COST-EFFICIENT TAXATION MECHANISMS

V. N. Burkov and V. I. Khokhlov

UDC 336.2

Taxation mechanisms are of central importance in a market economy. This paper examines so-called cost-efficient taxation mechanisms, i.e., mechanims that encourage producers to increase quality and reduce costs (weak cost-efficiency) and mechanisms that encourage producers to reduce prices following cost reduction even in a monopolistic setting (cost-efficiency). A general form of cost-efficient tax scales is derived for various tax systems.

Government regulation takes different forms during transition to a market economy (price formation, taxation, state orders, etc.). Taxation is one of the main levers used by government. On the one hand, taxation should be sufficiently "soft" so as not to reduce business activity and provide incentives to business firms to increase production efficiency and cut costs; on the other hand, taxation should supply the required revenues to the state budget and also serve anti-monopolistic functions (eliminate excess profits, encourage competition). The paper examines a class of taxation mechanisms that have the property of cost-efficiency, i.e., mechanisms that encourage higher quality (a useful effect) and lower costs (weak cost-efficiency) and also mechanisms that encourage price reduction even for a monopolistic producer (cost-efficiency).

1. TYPES OF TAX SYSTEMS

There are different kinds of tax systems. First, tax can be levied on profit or income. Second, the tax rate may be a function of some indicator, such as again profit or income, or alternatively some relative measure, such as profit rate, average income per worker, income-to-labor cost ratio, etc. Finally, the business (residual) profit (income) may be a monotone increasing function of price and a decreasing function of cost, reaching a point of maximum by these two indicators. Tax systems of the first kind are called soft, in distinction from tax systems of the second kind that are called rigid. We will give a formal description of different types of tax systems. For ease of presentation, we denote the type of the tax system by two words: the first word is the taxed indicator and the second word is the indicator that determines the tax rate. Thus, the "profit–profit rate" system is a tax system which taxes the profit and in which the tax rate depends on the profit rate. As a rule, we consider only progressive tax systems, in which the tax rate is a nondecreasing function of the corresponding indicator. We will now describe the most common forms of tax systems.

2. "PROFIT–PROFIT RATE" TAX SYSTEM

This system taxes the profit and the tax rate is an increasing (nondecreasing) function of the profit rate. Profit is defined as

$$\Pi = \rho C,$$

where $\rho = (P - C)/C$ is the profit rate (P is price and C is cost). The tax is thus calculated as

$$T = \mu(\rho) \quad \Pi = \mu(\rho)\rho C, \tag{1}$$

where $\mu(\rho)$ is the tax rate.

The residual after-tax profit is

$$\Pi_0 = \Pi - T = [1 - \mu(\rho)]\rho C. \tag{2}$$

With a soft tax system, Π_0 is an increasing function of price or, equivalently, an increasing function of profit rate, because $P = (1 + \rho)C$.

Institute of Control Problems, Moscow. "Telekom" Concern, Moscow. Translated from Avtomatika i Telemekhanika, No. 5, pp. 89-97, May, 1992. Original article submitted May 4, 1991.

0005-1179/92/5305-0703$12.50

On the other hand, Π_0 is a decreasing function of cost. Substituting $C = P/(1 + \rho)$ in (2), we obtain

$$\Pi_0 = [1 - \mu(\rho)] \frac{\rho}{1+\rho} P. \tag{3}$$

If Π_0 is a decreasing function of cost, then (3) is obviously an increasing function of profit rate. Note that this is a stronger requirement than increase of (2) as a function of profit rate: if (3) is an increasing function of ρ, then (2) is also an increasing function of ρ. Differentiating (3) with respect to ρ, we obtain a condition of "softness" of the tax system:

$$\rho(1+\rho) \frac{d\mu}{d\rho} + \mu \leqslant 1. \tag{4}$$

Let us derive a general expression for soft tax scales. To this end, denote

$$h(\rho) = \mu(\rho) + \rho(1+\rho) \frac{d\mu}{d\rho} \tag{5}$$

and consider the differential equation (5), where $h(\rho) \leq 1$. Its solution is

$$\mu(\rho) = \frac{1+\rho}{\rho} \int_0^{\rho} \frac{h(x)}{(1+x)^2} dx. \tag{6}$$

For a soft tax system to be progressive, i.e., for $\mu(\rho)$ to be an increasing function, we must additionally have $d\mu/d\rho \geq 0$. Differentiating (6), we obtain

$$\frac{d\mu}{d\rho} = -\frac{1}{\rho^2} \int_0^{\rho} \frac{h(x)}{(1+x)^2} dx + \frac{h(\rho)}{\rho(1+\rho)} \geqslant 0$$

or, using (6),

$$\mu(\rho) \leqslant h(\rho). \tag{7}$$

Using relationships (6), (7), we can design soft progressive tax scales that satisfy the given requirements.

Example 1. Consider the progressive tax system

$$\mu(\rho) = \begin{cases} k\rho, & \text{if} \quad \rho \leqslant a; \\ ka, & \text{if} \quad \rho > a, \end{cases}$$

where $ka < 1$.

When is this a soft system? The system is obviously always soft for $\rho > a$, because

$$h(\rho) = \mu(\rho) = ka < 1.$$

If $\mu(\rho) = k\rho$, then

$$h(\rho) = k\rho(2+\rho) \leqslant 1.$$

A necessary and sufficient condition of softness of a tax system has the form

$$ka(2+a) \leqslant 1.$$

What happens if this condition is violated? For instance, let $k = 1/3$, $a = 1.5$, and $C = 1$. Find the maximum ρ for which the softness condition is still satisfied. Solving the equation

$$\rho_m(2+\rho_m) = 3,$$

we obtain $\rho_m = 1$. Thus, if the price offered by the buyer does not exceed $P = (1 + \rho_m)C = 2$, then the tax system is soft. The producer produces goods with a maximum cost and the residual profit is

$$\Pi_0(\rho) = (1 - {}^1\!/_3\,\rho)\rho.$$

For $\rho = 1$, we have $\Pi_0(1) = 2/3$. If the price exceeds $P = 2$, the softness condition is violated. In this case, the producer is better off maintaining the optimal profit rate $\rho^* = 1$ and increasing the cost with the increase of selling price by the formula

$$\tilde{C} = \frac{P}{1+\rho^*} = \frac{P}{2}.$$

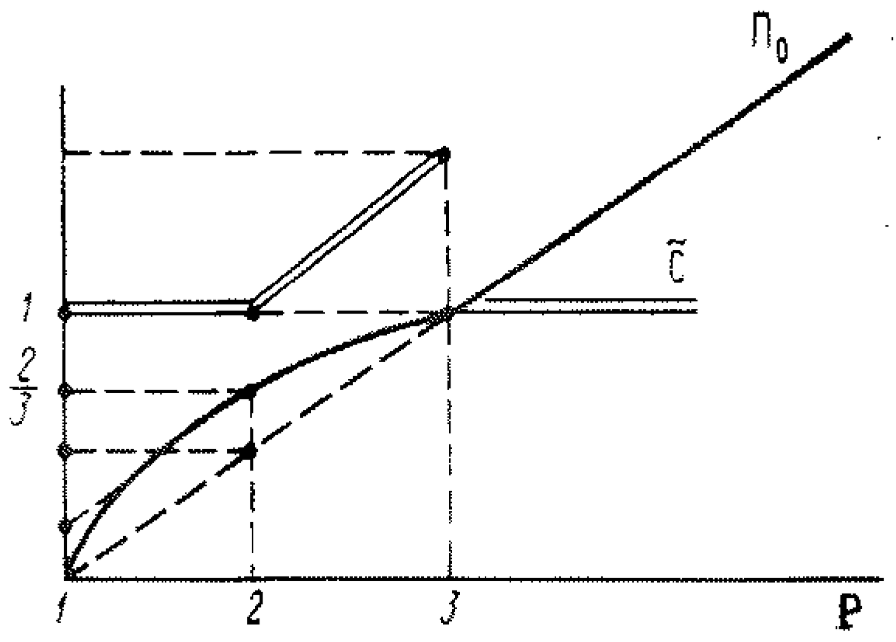

Fig. 1

The residual profit in this case is $\Pi_0 = P/3$ and it increases linearly with the increase of price. However, when $P > (1 + a)C = 2.5$, the producer has another opportunity. If the producer again shifts to production of goods with a minimum cost $C = 1$, the profit rate will exceed $a = 1.5$ and the tax rate will be constant $\mu = ka = 0.5$. In this case,

$$\Pi_0 = (1 - \mu)\rho C = (1 - \mu)(P - 1) = 0{,}5\,(P - 1).$$

If $0.5(P - 1) > P/3$, then the optimal strategy for the producer is to shift to production of minimum-cost goods. From the equation

$$P/3 = (P - 1)/2$$

we obtain the transition price $P^* = 3$.

Thus, for $P > 3$ the tax system again functions as a soft tax system. The variation of residual profit (solid curve) and cost (hollow line) as a function of selling price is shown in Fig. 1. Note the abrupt change of the economic price policy near $P = 3$. Indeed, if P is mainly expected to be lower than 3 (but higher than 2), the producer should follow the wasteful cost strategy, maintaining production costs at the level of 0.5 P. If P is expected to rise above 3, then it is better to reduce the cost to the minimum $C = 1$. This uncertainty obviously has a negative impact on the normal operation of the firm.

3. PIECEWISE-CONSTANT TAX SYSTEM

A piecewise-constant tax scale is quite common. Some profit rates are fixed $0 = \rho_0 < \rho_1 < \rho_2 < \ldots < \rho_k$ and a tax rate μ_i is established for each half-open interval $[\rho_{i-1}, \rho_i)$, $i = 1,\ldots,k$. The profit is taxed at this tax rate when the profit rate exceeds ρ_{i-1}. The tax rates are such that $\mu_1 < \mu_2 < \ldots < \mu_k$. Let us derive a softness condition for this tax system. We start with the case $k = 1$. The tax rate is μ_1 up to the profit rate ρ_1; when the profit rate exceeds this threshold level, the tax is given by

$$T = \mu_1\rho_1 C + \mu_2(\rho - \rho_1)C.$$

Suppose that with a minimum cost C we have $\rho > \rho_1$. The producer is faced with two alternatives. The first alternative is not to exceed ρ_1 and to pay tax at the minimum rate μ_1. To this end, the cost should be raised to the level $C = P/(1 + \rho_1) = (1 + \rho)C/(1 + \rho_1)$. The residual profit in this case is

$$\Pi_0^1 = (1 - \mu_1)\,\frac{\rho_1}{1 + \rho_1}\,P.$$

The second alternative is to produce at a minimum cost C, paying tax at the higher rate. The residual profit in this case is

$$\Pi_0^2 = (1 - \mu_1)\rho_1 C + (1 - \mu_2)(\rho - \rho_1)C,$$

where $\rho = (P - C)/C$.

With a soft tax system we should have $\Pi_0^2 \geq \Pi_0^1$. Let us investigate the behavior of Π_0^2 as a function of ρ, noting that $C = P/(1 + \rho)$. Differentiating with respect to ρ, we obtain

$$\frac{d\Pi_0^2}{d\rho} = \frac{(1 - \mu_2)(1 - \rho_1) - (1 - \mu_1)\rho_1}{(1 + \rho)^2}.$$

If $(1 - \mu_2)(1 + \rho_1) - (1 - \mu_1)\rho_1 \geq 0$, then Π_0^2 is an increasing function of ρ and the inequality $\Pi_0^2 \geq \Pi_0^1$ always holds. If $(1 - \mu_2)(1 + \rho_1) - (1 - \mu_1)\rho_1 < 0$, then Π_0^2 is a decreasing function of ρ and the inequality does not hold, because for $\rho = \rho_1$ we have $\Pi_0^2 = \Pi_0^1$. Thus, a necessary and sufficient condition of softness of this tax system is

$$\frac{1-\mu_2}{1-\mu_1} \geqslant \frac{\rho_1}{1+\rho_1}$$

or

$$\mu_2 \leqslant \frac{1+\mu_1\rho_1}{1+\rho_1}.$$

For instance, if $\rho_1 = 0.3$ and $\mu_1 = 0.1$, then

$$\mu_2 \leqslant 0{,}8.$$

Extending this argument to the general case, we obtain that the increase of Π_0^j with ρ is a necessary and sufficient condition for a soft tax system. In order to express this condition in an explicit form, let us write the expression for Π_0^j. Noting that $C = P/(1 + \rho)$, we obtain

$$\Pi_0^j = [\sum_{i=1}^{j-1} (1-\mu_i)(\rho_i - \rho_{i-1}) + (1-\mu_j)(\rho - \rho_{j-1})](1+\rho)^{-1}\mathrm{P}.$$

To simplify the algebra, we denote $A_{j-1} = \Sigma_{i=1}^{j-1} (1 - \mu_i)(\rho_i - \rho_{i-1})$:

$$\Pi_0^j = \frac{A_{j-1} + (1-\mu_j)(\rho - \rho_{j-1})}{1+\rho}\mathrm{P}.$$

Differentiating with respect to ρ, we obtain

$$\frac{d\Pi_0^j}{d\rho} = \frac{(1-\mu_j)(1+\rho_{j-1}) - A_{j-1}}{(1+\rho)^2}\mathrm{P},$$

The softness condition takes the form

$$(1-\mu_j)(1+\rho_{j-1}) \geqslant A_{j-1} \tag{8}$$

or

$$(1-\mu_j)(1+\rho_{j-1}) \geqslant \sum_{i=1}^{j-1} (1-\mu_i)(\rho_i - \rho_{i-1}). \tag{9}$$

Note that if $\mu_1 < \mu_2 < \ldots < \mu_k$, then condition (8) should be a strict inequality for $j < k$. Indeed, assume that for $s < k$ we have

$$(1-\mu_s)(1+\rho_{s-1}) = \sum_{i=1}^{s-1} (1-\mu_i)(\rho_i - \rho_{i-1}).$$

Take $j = s + 1$:

$$(1-\mu_{s+1})(1+\rho_s) \geqslant \sum_{i=1}^{s} (1-\mu_i)(\rho_i - \rho_{i-1}) = \sum_{i=1}^{s-1} (1-\mu_i)(\rho_i - \rho_{i-1}) +$$
$$+ (1-\mu_s)(\rho_s - \rho_{s-1}) = (1-\mu_s)(1+\rho_{s-1}) + (1-\mu_s)(\rho_s - \rho_{s-1}) = (1-\mu_s)(1+\rho_s).$$

Hence it follows that $\mu_{s+1} \leq \mu_s$, a contradiction.

Example 2. Let $\rho_1 = 0.3$, $\rho_2 = 0.4$, $\rho_3 = 0.5$, $\rho_4 = 0.6$. Denote $\eta_j = 1 - \mu_j$ and write the system of inequalities defining a soft progressive tax system:

$$1 > \eta_1 > \eta_2 > \eta_3 > \eta_4 > \eta_5 > 0,$$
$$1{,}3\eta_2 \geqslant 0{,}3\eta_1,$$
$$1{,}4\eta_3 \geqslant 0{,}3\eta_1 + 0{,}1\eta_2,$$
$$1{,}5\eta_4 \geqslant 0{,}3\eta_1 + 0{,}1\eta_2 + 0{,}1\eta_3,$$
$$1{,}6\eta_5 \geqslant 0{,}3\eta_1 + 0{,}1\eta_2 + 0{,}1\eta_3 + 0{,}1\eta_4.$$

Let us construct a tax system with a uniformly increasing tax rate, i.e., $\eta_j = \eta_1 - \epsilon(j - 1)$, $j = 2, 3, 4, 5$. Substituting in the system of inequalities, we reduce it to a simple form

$$\eta_1 \geqslant 5{,}8\epsilon.$$

Take $\eta_1 = 0.7$. Then the tax system with maximum uniform increase of the tax rate has the form

$$\mu_1 = 0{,}3,\ \mu_2 = 0{,}42,\ \mu_3 = 0{,}54,\ \mu_4 = 0{,}66,\ \mu_5 = 0{,}78.$$

4. RIGID PROGRESSIVE TAX SYSTEM

In a rigid tax system, the residual profit

$$\Pi_0 = [1 - \mu(\rho)] \frac{\rho}{1+\rho} P,$$

where $\rho = (P - C)/C$, C is the minimum cost to produce a volume of goods with selling price P, has a maximum as a function of ρ at some point ρ^*. If the selling price exceeds $P^* = 1 + \rho^*$, then the firm falls within the scope of a rigid tax system. Its optimal strategy in this case is to increase the cost to $\tilde{C} = P/(1 + \rho^*) > C$, i.e., to a level that ensures the optimal profit rate ρ^*.

5. "PROFIT–PROFIT" TAX SYSTEM

In this system, profit is taxed at a rate that increases with the increase of profit, i.e., $\mu(\Pi) = \mu(P - C) = \mu(\rho C)$ is an increasing function of Π. The residual profit

$$\Pi_0 = [1 - \mu(\Pi)]\Pi = [1 - \mu(P - C)](P - C)$$

should be an increasing function of price and a decreasing function of cost. This is equivalent to a single condition: the residual profit is an increasing function of profit, $d\Pi_0/d\Pi \geq 0$, or

$$\Pi \frac{d\mu}{d\Pi} + \mu(\Pi) < 1.$$

As previously, let

$$\Pi \frac{d\mu}{d\Pi} + \mu(\Pi) = h(\Pi).$$

We will obtain the general form of soft progressive "profit–profit" tax systems by solving the differential equation

$$x \frac{d\mu}{dx} + \mu(x) = h(x),$$

where $h(x) < 1$.

The solution of this equation is

$$\mu(\Pi) = \frac{1}{\Pi} \int_0^{\Pi} h(x)dx.$$

Since the tax system is progressive $d\mu/d\Pi \geq 0$, we obtain the condition $\mu(\Pi) \leq h(\Pi)$.

6. "INCOME–INCOME-TO-LABOR COST RATIO" TAX SYSTEM

This system taxes the income before labor costs (E) at a rate that increases with the ratio of income to labor costs. Denoting the labor cost by a and the income-to-labor cost ratio by d, we obtain

$$d = E/a = (P - S)/a,$$

where S are material and material-equivalent costs. As the objective function of the firm, we take the residual (after-tax) income per ruble of labor costs:

$$x/a = (1 - \mu(d))d.$$

The general form of a soft progressive "income–income-to-labor cost ratio" tax system is derived similarly to the previous case of a "profit–profit" system:

$$\mu(d) = \frac{1}{d} \int_0^d h(x)\,dx,$$

where $h(x) < 1$.

Other alternatives are analyzed along the same lines.

Note the main conclusion: a progressive tax system (soft or rigid) encourages the increase of prices with cost reduction (if the system is soft) or the increase of both prices and costs (if the system is rigid). A soft system may lead to inflationary effects, especially in case of a monopolistic producer; a rigid system may lead to low efficiency due to the tendency to increase the costs. These shortcomings of progressive taxation can be eliminated by introducing cost-efficient tax systems, which are considered in the next section.

7. COST-EFFICIENT TAX SYSTEM FOR SCIENTIFIC INSTITUTIONS

The principles of cost-efficient taxation mechanisms have been examined in [1]. Here we consider a specific implementation of these principles. The main idea of cost-efficient tax systems is to create two money flows that are extracted from the firm. The first money flow consists of ordinary tax payments to the budget. The second money flow involves compensation of the buyer through price reduction. If we consider the total money flow extracted from the firm, then the residual income or profit varies as with a soft progressive tax system, encouraging the firm to produce a greater volume at a lower cost. However, the presence of the second flow – partial reimbursement of the purchase price to the buyers – essentially distinguishes the cost-efficient tax system from a soft progressive tax system insofar as cost reduction also reduces the product price.

We will consider two cost-efficient tax systems: "income–average income" and "profit–profit rate". The first alternative corresponds to the experimental tax systems for scientific institutions adopted by GKNT SSSR in 1989-1990. The second alternative is close to the prevailing uniform tax system and requires minimum changes.

8. "INCOME–AVERAGE INCOME" COST-EFFICIENT TAX SYSTEM

Tax is levied on the income before labor costs $E = W - S$, where W is the output and S are the material and material-equivalent costs (financing expenses are subtracted from the revenue). Denote by $d = E/N$ the average income per employee (N is the number of employeesin the firm). An average income norm d_0 has been established for firms participating in the experiment. If the actual average income d does not exceed d_0, then the profit rate is μ_0. If the average income exceeds the norm, then the average income threshold is calculated as

$$\eta = d_0 + k(d - d_0),$$

where $0 < k < 1$. The entire income in excess of the threshold is extracted to the budget and there is no additional penalty for exceeding the threshold. This policy obviously deters the firm from achieving an actual average income in excess of the threshold level. As a result, there is an incentive to return a part of the excess income to the buyer (by reducing the job or product price) or to make a contribution to charitable organizations (the experiment did not distinguish between the two courses of action). Thus, the optimal actual income after price adjustment (or after a contribution to charity) is given by

$$\eta N = [d_0 + k(d - d_0)]N = E_0 + k(E - E_0), \text{ where } E_0 = Nd_0.$$

The business income in this case is given by

$$(1 - \mu_0)\eta N = (1 - \mu_0)[E_0 + k(E - E_0)].$$

Remark. In the experiment, the excess income is not extracted and the firm simply faces a higher tax rate for the entire period:

$$\mu = \mu_0 + (1 - \mu_0)(\frac{d_a}{\eta} - 1),$$