TWO-LEVEL ACTIVE SYSTEMS

I. BASIC CONCEPTS AND DEFINITIONS

V. N. Burkov and V. V. Kondrat'ev

UDC 65.012.122

A model of a two-level active system is described and the capabilities of the Center for forming its functioning mechanism are analyzed.

The basic problem in the theory of active systems is the estimation of the effectiveness of the functioning mechanisms of organization for the purpose of synthesizing the most effective mechanisms (in economic systems these are planning procedures, motivational mechanisms, competitions, etc.). Investigations in the theory of active systems have been carried out at the Institute of Control Problems since 1968 [1]. The first papers were devoted to the evaluation of a number of concrete functioning mechanisms (the principle of open control [2], the principle of coordinated control [3-5]), to the stimulation of reliable information [6], and to the development of methods for solving the corresponding optimization problems [2, 3, 5-7]. The problem of choosing an optimal control law in active systems was first proposed in 1972 [8, 9] by example of the problem of resource distribution. The statement of the problem of choosing the control law in active systems in a sufficiently general form was considered in [10]. The statements of a number of other control problems in active systems were discussed in [11] by example of the resource distribution problem. A game-theoretic analysis of the functioning of active systems by example of a number of models was carried out in [8-12] and elsewhere (the analysis of Nash-equilibrium situations) and in [4, 5, 13] (the analysis of the guaranteed result principle). A survey of the results in the theory of active systems up to 1974, inclusive is given in [14] where an exhaustive bibliography is presented. The present cycle of articles is a generalization and a development of the papers on the investigation of two-level active systems not containing random parameters (deterministic two-level active systems [5, 8-13, 15]). In the present article we describe a model of a two-level active system and the mechanism of its functioning. In the remaining papers of the cycle we shall consider a game-theoretic statement of the control problem for active systems (AS) and we shall present a series of results.

1. Model of a Two-Level AS

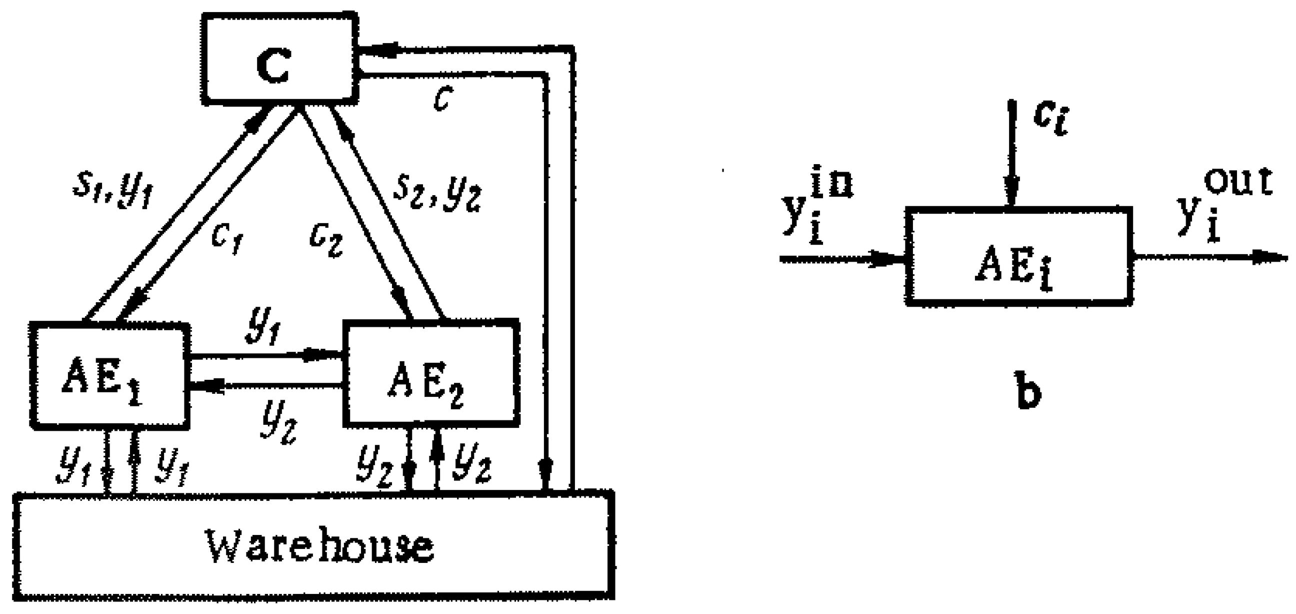
The structure of a two-level AS is formed by a Center (C), by n active elements (AE) subordinate to the Center, and by variables describing the state of the system. In order to account for the "external" connections of the system's elements we introduce the structural element "environment." To the environment we refer also certain "passive" elements of the system (for example, the centralized warehouses). In the deterministic models we shall assume that both the C as well as the AE know the state of the environment. Figure 1a shows an AS consisting of a Center, a centralized warehouse, and two active elements.

For the i-th AE we specify the state vector y_i (the realization vector in the economic interpretation), the control vector c_i, and the sets of their possible values: $y_i \in Y_i$ and $c_i \in C_i$, $i \in I = \{i \mid i = 1, 2, \ldots, n\}$.

Example 1. In the mathematical-economics literature there are a large number of models of the "expenditure–output" type, in which the element realization vector y_i is specified by means of an input realization vector y_i^{in} and an output realization vector y_i^{out}: $y_i = (y_i^{in}, y_i^{out})$, while the sets of possible realizations of the element is specified in the following way: $y_i^{in} \in Y_i^{in}$ and $y_i^{out} \in Y_i^{out}(y_i^{in})$, i.e., we specify the set of possible input realizations and the set of output realizations as a function of the input realizations (see Fig. 1b).

For the whole AS we specify the collection of realization vectors $y = \{y_i, i \in I\}$ (realizations of the AS), the collection of control vectors $c = \{c_i, i \in I\}$ (controls of the AS), and the sets of possible values of these quantities:

Moscow. Translated from Avtomatika i Telemekhanika, No. 6, pp. 64-72, June, 1977. Original article submitted May 6, 1976.



a b

Fig. 1

$y \in Y$ and $c \in C$. In the general case $Y = Y^g \cap \left(\prod_{i=1}^{n} Y_i\right)$, where Y^g are global constraints on the realization y of the AS: $y \in Y^g$. Analogously, $C = C^g \cap \left(\prod_{i=1}^{n} C_i\right)$. The presence of a purpose in the AS will be associated with the presence of the system's target function $W = \Phi(c, y)$ (for example, the economic utility– income function, the profit, the expenditures, etc.); the C will be reckoned as the administrative organ and it will be described in terms of the description of its actions with respect to the control of the AS. We shall also take it that the C's target function coincides with the AS's target function. Thus, the model of the AS can be represented in the form: $W = \Phi(c, y)$ is the target function of the AS, where $c = \{c_i, i \in I\}$, $y = \{y_i, i \in I\}$:

$$c_i \in C_i, \quad i \in I, \quad c = \{c_i\} \in C = C^g \cap \left(\prod_{i=1}^{n} C_i\right),$$

$$y_i \in Y_i, \quad i \in I, \quad y = \{y_i\} \in Y = Y^g \cap \left(\prod_{i=1}^{n} Y_i\right).$$

2. Description of the Center's Actions with Respect to Forming the Functioning Mechanism of the AS

The method of organizing the functioning of the AS's model will be called its functioning mechanism. Let us describe a number of components of the functioning mechanism of the model of a two-level AS and the capabilities of the C with respect to forming (changing) them.

Control by Means of Introducing Constraints. One of the properties of "activeness" is that the AE has a freedom of choice of the realization y_i from the set Y_i of possible realizations. In hierarchical systems the C can "judge" the set of possible realizations of each system elements by introducing for it a centrally established set of possible realizations B_i ($i \in I$). The set B_i of the i-th element can depend, firstly, on the controls c_i established by the C and, secondly, on the realizations y_j, $j \neq i$, chosen by the other elements of the system (this is frequently due to the presence of "horizontal connections" between the system's elements, e.g., reciprocal deliveries). First of all we consider the case when the AE are independent in the sense that the set B_i of possible realizations of each AE depends only on the control c_i and does not depend upon the realizations y_j, $j \neq i$, selectable by the other AE: $B_i(c_i)$, $i \in I$. It is obvious that when forming the sets $B_i(c_i)$ the C must satisfy the condition $\forall c_i \in C_i: B_i(c_i) \cap Y_i \neq \varphi$, $i \in I$. The introduction of sets $B_i(c_i)$ by the Center is called control by means of introducing constraints.

Procedures for Forming the Estimates. As a rule the functioning of hierarchical systems takes place under conditions of lesser information available to the C on the models of the elements subordinate to it. This situation is reflected in the theory of AS formally in the following manner. It is assumed that the sets $Y_i(r_i)$, $B_i(c_i, r_i)$, and $Y(r)$, where $r = \{r_i, i \in I\}$, are given in a parametric form known to the Center. Concerning the values of the vector-valued parameters r_i it is assumed that their dimensions are finite and that C knows only the set Ω_i of possible values of r_i, i.e., $r_i \in \Omega_i$ ($i \in I$), whereas each AE knows precisely the values of its "own" vector-valued parameter r_i. If the knowledge of the parameters $r = \{r_i\}$ only to within the set $\Omega = \prod_{i=1}^{n} \Omega_i$ is inadequate for an effective control of the system, then C can organize a procedure for forming estimates $s = \{s_i\}$ of the parameters. A number of such procedures are described in the Appendix.

Control Law in the AS. This is a procedure for which **C** determines the AS's control $c = \{c_i\}$ on the basis of the information available to it [10]. We shall examine the following structure of the control vectors: c_i: $c_i = (\lambda, x_i)$, where x_i is the plan vector and the vector λ of components is called, as before, the control. The components of the control vector λ can be common for a part or for all AE (for example, costs). The plan x_i is the desired value, determined by the Center, of all or a part of the components of the realization vector $y_i = \{y_{ij}\}$ of the i-th AE. At first we consider the case when the dimensions of vectors y_i and x_i coincide (completely plannable realizations) for all AE ($i \in I$).

A plan x_i of the i-th AE is said to be realizable if there exists a realization $y_i \in Y_i(r_i)$ such that $y_i = x_i$: It is obvious that in an AS with completely plannable realizations the set $X_i(r_i)$ of realizable plans of each AE coincides with the corresponding set $Y_i(r_i)$ of possible realizations: $X_i(r_i) = Y_i(r_i)$ ($i \in I$). Analogously, for the set $X(r)$ of realizable plans of the whole AS: $X(r) = Y(r)$.

For simplicity in what follows we shall take it that for each AE the set B_i depends only on the plan x_i: $B_i(x_i, r_i)$. The problem statement and the results presented below generalize without difficulty to the case when the sets B_i depends on both the plan x_i as well as the control λ: $B_i(x_i, \lambda, r_i)$. Concerning the sets $B_i(x_i, r_i)$ it is natural to require the fulfillment of the following condition: if $x_i \in X_i(r_i)$, then $x_i \in B_i(x_i, r_i)$, i.e., if plan x_i is realizable, then the set $B_i(x_i, r_i)$ contains the realization $y_i = x_i$.

Suppose that under the definition of plan the control **C** uses a certain procedure for forming the estimates $s = \{s_i\}$ of the parameters $r = \{r_i\}$ of the models of the AE. It is obvious that $s \in \Omega$, but in the general case $s \neq r$. The control law in the AS can now be defined as the mapping $s \xrightarrow{\pi} \lambda$, $x: \pi(s) = (x(s), \lambda(s))$. A number of AS control laws are described in the Appendix.

Criterial Control. The presence in the AE of the right to make decisions leads to the appearance in them of their own targets. This circumstance is reflected in the introduction for each AE of a target function $W_i = f_i(\lambda, x_i, y_i)$ (for example, as in the case of the **C**, the economic utility function). The action of the **C** with respect to forming (changing) the target functions of the AE is called a criterial control [11].

The presence of individual targets in the AE can lead to a situation where the realization y_i chosen by the AE may not coincide with the corresponding plan x_i. We shall assume that the AE is penalized when the plan and the realization do not coincide. Formally this can be reflected by the following condition on the AE's target function:

$$f_i(\lambda, x_i, y_i) < f_i(\lambda, y_i, y_i), \quad \text{if} \quad x_i \neq y_i, \ i \in I, \tag{1}$$

i.e., for a given realization y_i the value of the target function of the i-th AE is maximal if the realization y_i was planned for. An analogous condition holds for the target function of the AS:

$$\Phi(\lambda, x, y) < \Phi(\lambda, y, y), \quad \text{if} \quad x \neq y. \tag{2}$$

In practice the possibility of forming the target functions of the elements is connected with the possibility of forming a payment system, of introducing penalties and encouragements, of organizing competitions, and of making awards depending upon the place occupied, etc., which in the economic interpretation corresponds to the forming of a motivation system.

Functioning Mechanism of an AS. This is said to be realizable if any set of locally admissible realizations of the AE satisfies global constraints, i.e., $\forall s \in \Omega$, $\forall y_i \in B_i(x_i(s), r_i)$, $i \in I$: $y = \{y_i\} \in Y^g(r)$. A sufficient condition for the realizability of an AS's functioning mechanism is the condition of independence of the system's elements, viz.:

$$\forall s \in \Omega: \sum_{i=1}^{n} B_i(x_i(s), r_i) \subset Y(r). \tag{3}$$

Indeed, in this case any choice of realizations $y_i \in B_i(x_i(s), r_i)$, $i \in I$, yields a realization $y \in Y(r)$ of the system. We denote the set of plans x satisfying (3) by Z. Obviously, in a system with independent elements the planning procedure in the Center must be such that the plans obtained belong to Z: $x \in Z$. The independence of the system's elements in the sense indicated, as a rule, simplifies the investigation and utilization of the control in the system. Examples of systems with independent elements are presented in the Appendix.

3. Accounting for the Future in AE Effectiveness Criteria

The target function $W_i = f_i(\lambda, x_i, y_i)$ introduced above allows us to formalize the presence of a target in the case when the AE attempts to optimize its own utility function only in the functioning period being examined, without accounting for the future consequences of the decisions made "today." This is justified if the decisions made in a given period of functioning do not affect the future periods of functioning (more precisely, do not affect the plan x_i, the control λ, and the set $B_i(x_i, r_i)$ of possible realizations in the future periods). However, if such an influence exists, then it is natural to accept that the AE predicts the consequences of the decisions made (another property of "activeness"). As an example we point to the "planning from achievement" principle well known in economics, when the production output of an enterprise in a given period influences the plans for future periods. Under these conditions it may turn out advisable for the enterprises to lower the work effectiveness "today" so as to ensure advantageous work conditions "tomorrow." The method and the extent of accounting for the future for the various elements are determined mainly by the subjective characteristics of the managers. The function which reflects the subjective target of an AE in a given functioning period with due regard to future periods will be called the effectiveness criterion of the AE, allowing for the future functioning periods in contrast to the target function f_i which determines the economic effect of the AE only in the "current" period. Thus, as the effectiveness criterion of the AE, taking account in the k-th functioning period of N_i future periods, we can take

$$W_i^k = f_i(\lambda^k, x_i^k, y_i^k) + \sum_{q=k+1}^{k+N_i} f_i(\lambda^q, x_i^q, y_i^q). \tag{4}$$

The number N_i is called the "degree of foresight" of the i-th AE.

Another form of the reflection of the future can be the presentation of the effectiveness criterion for the AE as a sum of the element's target function in the current period and of the weighted sum of the element's target functions in the succeeding periods:

$$W_i^k = f_i(\lambda^k, x_i^k, y_i^k) + \sum_{q=k+1}^{\infty} \delta_i^{q-k} f_i(\lambda^q, x_i^q, y_i^q). \tag{5}$$

In principle we can admit the case when the summation extends only over N_i succeeding periods. The coefficient δ_i, called the "discount coefficient" in economic papers, characterizes the degree of foresight of the element. It is usually assumed that $0 < \delta_i < 1$. The peculiarity of effectiveness criterion (5) is its sliding nature. Indeed, $W_i^k = f_i(\lambda^k, x_i^k, y_i^k) + \delta_i W_i^{k+1}$, i.e., the effectiveness criterion in period k is the sum of the target function in period k and the weighted effectiveness criterion of the next period.

Other methods for accounting for the future can be proposed [10]. Let us stress only the following important detail. The Center does not know the AE's effectivity criterion even if it knows the form of its target function in an individual functioning period. The difficulties in determining the extent of allowing for the future in the effectiveness criterion of the elements concerns not only the Center but also the AE themselves, since the prediction of the consequences of the decisions made is a rather complex problem. Futhermore, the extent of allowing for the future in the effectiveness criteria of the AE can change from one functioning period to another. Therefore, a serious requirement on the control law is the independence (or weak dependence) of the behavior of an element (the decision-making principle) from (on) the method of taking the future into account in the element's effectiveness criterion.

4. Conclusion

Let there be given a model of an AS and its functioning mechanism. The functioning of such an AS consists of separate periods. Each period includes three stages: formation of the estimates, planning, and realization of the plan. At the stage of forming the estimates the C forms the estimate $s = \{s_i\}$ of the parameters $r = \{r_i\}$. At the planning stage the C determines the control $\lambda(s)$ and the plan $x(s)$ of the AS by the control law $\pi(s)$ and communicates them to the AE. At the realization stage each AE chooses a realization $y_i \in B_i(x_i(s), r_i)$, $i \in I$ after which the achieved value of the target functions of the elements and of the center are determined.

Let us enumerate the properties of the "activeness" of the organizational subsystems formalized in the description of an active element:

a) the presence of a purpose and the accounting for the future consequences of the decision made. Formally this property is reflected in that in the expression for the effectiveness criterion of the given period there occurs the target functions of future periods;

b) a definite freedom of action in communicating information on and realizations of the plans. Indeed, each element can communicate any estimate s_i from set Ω_i and choose any realization y_i from the set $B_i(x_i(s), r_i)$;

c) the knowledge of the structure and of the functioning mechanism of the system.

The AS described is a multicriterion system in which both the Center as well as the elements have the right to make independent decisions (the C can form or change the functioning mechanism of the AS). The situation is of a game (conflict) nature and calls for a game-theoretic approach.

APPENDIX

1. Certain Methods of Forming the Estimate $s = \{s_i\}$ of the Parameters $r = \{r_i\}$. In an adaptive method the estimates $s^{ad} = \{s_i^{ad}\}$ are determined by the Center on the basis of observations during the past functioning periods of the AS, for example, on the basis of some operator $s_i^k = \xi_i(s^{k-1}, y_i^{k-1})$, $i \in I$ (the C looks backward by one period) [11]. Here k is the number of the functioning period. Examples of the adaptive method for forming the estimates are the procedures, very well known in economics, of "planning from achievement," "planning from the mean index after a number of preceding periods," etc.

A counter-method of forming the estimates consists in each element informing the Center of the estimate $s_i^c \in \Omega_i$ of the parameter r_i [11]. The counter-method of forming the estimates can be applied, for instance, for planning in those cases when the individual functioning periods of the system are weakly connected with each other in the sense of the continuity of the model. Situations of such kind arise when the planning organ distributes the resources on demands, in the case of planning scientific-research and experimental-designer developments, when determining the order portfolio of the construction combines, etc.

Combined methods of forming estimates also are possible. For example, a number of parameters can be estimated by the adaptive method of forming estimates, while the remaining ones are estimated by the counter method, or the vector-valued parameters are estimated as a combination of the estimates obtained by adaptive and counter methods. Thus, the procedure for the formation of "counter-plan" data is constructed as follows. The planning organ takes as the base estimates, characterizing the capabilities of the enterprise, the indices achieved in the past functioning periods. This is an adaptive method for forming the estimates. If, however, the enterprise proposes to achieve during the plan period indices exceeding the ones achieved and informs the planning organ of this, then these indices are taken as estimates of the enterprise's model. This is a counter method. Consequently, under such a procedure the estimates s_i are obtained as $s_i = \max(s_i^{in}, s_i^{ad})$ and are combined estimates.

2. Certain Control Laws for AS. At the present time the planning problem in an economic system is often understood as solving some optimization problem. The optimal planning principle is constructed as a problem of maximizing the Center's target function under the constraints X(s) existing in the system. As a rule the control λ does not enter into the optimization problem. Later on it is implicitly assumed that all components of the elements' realization vectors are planned and that the plans obtained as a result of solving the problem are realizable and will be fulfilled, i.e., $y = x \in Y(r)$. Under these conditions the optimal planning principle is expressed in the form

$$\Phi(x) \to \max, \quad x \in X(s). \tag{A.1}$$

A development of the optimal planning procedure is the principle of strict centralization (SC) [9]. When planning on the SC principle it is already taken into account that the AE have a freedom of choice of their own realizations. It is assumed that for a given plan x_i and control λ each AE chooses a realization $\hat{y}_i$ in an optimal way

$$\varphi_i(\lambda, x_i, r_i) = f_i(\lambda, x_i, \hat{y}_i) = \max_{y_i \in B_i(x_i, r_i)} f_i(\lambda, x_i, y_i), \qquad i \in I. \tag{A.2}$$

We introduce $T(\lambda, x, r)$, the set of realizations $\hat{y} = \{\hat{y}_i\}$, satisfying these conditions. We denote

$$\Psi(\lambda, x, r) = \min_{z \in T(\lambda, x, r)} \Phi(\lambda, x, z). \tag{A.3}$$

The SC principle is formally written in the form of the following optimization problem:

$$\psi(\lambda, x, s) \to \max, \tag{A.4}$$

$$x \in X(s), \quad \lambda \in L. \tag{A.5}$$

The set of solutions of this problem defines the set of SC laws. A concrete SC law is obtained in problem (A.4)-(A.5) is complemented by a procedure for the unique selection of the solution. In a number of papers [9, 16, 17] by simple examples it is shown that the Center's application of the SC principle can lead to uncertainty in the information communicated to the AE and, as a consequence, to a lower effectiveness of functioning of the AS.

The Open Control (OC) Principle [2]. To problem (A.4)-(A.5) we add on the condition of perfect coordination

$$\varphi_i(\lambda, x_i, s_i) = \max_{z_i \in X_i(s_i)} \varphi_i(\lambda, z_i, s_i), \quad i \in I. \tag{A.6}$$

Conditions (A.6) reflect the requirements of "utility" of the plans prescribed by the Center for the AE under estimates $s = \{s_i\}$. The set of optimal solutions of problem (A.4)-(A.6) defines the set of OC laws. A concrete OC law is obtained if a rule for the unique selection of the solution is defined. The estimate $\varphi_i(\lambda, x_i, s_i)$ of the target function $\varphi_i(\lambda, x_i, r_i)$ is called the preference function of the AE. An analysis of the OC principle for a number of problems [9-11, 13] showed that as a rule the OC laws stimulate the communication of reliable information. However, the system's functioning effectiveness can be lower if the target functions of the AE are "poorly coordinated" with the target function of the system.

As a generalization of the OC principle Burkov and Ivanovskii [3-5] formulated the principle of coordinated control (CC), which is defined as solution of problem (A.4)-(A.5) with additional "coordination conditions"

$$\Psi_i(\lambda, x_i, s_i) = \max_{z_i \in X_i(s_i)} \Psi_i(\lambda, z_i, s_i), \quad i \in I, \tag{A.7}$$

where $\Psi_i(\lambda_i, x_i, s_i)$ is the preference function of the i-th AE. Having determined the concrete form of the preference function and the rule for a unique selection of the solution (in case it is nonunique), the C obtains a certain CC law. We remark that when $\Psi_i = \varphi_i$, $i \in I$, we obtain an OC law, while if Ψ_i does not depend on x_i, $i \in I$, we obtain a SC law. A number of variants of the coordination conditions have been suggested: the conditions of ε coordination [3], the conditions of guaranteed coordination [5], the coordination conditions ensuring a minimal reasonableness of the CC laws [11]. Meaningfully, the problem of determining the plan x(s) and the control $\lambda(s)$ in CC laws is the problem of determining the best plan from the point of view of the system as a whole, under the condition that this plan coordinates with the interests of the subsystems, i.e., is a problem of optimal coordinated planning."

3. Criterial Control. We have already examined criterial control above by means of introducing penalties if the AE's realization deviates from the plan. We consider another criterial control method by means of introducing penalties for distortion of information.

If $y_i \notin B_i(x_i, s_i)$, then, obviously, the estimate $s_i \neq r_i$. In this case the C can apply a system of penalties for the deviation of the estimate from the reliable one. The complexity here is that the C must determine some estimate θ_i which is taken as being reliable. Obviously, the estimate θ_i must satisfy the condition: $y_i \in B_i(x_i, s_i)$. In the simplest case the estimate θ_i is communicated by the AE themselves together with the realization y_i. The case of severe penalties for the uncertainty of the data is defined by the condition $s_{ij} \leq r_{ij} \forall j$, which is equivalent to the constraint $y_i \in B_i(x_i, s_i)$ on the set of possible realizations. The introduction of penalties for information distortion was analyzed in [9, 11] by example of a resource distribution problem and in [13] in a general model of an AS. We present two examples of systems with independent elements.

Example 1 (resource distribution). Let us consider an AS consisting of a C, n enterprises, and a centralized warehouse of raw-material resources of m types. By R_j we denote the amount of resources of type j at the warehouse in the functioning period being examined. The processing of the resources at enterprise i yields a specific effect y_i^{out} (a scalar). By $V_i(y_i^{in}, r_i)$ we denote the maximal effect which can be obtained at enterprise i under the existing technology of production. Here $r_i = \{r_{ij}\}$. The parameters r_{ij} characterize the maximal effectiveness of processing a recourse of j-th type by the i-th enterprise.

The real effect y_i^{out} may be less than $V_i(y_i^{in}, r_i)$ for reasons of poor organization of labor, of small personal interest on the part of the enterprise's manager in developing all the production reserves, etc. Thus, the set $Y_i(r_i)$ of possible realizations of the i-th enterprise is described by the inequalities

$$0 \leq y_{ij}^{in} < \infty, \quad j = 1, 2, \ldots, m; \quad 0 \leq y_i^{out} \leq V_i(y_i^{in}, r_i).$$

The set Y(r) of realizations of the AS is determined by the local constraints on the set $Y_i(r_i)$, $i \in I$, of possible realizations of each enterprise and by the global constraints reflecting the limited amount of the exogenous resource in the AS:

$$\sum_{i=1}^{n} y_{ij}^{ex} \leq R_j, \quad j=1,2,\ldots,m.$$

In order to achieve the independence of the enterprises it is necessary to restrict the set of their admissible realizations. Let the components of the plan $x^{in} = \{x_i^{in}, i \in I\}$ correspond to the resource distribution over the enterprises planned by the C. We accept that the enterprises use all the resources distributed to them, i.e., $y_i^{in} = x_i^{in}$, $i \in I$. As a matter of fact, by introducing this condition, we give a constraint on the set $B_i(x_i, r_i)$ of possible realizations under plan x_i of the form:

$$y_i^{in} = x_i^{in}, \quad 0 \leq y_i^{out} \leq V_i(y_i^{in}, r_i).$$

It is easy to see that for sets $B_i(x_i, r_i)$, $i \in I$, specified thus, any plan $x = \{x_i\}$ satisfying the constraints

$$\sum_{i=1}^{n} x_{ij}^{in} \leq R_j, \quad j=1,2,\ldots,m, \tag{A.8}$$

belongs to the set Z in (A.7). In this case the AS is, by the same token, a system with independent elements. We remark that a change in sets $B_i(x_i, r_i)$ also changes condition (A.8). For example, suppose that the C gives the enterprises the right to overconsume each type of resource up to 10% of the planned amounts. In this case the sets $B_i(x_i, r_i)$ will be defined by the conditions

$$x_{ij}^{in} \leq y_{ij}^{in} \leq \frac{11}{10} x_{ij}^{in}, \quad j=1,2,\ldots,m; \quad 0 \leq y_i^{out} \leq V_i(y_i^{in}, r_i), \quad i \in I,$$

while the condition ensuring the membership of any plan $x = \{x_i\}$ to set Z is written as

$$\sum_{i=1}^{n} x_{ij}^{in} \leq \frac{10}{11} R_j, \quad j=1,2,\ldots,m. \tag{A.9}$$

When distributing resources the C must allow for constraint (A.9). However, if the C in this case takes into account the "natural" constraints (A.8), then for $\sum_{i=1}^{n} x_{ij}^{in} = R_j$ and for the choice $y_i^{in} = (11/10)x_i^{in}$, $i \in I$, the corresponding realization $y \notin Y(r)$. Consequently, now the AS will not be a system with independent elements.

Example 2 (severe penalties). A system with severe penalties is characterized by an abrupt decrease in the value of the AE's target function when the realization deviates from the plan. In other words, the punishment for deviation from the plan is so harsh that a reasonable behavior of the AE is an unconditional fulfillment of the plan assignments, i.e., $\forall i: y_i = x_i$. The well-posedness of such a definition of a system with severe penalties requires the realizability of plan x, i.e., $x \in Y(r)$. Here, obviously, the AS is a system with independent elements.

LITERATURE CITED

1. V. N. Burkov, "Planning of product production in problems of engineering–material supply," in: Operative Control of Production [in Russian], Nauka (1971), pp. 93-97.
2. V. N. Burkov and A. G. Ivanovskii, "Planning in an engineering–material supply system on the basis of the open control principle," Materials for the Symposium on Problems of Creating Automatic Control Systems for Engineering–Material Supply [in Russian], Nauchn. Tekhn. Inst. Material'no-Tekhnicheskogo Snabzheniya, Moscow (1970), pp. 20-22.
3. A. G. Ivanovskii, "Problems of coordinated planning for two branches," in: Control Systems [in Russian], Nauka (1973), pp. 135-137.
4. V. N. Burkov, "Principle of control of multilevel active systems," Internat. Symp. Problems Organizational Control and Hierarchical Systems. Reports Abstracts [in Russian], Part 1, Inst. Avtomat. Telemekh., Moscow (1972).
5. V. N. Burkov, "Principle of coordinated control," in: Active Systems [in Russian], No. 2, Inst. Avtomat. Telemekh., Moscow (1974), pp. 3-42.
6. A. G. Ivanovskii, "Problems of stimulating and obtaining objective data in active systems," Avtom Telemekh., No. 8, 112 (1970).
7. V. N. Burkov and I. A. Gorgidze, "Estimate of the solution of the coordinated planning problem for one class of active systems," Soobshch. Akad. Nauk Gruz. SSR, 61, No. 1, 553 (1971).

8. V. N. Burkov, S. V. Emel'yanov, and V. I. Opoitsev, "Control of active systems," Proc. First Schools–Seminar on Control of Large-Scale Systems [in Russian], Metsniereva, Tbilisi (1973), pp. 5-33.
9. V. N. Burkov and V. I. Opoitsev, "Resource distribution in an active system," in: Active Systems [in Russian], Inst. Avtomat. Telemekh., Moscow (1973), pp. 27-46.
10. V. N. Burkov and S. V. Emel'yanov, "Control of active systems," in: Active Systems [in Russian], Inst. Avtomat. Telemekh., Moscow (1973), pp. 3-26.
11. V. N. Burkov and V. I. Opoitsev, "A metagame approach to the control of active systems," Avtom. Telemekh., No. 1, 103 (1974).
12. V. N. Burkov and V. V. Kondrat'ev, "Quasioptimality of the open control principle in a resource distribution problem," Sixth All-Union Conf. Control Problems. Abstracts of Reports [in Russian], Part 1, Nauka (1974), pp. 357-360.
13. V. N. Burkov and V. V. Kondrat'ev, "Analysis of open control laws for active systems from the viewpoint of the principle of guaranteed result," Proc. Mosk. Fiz. Tekhn. Inst., Ser. Radiotekh. Elektron., No. 9, 18 (1975).
14. S. V. Emel'yanov and V. N. Burkov, "Theory of active systems (a survey)," in: Coordinated Control [in Russian], Inst. Avtom. Telemekh., Moscow (1975), pp. 3-39.
15. V. N. Burkov, "Theory of active systems," Systems Sci., 2, No. 1-2, 23-55 (1976).
16. V. N. Burkov, I. M. Makarov, and V. B. Sokolov, "Centralized control of active systems," Avtom. Telemekh., No. 8, 113-119 (1973).
17. V. V. Kondrat'ev, F. S. Nuriev, and A. V. Shchepkin, "Analysis of equilibrium situations in an active system for a loading problem," in: Active Systems [in Russian], No. 2, Inst. Avtomat. Telemekh., Moscow (1974), pp. 63-72.