2.

On the basis of these assertions, we have developed a high-speed algorithm (called here Algorithm A) which determines quasi-optimal plans for a fixed set $\{R_{kd}\}$ satisfying (18.3.3). The algorithm comprises a switching procedure using both criteria J_1 and J_2. Extensive simulation [124] shows that the algorithm converges well and is more efficient than various linear programming methods.

Thus, to solve reallocation *Problems RP1* and *RP2*, a combined *Algorithm B* has been developed [124], which is, in essence, a combination of two algorithms: the cyclic coordinate descent algorithm and *Algorithm A*. The idea of carrying out *Algorithm B* is as follows:

The cyclic coordinate descent algorithm undertakes the search with respect to each coordinate *only on the* nf *-dimensional set of variables* $\{R_{kd}\}$. After obtaining a routine search point $X = \{R_{kd}\}$, *Algorithm A* calculates for that point the additional nb coordinates $\{V_{kc}\}$ or $\{T_{kc}\}$ for *Problems RP1* and *RP2*, respectively. Since, in practice, the number of products b usually exceeds the number of resources f, value nb exceeds value nf. *Thus, we substitute the relatively slow coordinate descent algorithm in a high-dimensional area for the same algorithm in a lower dimensional area {* nf *versus* $(nb+nf)$*} in combination with a high-speed Algorithm I to calculate the additional* nb *coordinates*. For each search point X in the nf-dimensional area, the value of the objective is calculated according to $(nb+nf)$ coordinates $\{V_{kc}, R_{kd}\}$ or $\{T_{kc}, R_{kd}\}$, to determine the direction of the objective's increase. The iterative process terminates either at a local maximum point, or upon reaching a boundary point of set $\{R_{kd}\}$.

Thus, the input information for *Algorithm B* is the set of pregiven values $\{R_d\}$, $1 \le d \le f$, while at the output, we obtain either production plan $\{V_{kc}, R_{kd}\}$ or $\{T_{kc}, R_{kd}\}$. Note that the algorithm provides only a quasi-optimal solution, since the latter depends on the initial search point $\{R_{kd}\}^0$ which has to satisfy obvious restrictions

$$\begin{cases} \sum_{k=1}^{n} R_{kd}{}^{0} \le R_d \\ R_{kd}^{min} \le R_{kd} \le R_{kd}^{max}, \quad 1 \le d \le f \; . \end{cases} \tag{18.7.4}$$

Since the initial data $\{R_d\}$ satisfies, in turn,

$$\begin{cases} R_d \ge \sum_{k=1}^{n} R_{kd}^{min} \\ R_d \le \sum_{k=1}^{n} R_{kd}^{max}, \quad 1 \le d \le f , \end{cases} \tag{18.7.5}$$

it can be clearly recognized that one may obtain numerous initial search points $\{R_{kd}\}^0$ by implementing various trivial procedures. Thus, the algorithm's solution varies correspondingly.

Let us consider now in greater detail reallocation models (18.3.1-18.3.4), (18.5.1-18.5.4) and (18.5.6, 18.6.1-18.6.3). In the course of simulating the three-level control system, we solved problems (18.5.1-18.5.4) by implementing *Algorithm B*. As for the more complicated problems, e.g., problems (18.3.1-18.3.4) and their dual supplement (18.5.6, 18.6.1-18.6.3), they have been solved by using a newly developed approximate algorithm (call it *Algorithm C*). We shall illustrate the basic principles of the algorithm through the example of the two-level reallocation problem (18.3.1-18.3.4).

Algorithm C comprises two cycles - external and internal. In the external cycle, we implement the classical coordinate descent search algorithm with variables $\{R_d\}$ (to be optimized) and an objective

$$\underset{\{R_d\}}{Max}\, J = \underset{\{R_d\}}{Min}\left[\left\{\sum_{d=1}^{f}\left(C_d \cdot R_d \cdot T\right)\right\} + \delta\left(1 - J_1\right)\right], \tag{18.7.6}$$

where

$$\delta(x) = \begin{cases} 0 & if \quad x \le 0 \\ \Re & otherwise. \end{cases}$$

Here $\Re$ is a very large number (in the course of experimentation, we took it to be equal10^{17}), while J_1 is obtained by applying *Algorithm B* with the input $\{R_d\}$. Thus, introducing a quasi-optimal search by means of *Algorithm B*, forms the internal cycle. Note, in conclusion, that objective (18.7.6) automatically prohibits cases $J_1 < 1$, which do not lead to a feasible solution of problem (18.3.1-18.3.4). It must be also pointed out that $\underset{\{V_{kc}, R_{kd}\}}{Min} I \le T$ in (18.3.4) and $\underset{\{V_{kc}, R_{kd}\}}{Max} J_1 \ge 1$ in *Algorithm B* are equivalent relationships.

The dual reallocation problem (18.5.6, 18.6.1-18.6.3) has been solved by means of a small modification of *Algorithm C*.

The following conclusions can be drawn from the Chapter:

1. The developed three-level control algorithm can be successfully applied to real man-machine OS under random disturbances when the output can be measured only at pregiven control points. The algorithm can be used for controlling production systems with multiple resources and numerous products. The latter can be manufactured at several possible speeds.
2. The developed control algorithm is easy to handle: it can be implemented on a PC. Verifying the algorithm's performance by means of simulation for a factory comprising 24 production units on a PC, requires little computational time [124]. In our opinion, the three-level control model may be used for controlling man-machine production systems of any size.
3. The developed control algorithm enables determining the system's optimal structure, which results in minimizing the system's total expenses within the planning horizon. Calculating the system's cost parameters may be carried out by means of simulation.
4. It can be well-recognized that the active systems' techniques, similar to those outlined in the preceding chapter, can be applied for optimizing the multilevel production control model as well. Here human factors' influence can manifest itself by underestimating unit speeds v_{jkc} in order to work without intensity and, later on, returning to more realistic estimates accompanied by much-desired rewards. It goes

without saying that monitoring various multilevel OS under random disturbances by means of the active systems' theory results in different control models, yet the principal concepts (as well as the results) remain the same.

Conclusions

1. Modern man-machine organization systems (OS) are playing nowadays an important role in world's economics. Until now a lot of books and papers have been published referring to various particular problems in OS: human behavior, psychology, sociology, environment, construction, production management, safety engineering, health care, project management, etc. However, as yet there are no publications where for a standard man-machine OS a model would be created to enable monitoring and controlling the system as a whole, within the system's functioning. The purpose of our book (which is actually not a text-book, but rather a research monograph) was creating and presenting such a cybernetic model (usually a multilevel one) that would facilitate planning and controlling a hierarchical OS from bottom to top, until the system would reach its target. We have created such a model for various important OS related to project management, construction, production management, safety engineering, and maintenance systems. We have done our best to develop this research in conjunction with concepts of the Theory of Active Systems based on human behavior and socio-economics.

2. Problems of estimating the utility of complicated and usually multilevel OS are very urgent, especially for organization systems with a variety of quality parameters. Applications of the utility theory in recent publications are restricted to market competitive models and do not deal as yet with complicated hierarchical systems' functioning. The nowadays existing multi-attribute utility theory can be applied only to the stage preceding the product's design and determining the objectives for future market competition. We suggest implementing the utility concept as a generalized system's quality estimate which takes into account several essential parameters. The latter usually define the quality of the system as a whole. We have developed a generalized harmonization problem in order to maximize the system's utility. The corresponding model is optimized by means of a two-level heuristic algorithm. At the upper level (the level of independent parameters) a relatively simple search procedure, e.g., the cyclic coordinate algorithm, has to be implemented. At the lower level partial harmonization problems to optimize the dependent parameters, have to be used. Note, that nowadays there is no formalized linkage between the system's parameters and attributes and, thus, no optimization problem can be put and solved in order to maximize the product's utility within its specific life cycle [7, 49]. The developed research enables implementing such a linkage, in future, on the stages of both designing and creating new products and, later on, on the stage of marketing the product. Besides optimizing and calculating the system's utility, harmonization models are used in determining various reliability parameters. Thus, those models can be implemented in risk assessment analysis.

3. The newly developed harmonization models are examined and verified by considering various examples of organization systems. For stochastic PERT-COST network projects three parameters are implemented in the model: the budget assigned to the project, the due date and the project's reliability to meet the due date on time. The

harmonization model's solution is achieved by means of implementing a two-level heuristic algorithm. At the upper level a cyclic coordinate search algorithm to determine the quasi-optimal couple (budget – due date) is suggested. At the bottom level a high-speed heuristic procedure serving as a partial harmonization sub-model, is implemented: on the basis of input values (the assigned budget and the set due date) to maximize the probability of meeting the deadline on time by undertaking optimal budget reallocation among the project's activities. For the case of several stochastic network projects the developed theory enables determining optimal parametrical values for all projects in order to achieve the maximal utility level for the unification of all projects. The developed algorithm to optimize the harmonization model for a hierarchical project management system in R&D design offices, presents a two-level heuristic procedure. At the upper level a cyclic coordinate search algorithm together with a subsidiary model to verify the feasibility is implemented. At the lower level certain linear programming techniques can be applied to obtain an approximate solution. The harmonization model can be used both for projects with different priorities and for projects of equal significance.

4. Safety engineering concepts have been also implemented in the project's harmonization model for PERT-COST type stochastic network projects. Four parameters are imbedded in the model: the budget assigned to the project, the due date, the project's reliability to meet the due date on time and the hazardous failure probability. The latter criterion is difficult to be formalized and requires human judgment, rating schemes and other expert estimates in order to turn qualitative information into quantitative estimates. Thus, expert systems have been used to obtain the required information. Implementing an additional safety engineering related parameter results in an essential increase of the project's utility. This fully corresponds with the modern requirements of strengthening safety parameters as much as reasonably possible. Harmonization approaches in reliability and safety engineering have been successfully used to develop various cost – reliability optimization models. The latter are applicable to a broad spectrum of hierarchical technical systems with a possibility of hazardous failure at the top level and a pregiven multi-linkage of failure elements at different levels of the fault tree. In order to obtain quasi-optimal solutions of harmonization problems in reliability and safety engineering, we have implemented the sensitivity analysis in the corresponding optimization algorithms. Sensitivity values (e.g. cost-reliability sensitivity) have been successfully utilized for developing heuristic computational techniques. The newly developed harmonization models in project management and in reliability and safety engineering cannot be compared with any similar research outlined in former publications in the regarded area. The existing references do not cover multi-parametrical optimization both for multilevel PERT-COPST projects and hierarchical production plants with the possibility of hazardous failures at the top level.

5. Besides project management and safety engineering, harmonization models can be applied directly to all kinds of man-machine OS under random disturbances, e.g., construction and maintenance systems. The latter represent an essential part of

existing OS and require a high quality monitoring. For such OS we suggest using the developed harmonization techniques both for estimating the system's utility and for implementing regular control actions at inspection points to enhance the progress of the OS in the desired direction. Being a regulation model, harmonization can be implemented (in a random disturbances environment) as a risk assessment tool as well. Thus, for this class of OS, harmonization, controlling and risk assessment usually meet.

6. We have undertaken extensive experimentation to verify the fitness of the developed harmonization theory, especially in project management and safety engineering (see §§8.5, 9.2, 10.2 and 11.9). Besides justification purposes, the aim of such an experimentation was to show to the general reader the effectiveness of the developed approach, i.e., the comparative importance of the targets achieved versus non-complicated computations available on a common PC.

7. We have developed and presented in our book various man-machine OS being capable of approaching the goal with different speeds, depending on the intensity of the system's functioning. Those OS mainly cover production systems, especially building systems, mining enterprises, R&D projects, etc. We have developed cost-optimization models to monitor those systems. The fitness of the models has been verified by experimentation [173].

8. We have developed and presented several cost-optimization models referring to different cases of planning and controlling construction OS, especially for long-term innovative construction projects under random disturbances. The latter are usually characterized by a high level of uncertainty (e.g., by undertaking periodical geological surveys), which, in turn, leads to unpredictable alternative outcomes. The corresponding network projects, thus, comprise branching nodes in key events both of random and deterministic nature. To control such complicated projects we developed a special controlled alternative activity network (CAAN) model which enables decision-making in deterministic branching nodes [67-70, 83-84]. We also developed decision-making rules for undertaking capital investments (including contracting procedures) for long-term alternative innovative projects under random disturbances [25].

9. For various maintenance OS (especially in safety engineering) we have developed heuristic cost-reliability algorithms which enable solving two main problems:

   - to maximize the reliability of OS subject to cost-restriction (the direct problem) and to minimize the costs of maintenance subject to the system's reliability level (the dual problem).

   To solve those problems we applied a combination of cost-reliability sensitivity and predicting models.

10. We have undertaken intensive research to develop multi-attribute harmonization models in strategic management. A company engaged in designing and creating a new

product and, later on, delivering the latter in large quantities to the market, is considered. The product is composed of several subproducts, each of them, in turn, being a subject of several possible versions. The product's utility comprises both the utility of designing and creating the product's pattern example as well as the competitive utility to gain the future commercial success. The problem is to determine the input versions of designing subproducts in order to maximize the product's competitive utility subject to restrictions related to the design process. A two-level search algorithm of the problem's solution is suggested. The internal level is faced with optimizing the product's competitive utility by means of experts' information, while the external level centers on obtaining a routine feasible solution from the point of designing process.

11. Another important and urgent problem considered in the monograph refers to creating strategic hierarchical optimization models for complex holding corporations. A large-scale holding corporation comprising several subsidiary corporations, is considered. Each subsidiary corporation is engaged in designing and creating simultaneously several new products or providing services and, later on, delivering the latter in large quantities to the market. The product's utility comprises its marketability, i.e., the competitive ability to gain future commercial success on the market. The ability estimate can be forecast and determined by using scaled quantitative measures. In order to honor the company's good name for each product and service, their marketability has to be restricted from below. Given the holding corporation's budget, the problem boils down to optimal budget reallocation among subsidiary corporations and, later on, between individual projects. The objective is to maximize the holding corporation's marketability subject to the projects' marketability restrictions. This is a very complicated optimization model comprising three hierarchical levels - the holding corporation level, the subsidiary corporation level, and the project level. A heuristic three-level optimization algorithm based on the combination of a cyclic coordinate search method and the newly developed couple-reverse procedure, is suggested.

12. We have developed and outlined a new strategic optimization model to maximize products marketability and corporate sustainability. A company is faced with the problem of developing a new product intended for mass production and delivering in large quantities to the market. The product's utility expresses its marketability, i.e., the ability to gain future commercial success on the market. The R&D project at the development stage can be expressed by means of a stochastic network model with random activity durations. The due date of meeting the project's target as well as the desired probability of meeting the due date on time, are pregiven. The newly designed product comprises several competitive quantitative attributes with the corresponding restriction estimates. All attributes' values fully determine the product's marketability by means of experts' subjective judgment. Each attribute value depends on the corresponding part of the budget assigned to that attribute. The problem is to determine the minimal total R&D project's budget which enables accomplishing the project on time subject to the reliability constraint, and results in the maximal possible

marketability value. This means that assigning additional budget does not result in further improving the product’s marketability.

Another, not less important problem is as follows: the project of designing a new product is considered. The problem is to reallocate the company's expenses within the product's life cycle in order to maximize the company's profit. Thus, we deal with optimal budget reallocation within product's life cycle, including the sub-period of designing and creating the new product, as well as the sub-period of distributing the manufactured product on the market. Here the problem deals mostly with determining the product's selling price.

The results obtained can be directly applied to some urgent marketability problems (see, e.g., [118, 178]).

13. We have created and outlined in the book two multilevel complicated on-line control models for several important OS, namely:

   A. *Project management OS* (a portfolio of several stochastic PERT-COST projects) comprising a three-level on-line control model in conjunction with a resource delivery model (the fourth level).
   B. *Production management OS* (a three-level man-machine semiautomated control model).

   The following general idea is implemented in both on-line control models (see [70, 102, 104, 109, 124]) which comprise three optimization problems. *Problem I*, at the company level, enables optimal budget reassignment among the projects. The problem's solution, i.e., the budget assigned to each project, serves as the initial data for *Problem II* (at the project level), where budget is reallocated among the project's activities to maximize the probability of meeting the project's deadline. The solution of *Problem II* serves, in turn, as the initial data for *Problem III*, which carries out on-line control, i.e., determines optimal control points to inspect the progress of the project. This is done by determining the planned trajectories that must be repeatedly corrected in the course of the project's realization. If, at any control point, it turns out that a project deviates from the planned trajectory, an error signal is generated, and decision-making is based on solving *Problem II* to reassign the remaining budget among the remaining project's activities to maximize the probability to meet the deadline. If the problem's solution enables the project's deadline to be met, subject to the chance constraint, a corrected planned trajectory is determined, and *Problem III* is resolved to determine the next control point. Otherwise an emergency signal is generated, and decision-making is carried out at the company level. *Problem I is* resolved under emergency conditions to reassign the remaining budget among the non-accomplished projects. Thus, in the course of controlling a group of projects, the latter are first optimized on line from "top-to-bottom". In the case of an emergency, the generated "bottom-top" signals are converted into control actions to enable the projects' due dates to be met on time.

This general idea can be applied both to OS for project- and production management. One has only to substitute “design office” by “factory”, “project” by “section”, and “activity” by “production unit”, but the ideology of controlling a hierarchical OS remains the same. Extensive experimentation to verify the fitness of both models has yielded good results [109, 124].

14. When creating multilevel on-line control models for different OS, we have incorporated cost-optimization problems at the bottom hierarchical level, e.g., for controlling production- and project units. Those problems have been solved by introducing the newly developed *chance constraint principle* [87] to replace our former less effective *risk averse principle* [79].

15. Research has been undertaken to combine the general ideas of the outlined above multilevel on-line control models (see §§8.2-8.4) for several stochastic network projects, on one hand, with certain basic approaches and developments of the theory of active systems (§§13.4, 17.7), on the other hand. A conclusion can be drawn that such unification, being non-antagonistic to the human personnel’s rights, improves the quality of the project’s portfolio.

16. It can be well-recognized that the majority of OS considered in our monograph, have a strong innovative tendency. Indeed, in Chapters 9-11 research has been undertaken in the area of improving modern safety engineering devices which are subject to hazardous accidents. In Chapter 12 long-term construction projects with random alternative outcomes in key events and deterministic decision-making nodes are analyzed and controlled. Both chapters, based on a high level of indeterminacy, are taking aim at developing novel solutions in construction industry and in creating new engineering or technological devices. Chapters 8, 13-18 deal with complicated hierarchical on-line control models in project and strategic management which have not been described as yet in any publications. Thus, a conclusion can be drawn that the presented monograph covers OS which are innovative in nature.

17. To close up, we recommend to apply the basic general ideas of our research to *any* multilevel man-machine OS under random disturbances which:

    - can be formalized, and
    - requires monitoring and controlling,

    in order to reach its target subject to restrictions and chance constraints. We deal with such systems in our daily life.

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