TWO-LEVEL ACTIVE SYSTEMS

IV. THE COST OF DECENTRALIZATION OF OPERATING MECHANISMS

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UDC 62-50:65.012.2

The article deals with analysis and synthesis of performance mechanisms of a two-level active system when the center has complete information about the models of lower-level elements. Optimal performance mechanisms are obtained for different degrees of centralization. Working expressions are derived for the cost of partial and complete decentralization of planning.

1. INTRODUCTION

In [1] we described the model of a two-level system of active goal-directed elements and possible ways of organizing its operation (operating mechanisms). In [2], game-theoretic methods were applied to problems of analysis and synthesis of the operating mechanisms of a two-level system assuming that the upper-level element (the center) has incomplete information about some of the parameters of the subordinate element models and uses the counterflow data generation method to reconstruct the missing parameters. The problems formulated in [2] were investigated in [3].

Yet in some cases we may assume that the center has complete information about the models of the subordinate elements. Situations of this kind are frequently considered in publications on organization system management. In this article we investigate some problems of analysis and synthesis of performance mechanisms of a two-level active system when the center has complete information about the models of lower-level elements. Optimal performance mechanisms are obtained for various degrees of centralization. Expressions are derived for calculating the cost of partial and complete decentralization of planning.

We use the definition of the degree of centralization of performance mechanisms introduced in [4] and the classification of performance mechanisms by degree of centralization introduced in [5, 6]. A general survey of these topics sufficient for the purpose of this article will be found in [6].

Moscow. Translated from Avtomatika i Telemekhanika, No. 6, pp. 110-117, June, 1980. Original article submitted June 21, 1979.

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2. ANALYSIS AND SYNTHESIS OF OPERATING MECHANISMS WITH COMPLETE INFORMATION AT THE CENTER

If the center has complete information about the sets Y_i of the possible states y_i of every active element $i \in I$, we can naturally omit the data generation stage intended to supply the missing data on Y_i, so that the procedures generating the control parameters (control laws) no longer use information received from the active elements. Every performance period of the system is divided into two stages: the planning stage and the implementation stage.

Following the methodology of [2], we develop a game-theoretic description of a two-level active system and formulate the control problems.

The center makes the first move and communicates it to the elements. The center strategy is to choose an operating mechanism $\Sigma = \langle W, \pi \rangle$ from a feasible set G_Σ, where $W = \{W_i\}$, $W_i = f_i(\lambda, x_i, y_i)$ is the objective function of the i-th element, λ is the control (common for all elements), x_i is the plan (i.e., centrally required values of the planned components y_i^p of the state vector $y_i = (y_i^p, y_i^u)$, where y_i^u are the unplanned components), $\pi = (\lambda, x)$ is the control law ($x = \{x_i\}$).

Centrally imposed penalties on deviations of the operation y_i^p from the plan x_i may be formally expressed by writing the objective function of the element in the form $f_i(\lambda, x_i, y_i) = \mu_i(\lambda, y_i) - \eta_i(\lambda, x_i, y_i^p)$, where $\mu_i(\lambda, y_i) = f_i(\lambda, y_i, y_i)$ and $\eta_i(\lambda, x_i, y_i^p)$ is the penalty function for failing to meet the plan. We assume that $\eta_i(\lambda, x_i, y_i^p) \geq 0$, $\eta_i(\lambda, x_i, x_i) = 0$, and the functions $\mu_i(\lambda, y_i)$ attain their maxima on the sets Y_i, $i \in I$.

For a given mechanism Σ, the operation of the system may be regarded as a game of n active lower-level elements. In every operation period (i.e., in every session of the game), a given element makes only one move: It selects a particular state y_i from the set Y_i of feasible states, attempting to maximize its objective function, i.e., the strategy of the element is to choose one of the locally optimal states y_i^* which satisfies the condition

$$y_i^* \in R_i(\lambda, x) = \operatorname{Arg} \max_{y_i \in Y_i} f_i(\lambda, x_i, y_i), \tag{1}$$

where $R_i(\lambda, x)$ is the set of locally optimal states of the element.

We assume that the selected operating mechanism Σ satisfies the condition

$$R(\Sigma) = \prod_{i \in I} R_i(\lambda, x_i) \subset Y^{gl}, \tag{2}$$

where Y^{gl} is the set of global constraints on the states of the entire system, $y = \{y_i\}$, $y \in Y^{gl}$. This is the so-called implementability condition on the performance mechanism Σ.

An efficiency measure of the operating mechanism $\Sigma = \langle W, \pi \rangle$ is defined as

$$K(\Sigma) = \min_{y \in R(\Sigma)} \Phi(\lambda, x, y) / \max_{\lambda \in \Lambda, y \in Y} \Phi(\lambda, y^p, y), \tag{3}$$

where the numerator gives the assured value of the central objective function $\Phi(\lambda, x, y)$ on the solution set of the game $R(\Sigma)$, and the numerator is the maximum value of the central objective function on the set of all the feasible system sets $Y = Y^{gl} \cap \prod_{i \in I} Y_i$ and the set of controls Λ.

The analysis problem for a given operating mechanism Σ calls for investigating the properties of the solutions of the game $R(\Sigma)$ and determining the efficiency measure $K(\Sigma)$; the synthesis problem for operating mechanisms is concerned with constructing mechanisms which satisfy the given constraints ($\Sigma \in G$); the optimal synthesis problem seeks to find $\Sigma^* \in G$ with the maximum efficiency measure

$$K(\Sigma^*) = \max_{\Sigma \in G} K(\Sigma), \tag{4}$$

where G is the given set of operating mechanisms.

3. MECHANISMS WITH COMPLETELY AND PARTIALLY CENTRALIZED PLANNING

Completely Centralized Planning. Mechanisms with completely centralized planning are characterized by the following properties. First, all the performance components are planned, $y_i^p = y_i$, $i \in I$. Second, by generating the objective functions of the elements $W = \{W_i\}$, the center ensures that the actual operation y coincides with the plan x (this can be achieved, say, by introducing severe penalties for deviations from the plan into the objective functions of the elements [1]).

The optimal solution of the synthesis problem (4) on the set of performance mechanisms with completely centralized planning is easily obtained as the operating mechanism with the optimal planning law [1]. Moreover, this operating mechanism is absolutely optimal, i.e., $K(\Sigma^*) = 1$.

Indeed,

$$K(\Sigma^*) = \frac{\min\limits_{y \in R(\Sigma^*)} \Phi(\lambda_{opt}, x_{opt}, y)}{\max\limits_{\lambda \in \Lambda, y \in Y} \Phi(\lambda, y, y)} = \frac{\max\limits_{\lambda \in \Lambda, x \in Y} (\lambda, x, x)}{\max\limits_{\lambda \in \Lambda, y \in Y} \Phi(\lambda, y, y)} = 1,$$

where x_{opt} and λ_{opt} are respectively the optimal plan and the optimal control. The first equality is based on the definition of the efficiency measure $K(\Sigma^*)$, and the second equality uses the fact that in systems with completely centralized planning $y = x$.

This result (derived here without considering the "activity" of the elements) was originally obtained by Kantorovich [7] and it supports the application of operating mechanisms with the optimal planning law under conditions of complete information at the center.

Practical utilization of such operating mechanisms is somewhat complicated because they require that the following specific conditions be met. First, all the components of the state vector of the system must be planned. However, if too may items are to be planned, the actual solution for x_{opt} may significantly delay the decision-making process based on planned data [8]. Second, the operating mechanism of a system with completely centralized planning should ensure that the elements select the states y_i which coincide in all their components with the corresponding plans x_i. However, there are cases when it is impossible to make the actual operation y coincide with the optimal plan x_{opt}, since the penalties introduced by the central authority into the objective functions of the elements are of necessity bounded.

One of the possible ways to overcome these difficulties calls for operating mechanisms with partially centralized planning.

Partially Centralized Planning. Operating mechanisms with partially centralized planning are characterized as follows: First, there is no need to plan all the components of y, and it suffices to plan only some of the components $y^p = \{y_i^p\}$; second, the operation of the planned components of y^p need not exactly coincide with the plan.

The optimal planning law cannot be directly applied to partially centralized systems, and so it should be modified to a certain extent. A suitable modification is provided by optimal planning with prediction of the system state, OPP in brief:

$$(\lambda_{opp}, x_{opp}) = \arg \max_{(\lambda, x) \in H(\Sigma)} \psi(\lambda, x), \tag{5}$$

where $\psi(\lambda, x) = \min\limits_{y \in R(\Sigma)} \Phi(\lambda, x, y)$ is a prediction of the assured value of the central objective function for given control parameters (λ, x), $H(\Sigma)$ is the set of controlled parameters which satisfy the implementability condition (2). Note that if $H(\Sigma) = \phi$, no implementable operating mechanism can be constructed for the given objective function W.

The OPP law essentially determines the control parameters (λ_{opp}, x_{opp}) that maximize the assured value of the central objective function on the solution set of the game $R(\Sigma)$. The efficiency measure $K(\Sigma_{opp})$ of the performance mechanism Σ_{opp} with the OPP law in partially centralized systems is given by

$$K(\Sigma_{opp}) = \frac{\min\limits_{y \in R(\Sigma_{opp})} \Phi(\lambda_{opp}, x_{opp}, y)}{\max\limits_{\lambda \in \Lambda, y \in Y} \Phi(\lambda, y^p, y)} = \frac{\max\limits_{(\lambda, x) \in H(\Sigma_{opp})} \min\limits_{y \in R(\Sigma_{opp})} \Phi(\lambda, x, y)}{\max\limits_{\lambda \in \Lambda, y \in Y} \Phi(\lambda, y^p, y)} \leqslant 1. \tag{6}$$

Consider the set G^W of implementable operating mechanisms with a fixed W and different control laws $\pi = (\lambda, x)$. It follows from (3), (6) that the OPP operating mechanism Σ_{opp} is an optimal solution of the synthesis problem (4) on G^W:

$$K(\Sigma_{opp}) = \max_{\Sigma \in G^W} K(\Sigma). \tag{7}$$

Note that the optimal mechanism Σ_{opp} in general does not guarantee that the elements meet their plans, although the requirement that the plans are met is an essential condition imposed on the operating mechanism.

It is thus interesting to consider those operating mechanisms that meet the plan, $y^p = x$. We call these mechanisms correct operating mechanisms.

The center has two options for constructing a correct operating mechanism: 1) by choosing an appropriate reward system W (e.g., by introducing high "penalties"); 2) by choosing control laws that only assign "rewarding" plans to the elements, namely plans that are certain to be filled under a given reward system W (the open control law [1] provides an appropriate example of this approach).

Correct operating mechanisms also may be defined as operating mechanisms in which the centrally designated control parameters satisfy the following conditions:

$$\forall i \in I,\ x_i \in S_i(\lambda) = \{x_i \mid (x_i, y_i^{u*}) \in Y_i,\ \max_{y_i \in Y_i} f_i(\lambda, x_i, y_i) = f_i(\lambda, x_i, y_i^*)\}, \tag{8}$$

where $y_i^* = (x_i, y_i^{u*})$ is a locally optimal strategy of the element i.

The set $S(\lambda) = \prod_{i \in I} S_i(\lambda)$ is called the set of consistent plans, and the control laws in which the control parameters are determined from the consistency conditions $x \in S(\lambda)$ are called consistent control laws.

Comparison of the efficiency measures (3) for correct operating mechanisms under various consistent control laws shows that the consistent control law obtained by solving the problem (5) with the additional consistency conditions (8) is optimal in the sense of criterion (3).

The operating mechanism $\Sigma_{co} \in G^W$ with consistent optimal control law is derived under the additional consistency constraints (8) imposed on the centrally designated control parameters, so that naturally it is less efficient than the operating mechanism $\Sigma_{opp} \in G^W$. We will now determine the conditions that the reward system W should satisfy in order for the correct mechanism Σ_{co} to have the same efficiency as the OPP operating mechanism, $K(\Sigma_{opp})$:

$$K(\Sigma_{co}) = K(\Sigma_{opp}). \tag{9}$$

To formulate these conditions, we introduce the following notation. Consider the set $P(\lambda)$ which is the union, over all the possible plans (for a given λ), of the sets of values of the planned components y^{p*} of locally optimal states y^*, or

$$P(\lambda) = \{y^p \mid y = (y^p, y^u) \in \bigcup_{x \in X(\lambda)} R(\Sigma)\},$$

where

$$X(\lambda) = \{x \mid (\lambda, x) \in H(\Sigma)\}, \quad R(\Sigma) = \prod_{i \in I} R_i(\lambda, x_i).$$

LEMMA 1. Let $S(\lambda_{opp}) = P(\lambda_{opp})$, then (9) holds.

The proof of this and the following lemma is given in the Appendix.

We further assume that the elements are "friendly" with the center when selecting the planned components y_i^p, i.e., if $y_i^*\ R_i(\lambda, x_i)$ and $y_i^* = (x_i, y_i^{u*})$, the i-th element selects either the state y_i^* or another state $y_i \in R_i(\lambda, x_i)$ such that $y_i = (x_i, y_i^u)$. Under these conditions, we have the following lemma.

LEMMA 2. A sufficient condition for $S(\lambda) = P(\lambda)$ is the "triangle" inequality

$$\eta_i(\lambda, x_i, y_i^p) \leqslant \eta_i(\lambda, x_i, z_i^p) + \eta_i(\lambda, z_i^p, y_i^p) \tag{10}$$

for every $i \in I$, $x \in X(\lambda)$, $y_i = (y_i^p, y_i^u) \in Y_i$, $z_i = (z_i^p, z_i^u) \in Y_i$.

Finally, combining Lemmas 1 and 2, we obtain the following theorem.

THEOREM 1. Let the penalties $\eta_i(\lambda, x_i, y_i^p)$, $i \in I$ for deviations of the performance y_i^p from the plan x_i satisfy the "triangle" inequality (10) for $\lambda = \lambda_{opp}$. Then the correct mechanism Σ_{co} is optimal on G^W, i.e., $K(\Sigma_{co}) = K(\Sigma_{opp})$.

The following fairly common penalty functions satisfy the conditions of Theorem 1:

1) linear penalties $\eta_i(\lambda, x_i, y_i^p) = \langle\alpha_i, |y_i^p - x_i|\rangle$, where $\langle\alpha_i, |y_i^p - x_i|\rangle$ is the scalar product of the nonnegative vector α_i and the vector $|y_i^p - x_i|$ whose components are the absolute deviations of the operation components y_i^p from the corresponding plan x_i;

2) "yes–no" penalties which are independent of the magnitude of deviation:

$$\eta_i(\lambda, x_i, y_i^p) = \begin{cases} 0, & \text{if } y_i^p = x_i, \\ C_i(y_i^p), & \text{if } y_i^p \neq x_i, \end{cases}$$

where $C_i(y_i^p) \geq 0$.

4. THE COST OF DECENTRALIZATION

It follows from the preceding section that by lowering the degree of centralization we may reduce the efficiency measure of the optimal operating mechanism compared to the completely centralized case, when $K(\Sigma^*) = 1$. Using the terminology of [8], we refer to this reduction in efficiency as the cost of decentralization.

Partially Decentralized Planning. The cost of decentralization in this case is

$$\Delta = 1 - K(\Sigma_{opp}). \tag{11}$$

Here $K(\Sigma_{opp})$ is given by (6).

Completely Decentralized Planning. By gradually reducing the list of planned components, we end up with $y = y^u$ and no x. The result is an operating mechanism with completely decentralized planning which is also known as the market-price mechanism [6, 9], where the central authority fixes the control λ (the prices) and does not fix any plans x. Let Λ^0, $\Lambda^0 \subset \Lambda$, be the set of all the controls λ for which all the system states $y^* = \{y_i^*\}$ obtained when the elements select the locally optimal states y_i^*, i.e., $y_i^* \in R_i(\lambda) = \operatorname{Arg}\max_{y_i \in Y_i} \mu_i(\lambda, y_i)$, satisfy the global constraints $y^* \in Y^{gl}$, or in a different notation $R(\Sigma) = \prod_{i \in I} R_i(\lambda) \subset Y^{gl}$. In the literature dealing with models of decentralized economy, the set $R(\Sigma)$ is known as the set of competitive equilibria [9].

Note that the expression for the efficiency measure (3) in this case does not contain x and y^p (these variables simply do not exist) and it is written in the form

$$K(\Sigma) = \min_{y \in R(\Sigma)} \Phi(\lambda, y) / \max_{\lambda \in \Lambda, y \in Y} \Phi(\lambda, y). \tag{12}$$

If the control λ^* is determined as the solution of the problem

$$\lambda^* = \arg\max_{\lambda \in \Lambda^0} \psi_m(\lambda), \tag{13}$$

where $\psi_m(\lambda) = \min_{y \in R(\Sigma)} \Phi(\lambda, y)$, we obtain the optimal market mechanism Σ_m with the efficiency measure

$$K(\Sigma_m) = \max_{\lambda \in \Lambda^0} \min_{y \in R(\Sigma_m)} \Phi(\lambda, y) / \max_{\lambda \in \Lambda, y \in Y} \Phi(\lambda, y). \tag{14}$$

Note that the control law (13) is an expression of the OPP principle in systems with completely decentralized planning.

The cost of decentralization is thus given by

$$\Delta = 1 - K(\Sigma_m). \tag{15}$$

5. DISCUSSION OF RESULTS

Our formulation of the analysis and synthesis problems for operating mechanisms under complete information at the center is close to the corresponding formulations in information theory of hierarchical systems [8] and in the theory of games of coalition [10–12].

The result concerning the optimality of OPP laws, although of fundamental importance, is quite obvious. If the choice of the control law is considered as the center's strategy, using the terminology of [10, 11] we conclude that the OPP law coincides with the well-known result on the center's optimal strategy in the game Γ_1.

Knowing the optimal control law under complete information we can compute the cost of decentralization as the numerical efficiency measure of the operating mechanism for a given degree of decentralization.

Important new results in our opinion are those relating to correct operating mechanisms, the construction of correct mechanisms from consistent planning laws, and the optimality conditions of correct mechanisms.

Using consistent control laws and reward systems satisfying the condition (10) for penalties, we can construct efficient operating mechanisms and ensure that the plan is fulfilled.

APPENDIX

1. Proof of Lemma 1. Let λ_{opp}, x_{opp} be the control parameters corresponding to the performance mechanism Σ_{opp}, and $y' = (y^{p'}, y^{u'})$ a state of the system from the solution set of the game $R(\Sigma_{opp})$. From the definition of the set $P(\lambda_{opp})$ it follows that $R^p(\Sigma_{opp}) \subset P(\lambda_{opp})$, where $R^p(\Sigma_{opp}) = \{y^p | y = (y^p, y^u) \in R(\Sigma_{opp})\}$. We thus have $y^{p'} \in P(\lambda_{opp})$. Finally, using the condition of the lemma, we get $y^{p'} \in S(\lambda_{opp})$ or $y^{p'} = x_{opp}$. Hence it follows that the OPP law is a consistent control law, i.e., $K(\Sigma_{opp}) \leq K(\Sigma_{co})$. Comparing this inequality with $K(\Sigma_{co}) \leq K(\Sigma_{opp})$, we obtain (9). QED.

2. Proof of Lemma 2. Clearly, if $X(\lambda) \neq \phi$, then $P(\lambda) \neq \phi$. We will show that $S(\lambda) \neq \phi$. Consider the i-th element. Let y_i^* be the vector on which the function $\mu_i(\lambda, y_i)$ attains its maximum on Y_i. Take the plan $x_i = y_i^{p*}$ and show that $x_i \in S(\lambda)$, i.e., $S(\lambda) \neq \phi$. To this end it suffices to show that $f_i(\lambda, x_i, y_i^*) = \max_{y_i \in Y_i} f_i(\lambda, x_i, y_i)$. Indeed, since $\eta_i(\lambda, x_i, y_i^{p*}) = 0$, we have $f_i(\lambda, x_i, y_i^*) = \mu_i(\lambda, y_i^*) = \max_{y_i \in Y_i} \mu_i(\lambda, y_i) \leq \max_{y_i \in Y_i} (\mu_i(\lambda, y_i) - \eta_i(\lambda, x_i, y_i^p)) = \max_{y_i \in Y_i} f_i(\lambda, x_i, y_i)$. Since i is arbitrary, we get $S(\lambda) \neq \phi$.

Now, since certainly $S(\lambda) \subset P(\lambda)$, it suffices to show that $P(\lambda) \subset S(\lambda)$. Let $v_i(\lambda, y_i^p) = \max_{y_i^u} \mu_i(\lambda, y_i)$, where $y_i^u \in \{y_i^u | y_i = (y_i^p, y_i^u) \in Y_i\}$, and assume the contrary, i.e., $P(\lambda) \not\subset S(\lambda)$.

This means that at least one element j for some plan x, $x \in P(\lambda) \setminus S(\lambda)$ selects a locally optimal state y_j^* such that $y_j^{p*} \neq x_j$. From the condition of selection of the locally optimal state y_j^* by the element j we may write

$$v_j(\lambda, y_j^{p*}) - \eta_j(\lambda, x_j, y_j^{p*}) > v_j(\lambda, x_j).$$

Here a strong inequality is written since we assume a "friendly" element. On the other hand, since $x \in P(\lambda)$ and from the definition of $P(\lambda)$ there is a plan x' for which the elements select a locally optimal state $y = (y^p, y^u) = (x, y^u)$, i.e., we have the inequality

$$v_j(\lambda, x_j) - \eta_j(\lambda, x_j', x_j) \geq v_j(\lambda, y_j^{p^*}) - \eta_j(\lambda, x_j', y_j^{p^*}).$$

Combining the two inequalities, we get

$$\eta_j(\lambda, x_j', y_j^{p*}) > \eta_j(\lambda, x_j', x_j) + \eta_j(\lambda, x_j, y_j^{p*}),$$

which contradicts the triangle inequality. Thus the assumption $P(\lambda) \not\subset S(\lambda)$ is false. QED.

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OPTIMAL CONTROL PROBLEM WITH DISCRETE VARIABLES

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UDC 62-505:622.323

An optimal control problem arising in operation of multilayer oilfields is considered, in which the oil recovery and flooding wells are redistributed in such a way as to maximize the yield of the entire field over a specific operating period. An effective algorithm is devised for solving the problem, which amounts to solving a linear programming problem in functional space and a discrete programming problem. The article is in essence a continuation of [1].

1. The Problem

Let Ω be a domain of three-dimensional space R^3, bounded by an external contour Γ_0 and interior contours Γ_i, $i \in S$, corresponding to the contours of the operational and water-injection wells; S is the set of well numbers; x is a point of space R^3 with coordinates (x_1, x_2, x_3); $b_l(x)$ is the distribution of the layer permeability along the l-th coordinate, $l = 1, 2, 3$; μ is the liquid viscosity; (t_0, t_N) is a given time interval; d is the layer porosity; and β_f, β_m are the coefficients of compressibility of the fluid and medium.

In the porous flow of a homogeneous compressible fluid, the variation of the pressure p(x, t) over volume Ω and in time satisfies the parabolic equation [2]

$$\sum_{l=1}^{3} \frac{\partial}{\partial x_l} \frac{b_l(x)}{\mu} \cdot \frac{\partial p(x,t)}{\partial x_l} = (d\beta_f + \beta_m) \frac{\partial p(x,t)}{\partial t}, \tag{1}$$

$$x \in \Omega,\ t \in (t_0, t_N)$$

under the following boundary conditions.

On the exterior boundary Γ_0, the pressure is assumed to be given:

$$p(x,t) = p_k(x,t),\ x \in \Gamma_0,\ t \in (t_0, t_N). \tag{2}$$

If a part Γ_0' of the exterior boundary of the layer is impermeable, then the boundary conditions have the form

$$\frac{\partial p}{\partial \nu}(x,t) = 0, \quad x \in \Gamma_0', \quad t \in (t_0, t_N),$$
$$p(x,t) = p_k(x,t), \quad x \in \Gamma_0 \backslash \Gamma_0', \quad t \in (t_0, t_N). \tag{2'}$$

The interior boundaries Γ_i consist of M_i nonintersecting parts Γ_{ij}, corresponding to different layers $\Gamma_i = \bigcup_{j \in M_i} \Gamma_{ij}$. On each part Γ_{ij}, we can assume, since they are small, that the pressure is independent of the space variable:

Moscow. Translated from Avtomatika i Telemekhanika, No. 6, pp. 118-128, June, 1980. Original article submitted January 29, 1979.

0005-1179/80/4106-0835$07.50 © 1980 Plenum Publishing Corporation

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