CONTROL LAWS OF "SUPPLIER – CUSTOMER" SYSTEMS

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The functioning is considered of simple models of "supplier–customer" systems in the process of marketing produced products. A comparative analysis is made of the laws of rigid centralization and of open control [1]. Theorems are proven on optimality of shipping plans of produced products in the solution of the corresponding games.

1. System Description

We consider a "supplier–customer" system for the case of a one-product model. We use the notation that b_t is the quantity of prepared products put up for sale in the interval t ($t = \overline{1, T}$, where T is the number of planning intervals). We suppose that the superior planning organization has already solved the problem of linking customers and suppliers and, consequently, for the given supplier there is determined the customers and the quantity of products $Q_{\rho T}$ to be shipped to customer ρ during the entire planning period T ($q = \overline{1, n}$, where n is the quantity of customers). With this,

$$\sum_{\rho=1}^{n} Q_{\rho T} = \sum_{t=1}^{T} b_t.$$

The supplier must concoct a graph of the shipments $u = (u_\rho) = \{u_{\rho t}\}$ of prepared production on the basis of demand from the customers, with account taken of their productive power. We assume here that each customer ρ communicates to the supplier an order in the form of an integral graph of shipments $Q_\rho = \{Q_{\rho t}\}$, where $Q_{\rho t}$ defines the quantity of production to be shipped to customer ρ during the first t intervals. The customer can also communicate information on the "urgency" of orders, for example, in the form of coefficients of loss from undelivered products $\beta_\rho = \{\beta_{\rho t}\}$ or of expenditures on storage of redundant products $\alpha_\rho = \{\alpha_{\rho t}\}$. Upon deviations of the actual integral graph of shipments $V_\rho = \left\{ V_{\rho t} = \sum_{1}^{t} u_{\rho t} \right\}$ from R_ρ the customer suffers a loss (in the case of $V_{\rho t} > R_{\rho t}$ these may be losses from the storage of redundant products, while when $V_{\rho t} < R_{\rho t}$ these may be losses due to shortages of raw material). We shall consider the simplest case of a piecewise-linear dependence of loss on the quantity of the deficit $\Delta_{\rho t} = R_{\rho t} - V_{\rho t}$, namely,

$$L_{\rho t} = \begin{cases} \alpha_{\rho t} \Delta_{\rho t}, & \text{if} \quad \Delta_{\rho t} \leqslant 0, \\ \beta_{\rho t} \Delta_{\rho t}, & \text{if} \quad \Delta_{\rho t} > 0, \end{cases}$$

where $\alpha_{\rho t}$ and $\beta_{\rho t}$ are loss coefficients.

We accept the total loss $L = \sum_{\rho=1}^{n} \sum_{t=1}^{T} L_{\rho t}$ as the criterion of effectiveness of functioning of the "supplier–customer" system. We consider the following system of interaction between suppliers and customers. The customer pays for products at the price c_t if shipment is performed during interval t. Of practical interest is the case when the total orders from the customer in any interval $t < T$ exceeds the quantity of products produced in this time, i.e., $\sum_{\rho=1}^{n} Q_{\rho t} \geqslant \sum_{1}^{t} b_\tau = B_t$. In this case $c_1 \geq c_2 \geq \ldots \geq c_T$, i.e., the later the production of the shipment, the higher the price. The price c_T can be considered as the wholesale price of production, while the difference

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$\lambda_t = (c_t - c_T)$ determines the overcharge due to urgency. In his turn, the supplier is penalized for breakdown in the schedule of production. Using a piecewise-linear form for the penalty function, we write the objective function of the supplier in the form

$$F = \sum_{\rho=1}^{n} \sum_{t=1}^{T} (c_T u_{\rho t} - f_{\rho t}),$$

$$f_{\rho t} = \begin{cases} \gamma_{\rho t}(V_{\rho t} - Q_{\rho t}), & \text{if} \quad V_{\rho t} \geqslant Q_{\rho t}, \\ \mu_{\rho t}(Q_{\rho t} - V_{\rho t}), & \text{if} \quad V_{\rho t} < Q_{\rho t}, \end{cases} \tag{1}$$

where $\gamma_{\rho t}$ and $\mu_{\rho t}$ are penalty coefficients.

Remark 1. In (1) the wholesale price of the product c_T is independent of the time of shipment since it is assumed that the additional profit from the overcharge due to urgency is entered into the government's budget. Moreover, there have not been included the component objective functions of the enterprises which do not depend on the chart of product shipments, and also, the time for the exchange of the documentation corresponding to the lapse of time between production and delivery of products is not taken into account.

The objective function of the customer includes the fee on productions and the losses upon deviation of the actual graph from the desired one of the shipments:

$$P_\rho = \sum_{t=1}^{T} (c_t u_{\rho t} + p_{\rho t}),$$

$$p_{\rho t} = \begin{cases} \alpha_{\rho t}(V_{\rho t} - R_{\rho t}), & \text{if} \quad V_{\rho t} \geqslant R_{\rho t}, \\ \beta_{\rho t}(R_{\rho t} - V_{\rho t}), & \text{if} \quad V_{\rho t} < R_{\rho t}. \end{cases} \tag{2}$$

Here we have not taken into account the expenditures of the customer on overhead costs on production transport since it is assumed that they depend only weakly on the dynamics of shipment.

Finally, we write the constraints determining the admissible shipment graph:

$$u_{\rho t} \geqslant 0 \ (\rho = 1 - n, \ t = 1 - T),$$

$$\sum_{\rho=1}^{n} V_{\rho t} \leqslant B_t \quad (t = 1 - T), \tag{3}$$

$$V_{\rho t} = Q_{\rho t} \ (\rho = 1 - n). \tag{4}$$

2. System Functioning

We shall assume that the losses to the customer from undelivered products essentially exceed the expenditures on storage of redundant products. Ignoring the latter, we write the customer's objective function (for customer ρ) in the form

$$P_\rho = \sum_{t=1}^{T} \left\{ c_t u_{\rho t} + \beta_{\rho t}(R_{\rho t} - V_{\rho t}) 1[R_{\rho t} - V_{\rho t}] \right\}, \tag{5}$$

where $1[x] = 1$ when $x \geq 0$ and $1[x] = 0$ when $x < 0$.

We assume that the supplier does not overship products above a specified quantity $Q_{\rho t}$, i.e., $V_{\rho t} \leq Q_{\rho t}$ for all ρ and t. Furthermore, we can neglect the components $\sum_{\rho=1}^{n} \sum_{t=1}^{T} c_T u_{\rho t} = c_T \sum_{\rho=1}^{n} Q_{\rho T}$ of the supplier's objective function as not depending on the delivery schedule of the produced products. Therefore, the supplier determines the plan for product shipment on the basis of the condition for minimization of penalties for breakdown of supplies or on the basis of the equivalent condition for maximization of the quantity

$$F = \sum_{\rho=1}^{n} \sum_{t=1}^{T-1} \mu_{\rho t} V_{\rho t}. \tag{6}$$

We now consider the functioning of the system. On the data-formation step, each customer communicates to the supplier the integral graph of production demands $Q_{\rho t}$ (orders) and, if possible, an estimate of the loss coefficient $\beta_{\rho t}$. We assume that penalty coefficient $\mu_{\rho t}$ is equal (or directly proportional) to this estimate. On the planning step the supplier determines the graph of product shipments $V = (V_\rho)$. On this same step the prices $c = (c_t)$ are determined (or corrected). There are possible diverse procedures for the formation of shipment plans and prices (diverse control laws). In this paper we investigate two control laws, i.e., the law of rigid centralization (RC) and the law of open control (OC). In the RC law, the prices $\{c_t\}$ are fixed (in particular, $c_t = c_w$, that is, the wholesale price of the product), while the shipment plan is determined as the optimal solution of the problem of maximization of (6) under conditions (3), (4), and

$$V_{\rho t} \geqslant V_{\rho, t-1} \quad (t=1-T, \rho=1-n, V_{\rho 0}=0), \tag{7}$$

$$V_{\rho t} \leqslant Q_{\rho t} \quad (t=1-T-1, \rho=1-n). \tag{8}$$

We write the problem of (3), (4), and (6)-(8) in another form by going to the variables $u_{\rho t} = V_{\rho t} - V_{\rho,t-1}$. We denote $S_{\rho t} = \sum_{\tau=t}^{T-1} \mu_{\rho\tau}$. After uncomplicated transformations we obtain the maximization problem

$$\sum_{\rho=1}^{n} \sum_{t=1}^{T-1} s_{\rho t} u_{\rho t} \tag{9}$$

with the constraints

$$\sum_{\rho=1}^{n} \sum_{1}^{t} u_{\rho\tau} \leqslant B_t \quad (t=1 \ T-1), \tag{10}$$

$$\sum_{1}^{t} u_{\rho\tau} \leqslant Q_{\rho t} \quad (t=1-T-1, \ \rho=1-n). \tag{11}$$

In the OC plan the deliveries and prices are determined as the result of solving one problem of optimal matched planning which is distinguished from the problem of (9)-(11) by supplementary conditions of complete matching [1]. These conditions reflect the interest of the customer in contributing to the delivery schedules. By virtue of the condition of complete matching, each customer obtains a matched shipment plan, i.e., a plan providing a minimum of the preference function

$$\psi_\rho = \sum_{t=1}^{T} \{c_t u_{\rho t} + \mu_{\rho t}(Q_{\rho t} - V_{\rho t}) 1[Q_{\rho t} - V_{\rho t}]\} \tag{12}$$

on the set of possible plans, determined by conditions (11). We remark that the coefficient $\mu_{\rho t}$ in (12) is the estimate of $\beta_{\rho t}$, while the order $Q_{\rho t}$ is the estimate of the desirable shipment $R_{\rho t}$ (in particular, when $\mu_{\rho t} = \beta_{\rho t}$ and $Q_{\rho t} = R_{\rho t}$ (12) coincides with (5)). Since $V_{\rho t} \le Q_{\rho t}$, then the condition of a minimum of (12) under constraints (11), is equivalent to the condition of a minimum of

$$\sum_{t=1}^{T-1} (c_t - s_{\rho t} - c_T) u_{\rho t} \tag{13}$$

with the same constraints.

We turn now to the investigation of the functioning of the systems with RC and OC laws from the position of game theory. The strategies of the players (customers) in the given case are shipment graphs $Q_\rho = \{Q_{\rho t}\}$ communicated to the supplier and, if possible, the loss coefficients $\mu_\rho = \{\mu_{\rho t}\}$, if communication of these coefficients is provided for in the scheme for system operation. The collection of the strategies $Q = \{Q_\rho\}$ (and, possibly, $\mu = \{\mu_\rho\}$) defines the game situation. We first consider the RC law and, thereafter, the OC law.

3. Analysis of Rigid Control Laws

We shall assume that $c_t = c_w$ (wholesale price of the product) for any interval. We initially consider the case when the loss coefficient $\beta_\rho = \{\beta_{\rho t}\}$ of the user are known to the supplier, where $\mu_{\rho t} = \beta_{\rho t} \bar{s}_{\rho t} = \sum_{\tau=t}^{T-1} \beta_{\rho\tau} \overset{\text{def}}{=} r_{\rho t} (t=1-T-1)$.

Moreover, let $\beta_{\rho t} \geq \beta_{\rho+1,\tau}$ for all t, τ, ρ = 1, n−1. In this case, it is not difficult to show that an optimal solution to the problem of (9)–(11) can be obtained by using a simple rule of customer priority: Products are shipped to customers in the order of their increasing ordinal numbers (i.e., one initially determines the shipment graph for the first customer, then for the second, etc.). The following theorem is true.

Theorem 1. The strategy $Q_\rho = R_\rho$ is absolutely optimal for any customer.

Proof. By the definition of an absolutely optimal strategy, it is necessary to prove that R_ρ is an optimal strategy for customer ρ under any strategies of the other customers. For the first customer, having the highest priority, this is obvious (we assume that $R_{\rho t} < B_t$ for all ρ and t). Let $\rho' > 1$. We take $Q_{\rho t} = 0$ for all $\rho < \rho'$. In this situation, $Q_{\rho'} = R_{\rho'}$ is the optimal strategy for customer ρ'. We can easily show analogously that $R_{\rho'}$ is an optimal strategy in any other situation. Consequently, $R_{\rho'}$ is an absolutely optimal strategy.

Corollary 1. If we take into account of losses on storage of superfluous products, the strategy $Q_\rho = R_\rho$ is the unique absolutely optimal strategy for any customer.

Proof. The proof to Corollary 1 follows from Theorem 1.

Thus, if the loss coefficients are such that there exists an ordering of the customers in the aforementioned sense, then the RC law provides optimality for the shipping plan without the introduction of auxiliary "urgency charges." If there is no such ordering, then the absolutely optimal strategy for each customer is $Q_{\rho t} = R_{\rho t}$ (t = 1, T), which corresponds to overstatement of the urgency (the losses on storage of superfluous products are not taken into account in the given case). It is clear that, in this case, the RC law does not even provide for optimality of the shipment plan of finished products.

We now consider the case when the loss coefficients $\{\beta_{\rho t}\}$ are unknown to the supplier while the estimates $\{\mu_{\rho t}\}$ of these coefficients are communicated by the customers. It is intuitively clear that, in this case, the customers will overstate the importance of urgent shipment of the products (communicate overstated values of $\mu_{\rho 1}$) and, correspondingly, will overstate the orders for shipment at earlier intervals.

Thus, the RC law, in the general case, does not solve the problems of designing optimal shipment plans.

4. Analysis of Open Control Law

We assume that the graph of R_ρ is known to the supplier, while the loss coefficients β_ρ are unknown. In an OC law the prices c_t are not fixed, but are determined, along with the shipment plan, as the result of having solved the problem of matched planning. Consequently, the vector of prices depends on situation s, i.e., on the collections of communicated estimates $\{s_{\rho t}\}$. However, with a sufficiently large number of customers, it is natural to believe that the effect of the estimates s_ρ of the individual customers on prices c_t will be insignificant. This makes plausible the following hypothesis of weak effect (WE): The customers do not take into account the influence of the communicated estimates on prices c_t. Initially, we make concrete the procedure for developing the prices. For this, we consider the problem, dual to that of (9)–(11), after having defined the dual variables $\bar{\lambda}_t \geq 0$, $\varkappa_{\rho t} \geq 0$ (t = 1–T−1, ρ = 1–n):

$$\sum_{t=1}^{T-1} \lambda_t B_t + \sum_{\rho=1}^{n} \sum_{t=1}^{T-1} \varkappa_{\rho t} R_{\rho t} \to \min \tag{14}$$

under the conditions

$$\sum_{\tau=t}^{T-1} \lambda_\tau + \sum_{\tau=t}^{T-1} \varkappa_{\rho\tau} \geqslant s_{\rho t} \quad (\rho=1-n, t=1-T-1). \tag{15}$$

We use the notation $\sum_{\tau=t}^{T-1} \lambda_\tau = c_t - c_T$, $\sum_{t}^{T-1} \varkappa_{\rho\tau} = \delta_{\rho t} (t=1-T-1)$. In the variables $c_t - c_T$ and $\delta_{\rho t}$ the conditions of (15) assume a simpler form:

$$(c_t - c_T) + \delta_{\rho t} \geqslant s_{\rho t} \ (\rho=1-n, t=1-T-1). \tag{16}$$

We now write the conditions relaxing the rigidity:

$$(\delta_{\rho t} + c_t - c_T - s_{\rho t}) u_{\rho t} = 0 \ (\rho=1-n, t=1-T-1), \tag{17}$$

$$(R_{\rho t} - V_{\rho t}) \varkappa_{\rho t} = 0 \ (\rho=1-n, t=1-T-1), \tag{18}$$

$$\left(B_t-\sum_{\rho=1}^{n}V_{\rho t}\right)\lambda_t=0 \quad (t=1\div T-1). \tag{19}$$

We mention that conditions (17) and (18), written for customer ρ, are the relations for rigidity relaxation for the problem of (11), (13) with the appropriate notation for the dual variables. Therefore, the selection for prices c_t with optimal values of the corresponding variables of the dual problem of (14), (15) is automatically reduced to the meeting of the conditions of complete matching (11), (13). We shall henceforth assume that $c_t=\sum_{t}^{T-1}\lambda_\tau^0+c_T$, where $\bar{\lambda}_T^0$ is the optimal value of the corresponding variables of the dual problem of (14) and (15). We set $c_T = c_W$ (the wholesale price of the products). We mention that, from the condition of nonnegativeness of $\bar{\lambda}_t$ ($t = 1, T-1$), it follows that $c_1 \geq c_2 \geq c_3 \geq \ldots \geq c_T$. Thus, the procedure for formulating shipment and price schedules is determined (if the solutions of the direct and dual problems are nonunique, the choice is then arbitrary). We turn now to the investigation of the equilibrium situation under the WE hypothesis. Sufficient conditions for equilibrium for customer ρ under the WE hypothesis have the form

$$\sum_{t=1}^{T-1}u_{\rho t}^*(c_t-c_W-r_{\rho t})=\min_{u_\rho}\sum_{t=1}^{T-1}u_{\rho t}(c_t-c_W-r_{\rho t}) \tag{20}$$

with the conditions $\sum_{\tau}^{t}u_{\rho\tau}\leqslant R_{\rho t}$ ($t = 1-T-1$).

Remark. Equilibrium situations may exist for which (20) does not hold. However, these situations are unstable, in the sense that the customers for whom (20) is violated will attempt to improve their positions. At the same time, the situations in which (20) is met for all customers are stable since, for each customer, there is guaranteed a minimum of this objective function under the WE hypothesis. We shall henceforth consider equilibrium situations which are stable in the aforementioned sense. An example of such a situation is $s = r$, i.e., communication by all the customers of plausible estimates.

Theorem 2. Any equilibrium situation corresponds to an optimal plan of product shipment.

Proof. We write the relationships for relaxed rigidity for problem (20), after having denoted the dual variables as $\varkappa_{\rho t}\geqslant 0$, $\delta_{\rho t}=\sum_{t}^{T-1}\varkappa_{\rho\tau}$:

$$(\delta_{\rho t}+c_t-c_W-r_{\rho t})u_{\rho t}=0 \quad (\rho=1-n, \quad t=1-T-1),$$

$$(R_{\rho t}-V_{\rho t})\varkappa_{\rho t}=0 \quad (\rho=1-n, \quad t=1-T-1).$$

After having added to these conditions (19) which are met in any situation, we obtain the relationships for relaxed rigidity for the problem of (9)–(11), where $s = 4$ and $Q = R$, which are necessary and sufficient for optimality of equilibrium of the shipping plan. Theorem 2 is proven.

Theorem 2 also remains valid in the case when the customer strategy is the communication of orders Q_ρ or orders and estimates (Q_ρ, s_ρ). Indeed, conditions (20) remain equilibrium conditions even for these cases.

Remark. It has already been mentioned that there may be several equilibrium situations. In particular, all situations of the form $s_{\rho t} = r_{\rho t} + q_\rho$ (q_ρ is any number, $\rho = 1, n, t = 1, T-1$) are equilibrium situations. From the point of view of the customer they are all equivalent. Therefore, with account taken of weak penalties on the communication of implausible estimates, one can assume that $s^* = r$ is the unique equilibrium situation.

We now investigate the OC law from the position of maximal guaranteed result.

Theorem 3. Any strategy $s_\rho \leq r_\rho$ is a guaranteeing one.

Proof. We denote by P the set of t such that $u_{\rho t} > 0$. By virtue of the matching conditions, $c_t \leq c_W + s_{\rho t}$ for all $t \in P$. In the least favorable case it is obvious that $c_t = c_W + s_{\rho t}$ for all $t \in P$. With this the guaranteed result equals the maximum of the quantity

$$\sum_{t\in P}(s_{\rho t}-r_{\rho t})u_{\rho t}$$

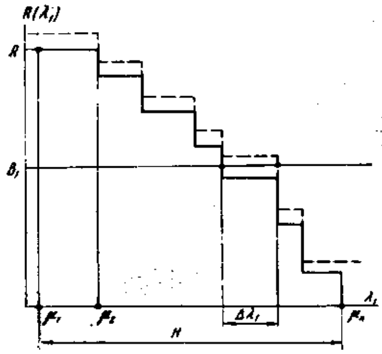

Fig. 1

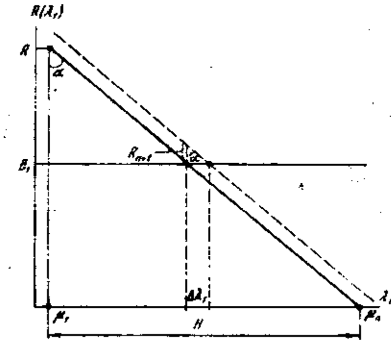

Fig. 2

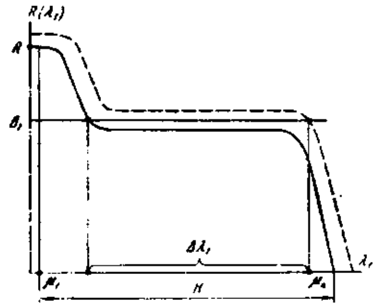

Fig. 3

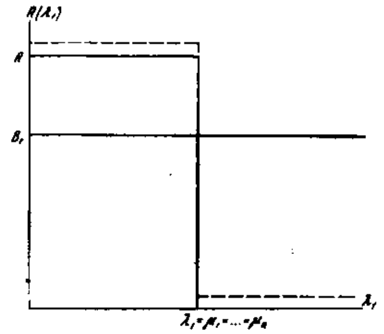

Fig. 4

under conditions (11) and is minimal when $s_{\rho t} \leq r_{\rho t}$, $t \in P$. Since any interval can belong to P, then $s_\rho \leq r_\rho$ is a guaranteeing strategy. The maximal guaranteed result equals 0. As in the case of equilibrium situations with penalties on discordant information taken into account, we can assume that r_ρ is the unique guaranteeing strategy. It remains to prove that there exist strategies of the other customers, such as $c_t = c_w + s_{\rho t}$ for all t. For this, it is sufficient to set $s_q = s_\rho$ for all $q \neq \rho$. Theorem 3 is proven.

5. The Weak-Effect Hypothesis

In investigating equilibrium situations, it was assumed that the customers do not take into account the effect of the communicated estimates μ_ρ on prices c_t (the hypothesis of weak effect). We now investigate the plausibility of this hypothesis. We restrict ourselves to the case when T = 2 (for example, annual planning with subdivisions into half-years). We denote by $\theta(\lambda_1)$ the set of ρ such that $\mu_{\rho 1} > \lambda_1$, and we construct the graph $R(\lambda_1) = \sum R_{\rho 1}$ for $\rho \in \theta(\lambda_1)$ (Fig. 1). We note that $\theta(\lambda_1)$ is the set of customers wishing to obtain products in the first interval with an increase of the price λ_1, while $R(\lambda_1)$ is the total of the orders of these customers. Obviously, the increase of λ_1 under the OC law is determined from the condition $R(\lambda_1) = B_1$ (cf. Figs. 1 and 2). We add one further customer and consider now λ_1 will change as estimate $\mu_{n+1,1}$ varies from zero to the maximal value (the dashed line on Fig. 1). This changes depends, as is easily seen, on order R_{n+1} and on the "speed of decrease" of $R(\lambda_1)$ and λ_1 increases. On Fig. 2 we smooth the curve for $R(\lambda_1)$. It is clear that $\Delta\lambda_1 \approx K \cdot R_{n+1}$, where K is the tangent of angle α. For example, let the loss coefficients $\mu_{\rho 1}$ be uniformly distributed in the interval of all possible values, i.e., $K = H/R$ ($R = \sum_{\rho=1}^{n} R_{\rho 1}$, $H = \mu_H - \mu_1$ — cf. Fig. 2). In this case $\Delta\lambda_1 \approx H \cdot R_{n+1,1}/R$, and the hypothesis of weak effect is sufficiently likely if the "weight" of the order of one customer is small in comparison with the total of all orders R (the absence of a "monopoly" customers). On Fig. 3 we show the example when the weight of the order of an individual customer is small but its effect on the price is significant. This is related to the fact that the other customers fall into two groups with significantly diverse loss coefficients. Finally, on Fig. 4, we show the example of the absence of effect of an individual customer on λ_1, when all the customers are identical.

Conclusions

Our investigation allows us to recommend the following operating mechanism in a "supplier–customer" system. The customers are unified into priority groups with essentially diverse loss coefficients in the diverse groups and with similar coefficients within any one of the groups. Distribution of products among the groups is performed in accordance with group priorities according to the principle of rigid centralization. Distribution of products among customers of one group is performed according to the principle of open control. With this, by virtue of the closeness of the loss coefficients of the customers of one group, it is possible to adopt the hypothesis of weak effect for a sufficiently large number of customers (on the order of five or more, as shown by experiments on business games). However, in equilibrium situations, the WE hypothesis guarantees an optimal plan of shipment (according to Theorem 2) and plausibility of the communicated information.

LITERATURE CITED

1. V. N. Burkov, Fundamentals of the Mathematical Theory of Active Systems [in Russian], Nauka (1977).