ELEMENTS OF OPTIMAL SYNTHESIS THEORY FOR FUNCTIONING MECHANISMS OF TWO-LEVEL ACTIVE SYSTEMS.

III. SOME PROBLEMS OF OPTIMAL COORDINATED PLANNING UNDER INCOMPLETE INFORMATION AT THE HEADQUARTERS

V. N. Burkov, A. K. Enaleev,
V. V. Kondrat'ev, and A. V. Tsvetkov

UDC 62-505.5

The results of [1, 2] are generalized to the case of incomplete information at the headquarters. Necessary and sufficient conditions are derived for optimality of correct functioning mechanisms when the varied components in the optimal synthesis problem are the planning procedure and the incentive system for the elements. Sufficient conditions for optimality of correct mechanisms are given. A number of new coordinated planning procedures are also considered.

1. In [1, 2] we developed a model describing the functioning of a two-level active system, formulated the optimal synthesis problem for the functioning mechanisms under the assumption of complete information at the headquarters, and derived necessary and sufficient conditions of optimality of correct mechanisms both on the set G_π with a fixed system objective function Φ and a fixed system of incentives f, and also on the set $G_{f,\pi}$ when only the system objective function Φ is fixed. Constructive sufficient conditions were derived for optimality of incentive systems ensuring that the plan is met.

In this article the results previously obtained in [1, 2] under complete information at the headquarters are generalized to the case when the headquarters has access to incomplete information about the goals of the elements, the system objectives, and the possible system states. Necessary and sufficient conditions are derived for optimality of correct functioning mechanisms on the set $G_{f,\pi}$ under incomplete information; constructive sufficient conditions are given under which incentive systems ensuring that the plan is met are optimal on some set of functioning mechanism. The optimal synthesis problem is generalized to one class of mechanisms with coordinated planning procedures, which include the correct functioning mechanisms, the partially coordinated planning mechanisms, the coordinated plan-exceeding mechanisms, and various combinations of these mechanisms. Previously mechanisms of this kind were considered in [3-5].

2. The model of a two-level active system functioning under incomplete information at the headquarters differs from the model described in [1] in that the system objective function Φ, the element incentives f, and the set of possible system states Y are known at the headquarters up to a set Ω of uncertain vector parameters r. It is assumed that the elements know the values of the parameter r when they select their state, but there is no exchange of information about the value of the parameter r between the elements and the headquarters. As before, we assume that the elements choose their states independently of one another [6].

Assume that the headquarters attempt to eliminate the uncertainty by using the guaranteed outcome principle [6]. Then the effectiveness of the system functioning mechanism is measured by the criterion $K(\Sigma) = \min K(\Sigma, r)$ over $r \in \Omega$. Here $K(\Sigma, r) = \min \Phi(x, y, r)$ over $y \in R(f, x, r)$ is the guaranteed estimate of the objective function Φ on the set of solutions of the game of the elements $R(f, x, r) = \{y \mid y = x, \text{ if } x \in P(f, x, r), \text{ else } y \in P(f, x, r)\}$ given the incentive system f, the plan x, and the parameter r and assuming friendly locally optimal behavior of the elements; $P(f, x, r) = \{y \in Y(r) \mid f_i(x_i, y_i, r) \geqslant f_i(x_i, z_i, r),\ z_i \in Y_i(r),\ i \in I\}$ is the set of locally optimal states of the elements.

Remark. The dependence of the system objective function Φ on the parameter r characterizes the variation of system goals. In some cases, even if the initial system objective function does not depend on the param-

Moscow. Translated from Avtomatika i Telemekhanika, No. 12, pp. 94-100, December, 1984. Original article submitted September 5, 1983.

 0005-1179/84/4512-1616$08.50

eter r, it is important for practical purposes to consider the synthesis problem with a "weighted" objective function $\Phi'(x, y, r) = \Phi(x, y)/\theta(x, r)$, which may correspond, say, to some multicriterial problem [7]. The function θ may be taken, say, in the form $\theta(x, r) = \max\Phi(x, y)$ over $y \in Y(r)$, $\theta(x, r) = \max \Phi(y, y)$ over $y \in Y(r)$ or $\theta(x, r) = \Phi(x, y(x, r))$, where $y(x, r)$ is some prespecified function of x and r.

The optimal system problem under incomplete information at the headquarters may be written (as under complete information) in the form

$$K(\hat{\Sigma}) = \max_{\Sigma \in G_\Sigma^r} K(\Sigma), \quad \hat{\Sigma} \in G_\Sigma^a \cap G_\Sigma^g. \tag{1}$$

Here and in what follows we assume that both max and min exist. We use the same notation as in the preceding articles [1, 2].

Let us consider some basic solutions of the problem (1). On the set G_π $(G_\Sigma{}^a = G_\pi)$, without any supplementary constraints $(G_\pi \subseteq G_\Sigma{}^g)$, the solution of the problem (1) is the *mechanism* $\Sigma^{OPP} = \langle \Phi, f, \pi^{OPP} \rangle$ with optimal planning procedure with state prediction π^{OPP} generalized to the case of incomplete information at the headquarters:

$$\pi^{OPP}: X^{OPP} = \operatorname{Arg} \max_{x \in X(f)} [\min_{r \in \Omega} \min_{y \in R(f,x,r)} \Phi(x, y, r)].$$

Here X^{OPP} is the set of plans generated using the procedure π^{OPP}. The procedure π^{OPP} exists regardless of the specific amount of information available at the headquarters, but it obviously *depends on the set* Ω. Moreover, as the set Ω becomes larger, the system functions less effectively with this planning procedure. Indeed, if $\Omega' \subseteq \Omega$, then

$$\min_{r \in \Omega'} K(\Sigma, r) \geqslant \min_{r \in \Omega} K(\Sigma, r).$$

As $G_\Sigma{}^g$ take the set of correct functioning mechanisms $\tilde{G}_\Sigma$. The set of perfectly coordinated plans under incomplete information at the headquarters given the incentive system f and the set Ω will be defined as $S(f, \Omega) = \bigcap_{r \in \Omega} S(f, r)$, $S(f, r) = \{x \in Y(r) \mid f_i(x_i, x_i, r) \geqslant f_i(x_i, y_i, r),\ y \in Y(r),\ i \in I\}$ is the set of perfectly coordinated plans for the case when the parameter r is known at the headquarters. The elements are motivated to implement plans from the set $S(f, \Omega)$ for any $r \in \Omega$. If $S(f, \Omega') \neq \phi$ and $S(f, \Omega') \cap S(f, r) = \phi$ for any $r \in \Omega \setminus \Omega'$, the elements are motivated to implement only those plans with $r \in \Omega'$.

The solution of the problem (1) on the set $G_\pi \cap \tilde{G}_\Sigma$ is the mechanism $\Sigma^{OPC} = \langle \Phi, f, \pi^{OPC} \rangle$ with optimal perfectly coordinated planning procedure π^{OPC} generalized to the case of incomplete information at the headquarters:

$$\pi^{OPC}: X^{OPC} = \operatorname{Arg} \max_{x \in X(f) \cap S(f,\Omega)} [\min_{r \in \Omega} \Phi(x, x, r)].$$

Here X^{OPC} is the set of optimal perfectly coordinates plans generated by the procedure π^{OPC}.

For Σ^{OPC}, unlike for Σ^{OPP}, the very possibility of constructing the procedure π^{OPC}, as well as the effectiveness of the functioning mechanism, depend on the degree of ignorance of the headquarters. As the headquarters ignorance increases, in general, the set of perfectly coordinated plans becomes more restricted: $\Omega' \subseteq \Omega$ implies that $S(f, \Omega) \subseteq S(f, \Omega')$. In the final analysis, increasing ignorance may lead to a situation such that $S(f, \Omega) = \phi$ and the procedure π^{OPC} does not exist at all. If $Y(\Omega) = \bigcap_{r \in \Omega} Y(r) \neq \phi$, the fact that $S(f, \Omega) = \phi$ is attributable to a poor choice of the incentive system. This difficulty in principle is avoidable within the framework of our model of functioning mechanisms. If $Y(\Omega) = \phi$, we have to apply additional data generating procedures in order to reduce the headquarters ignorance [6, 7].

LEMMA 1. Let $Y(\Omega) \neq \phi$ and for any $x \in Y(\Omega)$, $y \in Y(r)$, $r \in \Omega$, $i \in I$, $x \neq y$:

$$\chi_i(x_i, y_i) \geqslant \max_{r \in \Omega} [\max_{y_i \in Y_i(r)} h_i(y_i, r) - \max_{y_i \in Y_i} h_i(y_i, r)],$$

then $S(f, \Omega) \neq \phi$, where $h_i(y_i, r) = f_i(y_i, y_i, r)$ and $\chi_i(x_i, y_i) = f_i(y_i, y_i, r) - f_i(x_i, y_i, r)$.

The proof of the lemma is analogous to the proof of Lemma 2 in [2].

As we see, under incomplete information at the headquarters penalties for deviation of the element states from the plan in general are no longer sufficient to ensure that $S(f, \Omega) \neq \phi$.

Define the set of efficient perfectly coordinated plans $A(\Sigma, \Omega)$. This is the set of plans which, if performed, are no less effective than the plan x given the mechanism Σ and the set Ω:

$$A(\Sigma, \Omega) = \{z \in Y(\Omega) \mid \min_{r \in \Omega} \Phi(z, z, r) \geqslant K(\Sigma)\}.$$

Then the set $A(G_{f,\pi}, \Omega) = \bigcap_{\Sigma \in G_{f,\pi}} A(\Sigma, \Omega)$ is the set of all efficient plans with respect to the set of mechanisms $G_{f,\pi}$ and the set Ω.

Denote $Y(f, x, r) = \{z \mid z = x, \text{ if } x \in R(f, x, r), \text{ else } z \in Y(r)\}$.

Let us consider the optimality conditions of correct functioning mechanisms on the set $G_{f,\pi}$. We have

1°. $\exists \hat{f} \in \bar{G}_f : A(G_{f,\pi}, \Omega) \cap S(\hat{f}, \Omega) \cap X(\hat{f}) \neq \phi$;

2° $\exists \hat{f} \in \bar{G}_f,\ \hat{x} \in A(G_{f,\pi}, \Omega) \cap X(\hat{f}) : \forall y \in Y(r),\ i \in I,\ r \in \Omega$:

$$\hat{f}_i(\hat{x}_i, \hat{x}_i, r) \geqslant \hat{f}_i(\hat{x}_i, y_i, r); \tag{2}$$

3° $\exists \hat{f} \in \bar{G}_f,\ \hat{x} \in X(\hat{f}) \cap Y(\Omega) : \forall f \in \bar{G}_f,\ x \in X(f) : \exists z \in R(f, x, r'),\ r' \in \Omega : \forall y \in Y(r),\ r \in \Omega,\ i \in I$ inequality (2) and the inequality

$$\Phi(\hat{x}, \hat{x}, r) \geqslant \Phi(x, z, r') \tag{3}$$

4° $\exists \hat{f} \in \bar{G}_f,\ \hat{x} \in X(\hat{f}) \cap Y(\Omega) : \forall f \in \bar{G}_f,\ x \in X(f) : \exists z \in Y(f, x, r'),\ r' \in \Omega : \forall y \in Y(r),\ r \in \Omega,\ y' \in Y(r'),\ i \in I$ inequality (3) and the inequality

$$\hat{f}_i(\hat{x}_i, \hat{x}_i, r) - \hat{f}_i(\hat{x}_i, y_i, r) \geqslant f_i(x_i, y_i', r') - f_i(x_i, z_i, r')$$

are satisfied.

There is no analog of condition 5° from [2] under incomplete information.

THEOREM 1. The problem (1) has a solution if and only if any of the equivalent conditions 1°–4° is satisfied.

The proof of the theorem in general outline coincides with the proof of Theorem 1 from [2].

Condition 1° is the set-theoretic form of the optimality condition for correct functioning mechanisms under incomplete information at the headquarters.

Conditions 2°–4° are alternative forms of the condition 1°.

Conditions 1°–4° are valid under very general assumptions concerning the properties of the relevant functions and sets. Additional "useful" information about these properties generally allows constructive modification and simplification of the conditions 1°–4°, and especially 4°. For instance, let $r = (r_1, r_2, r_3)$, $r \in \Omega$, $\Omega = \Omega_1 \times \Omega_2 \times \Omega_3$ and assume that Φ depends only on a single component r_1, f depends only on r_2, and Y only on r_3. Then the form of problem (1) is not changed. In condition 1° we get $S(\hat{f}, \Omega) = S(\hat{f}, \Omega_2 \times \Omega_3)$. Condition 4° may be recast in the form: 4°a. $\exists \hat{f} \in \bar{G}_f,\ \hat{x} \in X(\hat{f}) \cap Y(\Omega_3) : \forall f \in \bar{G}_f,\ x \in X(f) : \exists z \in Y(f, x, r_2, r_3),\ r_2 \in \Omega_2,\ r_3 \in \Omega_3$:

$$\forall y \in \bigcup_{r_3 \in \Omega_3} Y(r_3),\quad y' \in Y(r_3),\quad i \in I:$$

$$\min_{r_1 \in \Omega_1} \Phi(\hat{x}, \hat{x}, r_1) \geqslant \min_{r_1 \in \Omega_1} \Phi(x, z, r_1),$$

$$\min_{p \in \Omega_2} [\hat{f}_i(\hat{x}_i, \hat{x}_i, p) - \hat{f}_i(\hat{x}_i, y_i, p)] \geqslant f_i(x_i, y_i', r_2) - f_i(x_i, z_i, r_2).$$

We see that when the headquarters has incomplete information only about the system objective function Φ, the problem reduces to the case of complete information in a system with the objective function $\Phi'(x, y) = \min \Phi(x, y, r_1)$ over $r_1 \in \Omega_1$. Similarly, if the center also has incomplete information about the goals of the elements and the possible states of the system, while the parameter r_1 is independent on the parameters r_2 and r_3, the problem (1) reduces to a synthesis problem with the objective function Φ'.

A particular case of the condition 4°a is obtained when the headquarters only has estimates of the system objective function Φ, the incentive system f, and the set of possible states Y:

$$\Phi\in\Phi^0=\{\Phi\,|\,\Phi^{\min}(x,y)\leqslant\Phi(x,y)\leqslant\Phi^{\max}(x,y)\},$$
$$f\in f^0=\{f\,|\,f_i^{\min}(x_i,y_i)\leqslant f_i(x_i,y_i)\leqslant f_i^{\max}(x_i,y_i),\quad i\in I\},$$
$$Y\in Y^0=\{Y\,|\,Y^{\min}\subseteq Y\subseteq Y^{\max}\}.$$

Then the necessary and sufficient condition 4°a takes the following form:

4°b. $\exists \hat f^0\in\bar G_f,\ \hat x\in X(\hat f^0)\cap Y^{\min}:\forall f^0\in\bar G_f,$

$$\begin{aligned}&x\in X(f^0):\exists f\in f^0,\quad Y\in Y^0,\quad z\in Y(f,x):\\&\forall y\in Y^{\max},\ y'\in Y,\ \hat f\in \hat f^0,\ i\in I:\\&\Phi^{\min}(\hat x,\hat x)\geqslant\Phi^{\min}(x,z),\\&\hat f_i(\hat x_i,\hat x_i)-\hat f_i(\hat x_i,y_i)\geqslant f_i(x_i,y_i')-f_i(x_i,z_i).\end{aligned}$$

3. Sufficient optimality conditions for correct functioning mechanisms derived in [1, 2] under complete information at the headquarters may be generalized to the case of incomplete information. Let us consider the sufficient conditions which follow from condition 4°. The cases corresponding to conditions 4°a and 4°b may be investigated along the same lines as condition 4°.

As in [2], we define sets of functioning mechanisms such that constructive solutions of the problem (1) exist on these sets or on their intersections.

The set of functioning mechanisms for which the system suffers losses as a result of the deviation of the actual states of the elements from the plan is denoted by $G_\Sigma^1=\{\langle\Phi,f,x\rangle\in G_\Sigma\,|\,\Phi(y,y,r)\geqslant\Phi(x,y,r),\ y\in Y(r),\ r\in\Omega\}$. We also denote $G_\Sigma^2=\{\langle\Phi,f,x\rangle\in G_\Sigma\,|\,\exists r\in\Omega,\ \hat f\in\bar G_f:\ Y(r)=Y(\Omega),\ \bigcup_{r\in\Omega}Y(r)\subseteq X(\hat f),\ S(\hat f,\Omega)\neq\emptyset\}$,

$$\begin{aligned}&G_\Sigma^3=\{\langle\Phi,f,x\rangle\in G_\Sigma\,|\,\exists r\in\Omega,\ \hat f\in\bar G_f:\ R(f,x,r)\cap Y(\Omega)\cap X(\hat f)=\emptyset,\\&S(\hat f,\Omega)\neq\emptyset\},\\&G_\Sigma^4=\{\langle\Phi,f,x\rangle\in G_\Sigma\,|\,\exists\hat f\in\bar G_f:\forall z\in X(f)\cap Y(\Omega),\ y\in Y(r),\ r\in\Omega,\\&i\in I:\\&\hat f_i(z_i,z_i,r)-\hat f_i(z_i,y_i,r)\geqslant f_i(x_i,z_i,r)-f_i(x_i,y_i,r)\}.\end{aligned}$$

The remaining notation is as in [1, 2].

<u>COROLLARY.</u> Correct functioning mechanisms solve the problem (1) if one of the following conditions is satisfied:

1^1. $\exists\hat f\in\bar G_f^{12}:\forall\hat x\in X(\hat f),\ y\in Y(r),\ r\in\Omega,\ i\in I:$

$$\hat f_i(\hat x_i,\hat x_i,r)-\hat f_i(\hat x_i,y_i,r)\geqslant 0;$$

2^1. $\exists\hat f\in\bar G_f^{12}:\forall f\in\bar G_f^{12},\ \hat x\in X(\hat f)\cap Y(\Omega),\ x\in X(f),\ y\in Y(r),\ r\in\Omega,\ i\in I:$

$$\hat f_i(\hat x_i,\hat x_i,r)-\hat f_i(\hat x_i,y_i,r)\geqslant f_i(x_i,\hat x_i,r)-f_i(x_i,y_i,r);$$

3^1. $\exists\hat f\in\bar G_f^{12}:\forall\hat x\in X(\hat f)\cap Y(\Omega),\ y\in Y(r),\ r\in\Omega,\ i\in I:$

$$\hat f_i(\hat x_i,y_i,r)=\begin{cases}\max\limits_{f\in\bar G_f^{12}}\max\limits_{x_i\in X_i(f)} f_i(x_i,y_i,r), & \text{if } \hat x_i=y_i,\\ \min\limits_{f\in\bar G_f^{12}}\min\limits_{x_i\in X_i(f)} f_i(x_i,y_i,r), & \text{if } \hat x_i\neq y_i;\end{cases}$$

4^1. $\exists\hat f\in\bar G_f^{124}:\forall\hat x\in X(\hat f)\cap Y(\Omega),\ y\in Y(r),\ r\in\Omega,\ i\in I:$

$$\hat f_i(\hat x_i,y_i,r)=\begin{cases}\max\limits_{f\in\bar G_f^{124}} f_i(\hat x_i,y_i,r), & \text{if } \hat x_i=y_i,\\ \min\limits_{f\in\bar G_f^{124}} f_i(\hat x_i,y_i,r), & \text{if } \hat x_i\neq y_i.\end{cases}$$

The corollary is proved along the same lines as the corollary in [2].

If the incentive system $f=(h,\chi)$ is characterized by a fixed function h, we obtain:

1^1a. $\exists\hat{\chi}\in\bar{G}_\chi^{12}(h):\forall\hat{x}\in X(\hat{\chi}),\ y\in Y(r),\ r\in\Omega,\ i\in I$:

$$\hat{\chi}_i(\hat{x}_i, y_i)\geqslant h_i(y_i, r)-h_i(\hat{x}_i, r);$$

2^1a. $\exists\hat{\chi}\in\bar{G}_\chi^{1}(h):\forall\chi\in\bar{G}_\chi^{1}(h),\ x\in X(\chi),\ \hat{x}\in X(\hat{\chi})\cap Y(\Omega),\ y\in Y(r),\ r\in\Omega,\ i\in I$:

$$\hat{\chi}_i(\hat{x}_i, y_i)\geqslant\chi_i(x_i, y_i)-\chi_i(x_i, \hat{x}_i)\geqslant\Delta_i(y_i, \Omega),\ \hat{\chi}_i(\hat{x}_i, y_i)\geqslant 0,$$

where

$$\Delta_i(y_i,\Omega)=\begin{cases}-\infty, & \text{if } y_i\in X_i(\chi)\cap Y_i(\Omega),\\ \max\limits_{r\in\Omega}\left[\max\limits_{i\in Y_i(r)} h_i(y_i,r)-\max\limits_{y_i\in X_i(\chi)\cap Y_i(\Omega)} h_i(y_i,r)\right], & \text{if } y_i\notin X_i(\chi)\cap Y_i(\Omega).\end{cases}$$

3^1a. $\exists\hat{\chi}\in\bar{G}_\chi^{12}(h):\forall\hat{x}\in X(\hat{\chi})\cap Y(\Omega),\ y\in Y(r),\ r\in\Omega,\ i\in I$:

$$\hat{\chi}_i(\hat{x}_i, y_i)=\begin{cases}0, & \text{if } \hat{x}_i=y_i,\\ \max\limits_{\chi\in\bar{G}_\chi^{12}(h)}\ \max\limits_{x_i\in X_i(\chi)}\chi_i(x_i, y_i), & \text{if } \hat{x}_i\neq y_i.\end{cases}$$

4^1a. $\exists\hat{\chi}\in\bar{G}_\chi^{124}(h):\ \forall\hat{x}\in X(\hat{\chi})\cap Y(\Omega),\ y\in Y(r),\ r\in\Omega,\ i\in I$:

$$\hat{\chi}_i(\hat{x}_i\, y_i)=\max_{\chi\in\bar{G}_\chi^{124}(h)}\chi_i(\hat{x}_i, y_i).$$

Under these conditions, the set $\bar{G}_\Sigma^{2}$ may be replaced with the set $\bar{G}_\Sigma^{3}$.

The condition 1^1a substantially differs from the analogous condition of high penalties under complete information [1, 6]. The inequality in condition 1^1a is no longer sufficient. In this case, additional assumptions are required concerning the set of the objective functions of the elements $\bar{G}_f$ and the set of possible states of the system Y. One of the requirements ensuring sufficiency of condition 1^1a is formulated in Lemma 1.

An analog of condition 2^1a under complete information is the condition of strong coordination [1, 6]. The difference is that there are constraints on the minimum penalties and also on the magnitude of their difference. Thus, under incomplete information, continuous penalty functions do not necessarily satisfy condition 2^1a.

4. The method proposed in [1, 2] for constructing the solutions of the optimal synthesis problem (1) in the class of correct functioning mechanisms under complete information may be generalized under incomplete information to represent coordination of more general type. The need for such a generalization is motivated by the specific functional features of systems mainly under incomplete information, when correct functioning mechanisms without communication are either insufficiently effective [6] or cannot be implemented at all (e.g., if $Y(\Omega)=\phi$.

The proposed generalization consists of the following. By analyzing the system performance and using the prevailing standards and regulations, the headquarters generates the set of functioning mechanisms G_Σ^g that satisfy the supplementary constraints. Let us consider the case when the constraints on functioning mechanisms may be represented as constraints on the system states and written as a set of coordinated states L dependent in general on the plan x and the parameter r. The elements will clearly strive to achieve the states of this set (assuming friendliness of the elements and the headquarters [6]) if the headquarters assigns plans from the set $S^L(f, r)=\{x|\,L(x, r)\cap P(f, x, r)\neq\phi\}$. The set $S^L(f, r)$ will be called the set of L-coordinated plans given the incentive system f and the parameter r. As with perfect coordination, we denote by $S^L(f,\Omega)=\bigcap\limits_{r\in\Omega}S^L(f,r)$ the set of L-coordinated plans given the incentive system f and the set Ω; by

$$A^L(\Sigma,\ \hat{f},\ \Omega)=\{x\,|\min_{r\in\Omega}\min_{y\in P(\hat{f},x,r)\cap L(x,r)}\Phi(x, y, r)\geqslant K(\Sigma)\}$$

we denote the set of plans which are not less effective than the mechanism Σ under L-coordination, given the incentive system $\hat{f}$ and the set Ω. The optimality conditions for L-coordinated mechanisms are constructed as in the case of perfect coordination considered above. For example, an analog of the optimality condition 1° has the form

1°c. $\exists\hat{f}\in\bar{G}_f:A^L(G_{f,\pi},\hat{f},\Omega)\cap S^L(\hat{f},\Omega)\cap X(\hat{f})\neq\phi.$

The specific form of the analogs of conditions 2°-4° and the form of the sufficient optimality conditions for L-coordinated mechanisms depends on the properties of the set of coordinated states L. Therefore, sufficient conditions should be derived separately for each particular case of coordination. Let us consider some sets of coordinated states which actually occur in organizations.

In case of perfect coordination, $L^1(x) = \{y \mid y = x\}$ for every $r \in \Omega$.

In case of partial coordination, such as that considered in [4], $L^2(x, r) = \{(y^1, y^2) \in Y(r) \mid y^1 = x^1\}$ for every $r \in \Omega$, where $x = (x^1, x^2)$. Here the plan should be met only with respect to some of the components with the superior index 1.

Coordination which allows to exceed the plan or to minimize the deviation of the actual state from the plan are particularly important under incomplete information at the headquarters, since even if $Y(\Omega) = \phi$, mechanisms with these types of coordination are realizable. Examples of coordinated plan-exceeding states are the following: $L^3(x) = \{y \mid y \geqslant x\}$, $L^4(x, r) = \{y \in Y(r) \mid \overline{\exists} z \in Y(r) : \exists i \in I,\ j \in J_i : z \geqslant y,\ z_{ij} > y_{ij}\}$, $L^5(x^i, x^f, r) = \{y \in Y(r) \cap [x^f; x^i] \mid y \geqslant z,\ z \in Y(r) \cap [x^f; x^i]\}$ and $L^6(x^f, x^i, r) = \{y \in Y(r) \cap [x^f; x^i] \mid \overline{\exists} z \in Y(r) \cap [x^f; x^i] : \exists i \in I,\ j \in J_i : z \geqslant y,\ z_{ij} > y_{ij}\}$, where $\geq$ is the partial order symbol [7], x^i is the initial plan, x^f is the final plan, $[x^i, x^f] = \{y \mid x^f \geq y \geq x^i\}$. The cases of L^4-coordinated and L^5-coordinated functioning mechanisms were considered in [3] and [5], respectively. In particular, the L^5-coordinated mechanism motivates the elements to achieve states that are "closest" in the sense of the given preference relation to the final plan x^f, from $[x^i, x^f]$.

LITERATURE CITED

1. V. N. Burkov, A. K. Enaleev, V. V. Kondrat'ev, and A. V. Tsvetkov, "Elements of optimal synthesis theory for functioning mechanisms of two-level active systems. I. Necessary and sufficient optimality conditions for correct functioning mechanisms under complete information at the headquarters," Avtomat. Telemekh., No. 10, 139-143 (1983).
2. V. N. Burkov, A. K. Enaleev, V. V. Kondrat'ev, and A. V. Tsvetkov, "Elements of optimal synthesis theory for functioning mechanisms of two-level active systems. II. Synthesis of optimal correct functioning mechanisms under complete information at the headquarters," Avtomat. Telemekh., No. 11, 86-92 (1984).
3. V. V. Kondrat'ev, "Synthesis of functioning mechanisms of active systems under uncertainty," in: Proc. 8th All-Union Seminar-Conf. on Management of Large Systems [in Russian], Kazakh. Politekh. Inst. im. V. I. Lenin, Alma-Ata (1983), pp. 2-4.
4. V. V. Kondrat'ev, A. A. Tikhonov, and A. V. Tsvetkov, "Partially coordinated planning under incomplete information at the headquarters," in: Proc. 8th All-Union Seminar-Conf. on Management of Large Systems [in Russian], Kazakh. Politekh. Inst. im. V. I. Lenin, Alma-Ata (1983), pp. 18-20.
5. A. V. Tsvetkov, "Coordinated planning in problems with meeting and exceeding the plan under uncertainty," in: Proc. 8th All-Union Seminar-Conf. on Management of Large Systems [in Russian], Kazakh. Politekh. Inst. im. V. I. Lenin, Alma-Ata (1983), pp. 61-63.
6. V. N. Burkov and V. V. Kondrat'ev, Functioning Mechanisms of Organizations [in Russian], Nauka, Moscow (1981).
7. N. N. Moiseev (ed.), Modern State of Operations Research Theory [in Russian], Nauka, Moscow (1979).