TWO-LEVEL ACTIVE SYSTEMS. II. ANALYSIS AND SYNTHESIS OF OPERATING MECHANISMS

V. N. Burkov and V. V. Kondrat'ev

UDC 65.012.122.2

The article presents formal statements of analysis and synthesis problems for the operating mechanisms of two-level active systems using the two-way method of data formulation [1]. To formalize the statements of the problems, the authors employ the theory of games with nonconflicting interests [2, 3] and the notions of equilibrium stability for many purposeful elements [4, 5].

1. Introduction

In [1] the authors described a model of a two-level active system (AS) and its operating mechanism. The approach elaborated in [1] can be employed in simulating economic mechanisms of hierarchical organizational systems (planning procedures, including setting of prices and standards, etc.; data-generation procedures for planning; systems for stimulating, centralizing, and decentralizing control). In this paper we offer formal statements of the analysis and synthesis problems for operating mechanisms of two-level active systems with the two-way method of data generation [1]. Operation of AS is regarded as a game with nonconflicting interests [2, 3], in which the participants are a center (C) and active elements (AE). The center has the first move; it involves choosing the AS operating mechanism. After C's move, a game among AE is played out. The strategies of the elements in the game involve communication of information to C and choice of state. Operating periods ("sets" of the game) can be repeated with the AS operating mechanism remaining unchanged. The analysis problem for the operating mechanism involves analyzing the decisions of the AE game for a specified operating mechanism, while the synthesis problem involves the determination of the AS operating mechanism that maximizes the efficiency criterion of the center defined on the set of solutions of the AE game. To formalize the notion of AE game solution we will draw up the theory of games with nonconflicting interests [2, 3] and the notion of stability of the dynamics of team behavior [4, 5].

In what follows we will assume everywhere that the minimum (or maximum) of the functions investigated is attained, and that the set of minima (or maxima) is compact and nonempty.

2. Game Decisions in One Class of Games

Consider an (n+1)-st-player game whose target functions are $W_i = f_i(z_0, z_1, \ldots, z_n)$, $i = 0, 1, 2, \ldots, n$. Here is the player number and z_i is the strategy of the player with number i. The sequence of moves is as follows. Player with number $i = 0$ has the right to one move (he selects strategy z_0 from the set of permissible strategies U_0; $z_0 \in U_0$), makes this move first, and communicates it to the remaining players. Then players with numbers $i \in I = \{i \mid i = 1, 2, \ldots, n\}$ choose their strategies z_i. If player $i = 0$ calculates and does indeed have information about the strategies of the remaining players, he can set up his strategy as a function of the strategies of the others $z_0 = \tilde{z}_0(z_1, z_2, \ldots, z_n)$ (game Γ_s, $s \geq 2$, in the terminology of [2, 3]). If player $i = 0$ does not have the requisite information, his strategy is independent of those of the others (game Γ_1). Each player with number $i \in I$, by substituting the strategy of player $i = 0$, $z = \tilde{z}_0(z_1, z_2, \ldots, z_n)$ into his own target function $W_i = f_i(z_0, z_1, \ldots, z_n) = f_i(\tilde{z}_0(z_1, z_2, \ldots, z_n), z_1, \ldots, z_n) = \tilde{f}_i(z_1, z_2, \ldots, z_n)$, can represent it as a function of strategies $z = (z_1, z_2, \ldots, z_n)$ of players with numbers $i \in I$. Consequently, after player $i = 0$ has moved and communicated the move to the others, we can consider an n-player game with target functions $W_i = \tilde{f}_i(z_1, z_2, \ldots, z_n)$ $i \in I$. Henceforth in this section we will everywhere consider the game of players with numbers $i \in I$ that arises after player $i = 0$ has moved.

Moscow. Translated from Avtomatika i Telemekhanika, No. 7, pp. 62-70, July, 1977. Original article submitted May 6, 1976.

We will consider games in which players with numbers $i \in I$ are entitled to two moves. The strategy of the first move of player i will be denoted by u_i, the strategy of the second move by v_i; thus $z_i = (u_i, v_i)$.† For player i the first move involves choosing the first-move strategy u_i from the set of permissible first-move strategies $U_i: u_i \in U_i$. The values of the first-move strategies $u = (u_1, u_2, ..., u_n)$ chosen by the players (or the aggregates of these strategies $\varkappa_i(u)$, $i \in I$, that determine the way in which the target functions of the players depend on the first-move strategies) are communicated to all players, after which players with numbers $i \in I$ make the second move, involving choice of second-move strategy v_i from the set of permissible second-move strategies $V_i(u)$; we will assume that set $V_i(u)$ depends on the first-move strategies u chosen by the players $v_i \in V_i(u)$. We will consider a particular case of the game described in which the target function of the i-th player $\tilde{f}_i$ depends on the first-move strategies of all players $u = (u_1, u_2, ..., u_n)$ and on the players' own second-move strategy v_i: $\tilde{f}_i(u, v_i)$. The players' choice of first- and second-move strategies involves a variable degree of informedness. In choosing first-move strategy $\hat{u}_i$, the i-th player may not know the first-move strategies $\hat{u}_j$, $j \neq i$ of the others' or his own second-move strategy $\hat{v}_i$. The i-th player chooses second-move strategy $\hat{v}_i$ under conditions of complete informedness and hence this can be done in an optimal fashion:

$$\tilde{f}_i(\hat{u}, \hat{v}_i) = \max_{v_i \in V_i(\hat{u})} \tilde{f}_i(\hat{u}, v_i) = \varphi_i(\hat{u}). \tag{1}$$

This makes it possible for the i-th player to predict the value of v_i from condition (1) in choosing the first-move strategy. The target function of the i-th player $\varphi_i(u)$ as "contracted" with allowance for the second-move strategy choice rule can be called the efficiency criterion of the i-th player with allowance for the second-move strategy choice rule (1).

The i-th player may choose first-move strategy u_i under conditions of indeterminacy. The principle of generating a rational strategy choice for the i-th player under conditions of indeterminacy [3] involves reduction by the player of a criterion φ_i to criterion φ_i^* that depends only on parameters known to him and on his own strategy, after which, by optimization of φ_i^* with respect to his own variables, he determines the strategy to be chosen in the game. The principle by which the i-th player chooses rational first-move strategies can be conveniently represented by the following diagram:

$$\varphi_i(u) \overset{\Pi_i}{\Rightarrow} \varphi_i^*(u_i, \beta_{\Pi_i}(u(i))) \underset{u_i \in U_i}{\longrightarrow} \max, \tag{2}$$

where $\overset{\Pi_i}{\Rightarrow}$ indicates a changeover from criterion φ_i to φ_i^* (rule for eliminating indeterminacy Π_i); $\beta_{\Pi_i}(u(i))$ are parameters known to the i-th player that form part of his efficiency criterion after rule for eliminating of indeterminacy Π_i has been employed; Π_i. $\underset{u_i \in U_i}{\longrightarrow} \max$ means that the i-th player chooses strategy u_i in attempting to maximize φ_i^* with respect to u_i on set U_i‡.

The situation $\hat{z} = (\hat{u}, \hat{v})$ is called a solution of the game if

$$\forall i: \varphi_i^*(\hat{u}_i, \beta_{\Pi_i}(u(i))) = \max_{u_i \in U_i} \varphi_i^*(u_i, \beta_{\Pi_i}(u(i))), \tag{3}$$

$$\forall i: \tilde{f}_i(\hat{u}, \hat{v}_i) = \max_{v_i \in V_i(\hat{u})} \tilde{f}_i(\hat{u}, v_i). \tag{4}$$

If several repeating "sets" are played, the players can also use information about past sets to eliminate indeterminacy. If this occurs (we will assume that only information from the preceding set is employed), the scheme by which the i-th player chooses a rational first-move strategy becomes

$$\varphi_i(u^k) \overset{\Pi_i}{\Rightarrow} \varphi_i^*(u_i^k, \beta_{\Pi_i}(u^{k-1}, u^k(i))) \underset{u_i^k \in U_i}{\longrightarrow} \max. \tag{5}$$

The definitions of solution z^k of the k-th set of the game are exact repetitions of those given above on the basis of conditions (3) and (4), the only difference being that (3) should be replaced by the condition

$$\forall i: \varphi_i^*(\hat{u}_i^k, \beta_{\Pi_i}(u^{k-1}, u^k(i))) = \max_{u_i^k \in U_i} \varphi_i^*(u_i^k, \beta_{\Pi_i}(u^{k-1}, u^k(i))). \tag{6}$$

†The game-theory literature also employs the terms "choice of first-move alternative," "choice of second-move alternative."

‡In the more general case, on some set $U_i' \subset U_i$.

Assume that in a game with repeating sets the player i = 0 entitled to the first move does not change his strategy from one set to the next. The situation $\hat{z} = (\hat{u}, \hat{v})$ is called a stable solution of the game in question† if

$$\forall i: \varphi_i^*(\hat{u}_i, \beta_{\Pi_i}(\hat{u}, \hat{u}(i))) = \max_{u_i \in U_i} \varphi_i^*(u_i, \beta_{\Pi_i}(\hat{u}, \hat{u}(i))), \tag{7}$$

$$\forall i: f_i(\hat{u}, \hat{v}_i) = \max_{v_i \in V_i(\hat{u})} f_i(\hat{u}, v_i). \tag{8}$$

Assume that in the initial set (k = 1) the players have chosen arbitrary permissible first-move strategies $u^1 \in U$, and assume that $\{\hat{u}^q, q \geq 2\}$ is some arbitrary sequence of first-move strategies that appear in the solution $\hat{z}^q\{\hat{u}^q, \hat{v}^q\}$ in sets with numbers q = 2, 3, ... The set of stable solutions R of the game will be called the set of globally stable solutions of the game if any sequence $\{\hat{u}^q\}$ converges to some stable first-move strategy $\hat{u}$ from R, i.e.,

$$\lim_{q \to \infty} \hat{u}^q = \hat{u}, \text{ where } z^* = (\hat{u}, \hat{v}) \in R. \tag{9}$$

Remark 1. In what follows, in speaking of solutions (or set of solutions of a game in the sense of one of the above definitions), we will frequently speak only about first-move strategies (or set of first-move strategies), bearing in mind that, for specified first-move strategies, the second-move strategies are determined by optimum selection rule (1).

3. Analysis and Synthesis of Operating Mechanisms of Two-Level Active Systems with Two-Way Method of Data Generation

1. Operation of AS with the Two-Way Method. Assume that we are given a model of an AS and its operating mechanism $\Sigma = \langle W, B, \pi \rangle$, i.e., we are given the AE target functions $W = \{W_i, i \in I\}$, constraints on the set of possible AE realizations $B = \{B_i, i \in I\}$, the two-way method of data generation $s = \{s_i, s \in I\}$, and the control law $\pi(s) = (x(s), \lambda(s))$. We will also assume that the operating mechanism Σ satisfies the condition of independence of the system elements [1.3]‡. Each operating period of the AS includes three stages: data generation by the two-way method, planning, and implementation of plan [1].

2. Game Description of AS Operation with the Two-Way Method. A. Game "Set" and Participants. In the game interpretation, a separate operating period is regarded as a set whose participants are C and n AE, for a total of (n + 1) participants.

B. Strategy and First-Move Capability of C (player i = 0). C is entitled to one move (on the permissible set G_Σ he chooses operating mechanism $\Sigma = \langle W, B, \pi \rangle$ with two-way method of data generation), makes his move first, and communicates it to the AE. If we are dealing with an AS with repeating operating periods, we will assume that C's strategies remain unaltered from one period to the next.

C. Strategies, Efficiency Criterion, and Selection Rule for Rational AE Strategies (players with numbers $i \in I$). The target function of the i-th AE in the k-th set is $W_i^k = f_i(\lambda(s^k), x_i(s^k), y_i^k)$.** Since the control law $\pi(s)$, $= (x(s), \lambda(s))$ is known to the AE, the i-th AE can represent his target function in the form $W_i^k = f_i(\lambda(s^k), x_i(s^k), y_i^k)$. It depends on the aggregates of data, or estimates, of all AE: plan $x_i(s^k)$ and control $\lambda(s^k)$, and also on the realization y_i^k of the AE. In each operating period the i-th AE makes two moves: The first-move strategy is the estimate $s_i^k \in \Omega_i$, communicated to C in the stage of data (estimate) generation, while the second-move

†Here we can draw a certain analogy with the notion of equilibrium of a dynamic system. So as not to confuse our concept with the established notion in game theory of equilibrium in the sense of Nash, it seemed desirable to employ the term "stable" solution of a game instead of "equilibrium" solution. An additional argument in favor of this is the subsequent requirement of global stability of the set of stable solutions of the game.
‡Here and henceforth, the first number indicates an article number in the bibliography, while the second number is the number of the formula in that article that is referred to.
**In the two-way method of data generation, estimates and realizations of AE of the k-th operating period have no effect on control λ^q or plan x^q in subsequent operating periods, and therefore there is no need to allow for the "far-sightedness effect" [1].

Assume that in a game with repeating sets the player $i = 0$ entitled to the first move does not change his strategy from one set to the next. The situation $\hat{z} = (\hat{u}, \hat{v})$ is called a stable solution of the game in question† if

$$\forall i:\ \varphi_i^*(\hat{u}_i, \beta_{\pi_i}(\hat{u}, \hat{u}(i))) = \max_{u_i \in U_i} \varphi_i^*(u_i, \beta_{\pi_i}(\hat{u}, \hat{u}(i))), \tag{7}$$

$$\forall i: f_i(\hat{u}, \hat{v}_i) = \max_{v_i \in V_i(\hat{u})} f_i(\hat{u}, v_i). \tag{8}$$

Assume that in the initial set ($k = 1$) the players have chosen arbitrary permissible first-move strategies $u^1 \in U$, and assume that $\{\hat{u}^q, q \geq 2\}$ is some arbitrary sequence of first-move strategies that appear in the solution $\hat{z}^q\{\hat{u}^q, \hat{v}^q\}$ in sets with numbers $q = 2, 3, \ldots$ The set of stable solutions R of the game will be called the set of globally stable solutions of the game if any sequence $\{\hat{u}^q\}$ converges to some stable first-move strategy $\hat{u}$ from R, i.e.,

$$\lim_{q \to \infty} \hat{u}^q = \hat{u}, \text{ where } z^* = (\hat{u}, \hat{v}) \in R. \tag{9}$$

Remark 1. In what follows, in speaking of solutions (or set of solutions of a game in the sense of one of the above definitions), we will frequently speak only about first-move strategies (or set of first-move strategies), bearing in mind that, for specified first-move strategies, the second-move strategies are determined by optimum selection rule (1).

3. Analysis and Synthesis of Operating Mechanisms of Two-Level Active Systems with Two-Way Method of Data Generation

1. Operation of AS with the Two-Way Method. Assume that we are given a model of an AS and its operating mechanism $\Sigma = \langle W, B, \pi \rangle$, i.e., we are given the AE target functions $W = \{W_i, i \in I\}$, constraints on the set of possible AE realizations $B = \{B_i, i \in I\}$, the two-way method of data generation $s = \{s_i, s \in I\}$, and the control law $\pi(s) = (x(s), \lambda(s))$. We will also assume that the operating mechanism Σ satisfies the condition of independence of the system elements [1.3]‡. Each operating period of the AS includes three stages: data generation by the two-way method, planning, and implementation of plan [1].

2. Game Description of AS Operation with the Two-Way Method. A. Game "Set" and Participants. In the game interpretation, a separate operating period is regarded as a set whose participants are C and n AE, for a total of $(n+1)$ participants.

B. Strategy and First-Move Capability of C (player $i = 0$). C is entitled to one move (on the permissible set G_Σ he chooses operating mechanism $\Sigma = \langle W, B, \pi \rangle$ with two-way method of data generation), makes his move first, and communicates it to the AE. If we are dealing with an AS with repeating operating periods, we will assume that C's strategies remain unaltered from one period to the next.

C. Strategies, Efficiency Criterion, and Selection Rule for Rational AE Strategies (players with numbers $i \in I$). The target function of the i-th AE in the k-th set is $W_i^k = f_i(\lambda(s^k), x_i(s^k), y_i^k)$.** Since the control law $\pi(s)$, $= (x(s), \lambda(s))$ is known to the AE, the i-th AE can represent his target function in the form $W_i^k = f_i(\lambda(s^k), x_i(s^k), y_i^k)$. It depends on the aggregates of data, or estimates, of all AE: plan $x_i(s^k)$ and control $\lambda(s^k)$, and also on the realization y_i^k of the AE. In each operating period the i-th AE makes two moves: The first-move strategy is the estimate $s_i^k \in \Omega_i$, communicated to C in the stage of data (estimate) generation, while the second-move

†Here we can draw a certain analogy with the notion of equilibrium of a dynamic system. So as not to confuse our concept with the established notion in game theory of equilibrium in the sense of Nash, it seemed desirable to employ the term "stable" solution of a game instead of "equilibrium" solution. An additional argument in favor of this is the subsequent requirement of global stability of the set of stable solutions of the game.
‡Here and henceforth, the first number indicates an article number in the bibliography, while the second number is the number of the formula in that article that is referred to.
**In the two-way method of data generation, estimates and realizations of AE of the k-th operating period have no effect on control λ^q or plan x^q in subsequent operating periods, and therefore there is no need to allow for the "far-sightedness effect" [1].

strategy is the realization $y_i^k \in B_i(x_i(s^k), r_i)$, chosen in this stage. It can be seen that the operation of the AS can be regarded as a game with nonconflicting interests as described in §2. Solutions of the AE game will be understood to be globally stable equilibrium situations $(\hat{s}, \hat{y})$, or, for brevity, globally stable equilibrium situations $\hat{s}$.

D. Hypothesis of Informedness and Efficiency Criterion for C. The efficiency of the k-th operating period of the AS is determined by the value of target function $\Phi(\lambda(s^k, x(s^k), y^k)$ that is attained in this period. If C has no information about future AE strategies $s_i^k \in \Omega_i$ and $y_i^k \in B_i(x_i(s^k), r_i)$ and parameters r_i, $i \in I$, it can determine the guaranteed value of AS operating efficiency: $\min_{r\in\Omega}\min_{s\in\Omega}\min_{y\in B(x(s),r)} \Phi(\lambda(s), x(s), y)$. The fact that C is entitled to the first move and knows the principles for choosing estimates s_i^k (5) and realizations y_i^k (1) for all AE ($i \in I$) permits I to determine set R of solutions of the AE game and to set up the guaranteed value of the AS target function, not on the set of all permissible AE strategies but only on the set of solutions of the AE game (principle of guaranteed result for player who makes his move first and has information about the selection principles for rational strategies on the part of the remaining participants [2, 3]). The set of solutions of the AE game can be written as $R = R_{(s)} \times \bigcup_{s(r)\in R_{(s)}} R_{(y)}(\hat{s}(r))$. Here $R_{(s)}$ is the set of first-move strategies, or estimates $\hat{s}(r)$ that are part of the set of solutions of the AE game; $R_{(y)}(\hat{s}(r))$ is the set of second-move strategies, or realizations $\hat{y}(r)$ that are part of the set of solutions of the AE game for first-move strategies $\hat{s}(r)$. Note that $R_{(y)}(s(r)) = \{\operatorname{Arg\,max} f_i(\lambda(s), x_i(s), y_i),\ i \in I\}$. The symbol $\operatorname{Arg\,max}_{z\in Z} f(z)$ denotes the set of all z^* such that $f(z^*) = \max_{z\in Z} f(z)$.

Let us consider the guaranteed value of the AS target function on set R of solutions of the AE game:

$$\tilde{\Psi}(r) = \min_{(\hat{s}(r),\ \hat{y}(r))\in R} \Phi(\lambda(\hat{s}(r)), x(\hat{s}(r)), \hat{y}(r)) = \min_{\hat{s}(r)\in R_{(s)}} \min_{\substack{\hat{y}(r)\in\{\operatorname{Arg\,max} f_i(\lambda(\hat{s}(r)), x_i(\hat{s}(r)), y_i),\ i\in I\}\\ y_i\in B_i(x_i(\hat{s}(r)), r_i)}} \Phi(\lambda(\hat{s}(r)), x(\hat{s}(r)), \tag{10}$$

$$\hat{y}(r)) = \min_{\hat{s}(r)\in R_{(s)}} \Psi(\lambda(\hat{s}(r)), x(\hat{s}(r)), r).$$

Here $\Psi(\lambda(s), x(s), r) = \min_{\substack{y\in\{\operatorname{Arg\,max} f_i(\lambda(s), x_i(s), y_i),\ i\in I\}\\ y_i\in B_i(x_i(s), r_i)}} \Phi(\lambda(s), x(s), y)$ denotes the guaranteed value of the AS target function on the set of locally optimal realizations of all AE for a specified control $\lambda(s)$ and plan $x(s)$. The value of $\tilde{\Psi}(r)$ can be conveniently compared with the maximum attainable value of the target function $\Psi_m(r)$ for the constraints specified in the system. This is defined as follows:

$$\Psi_m(r) = \max_{\substack{\lambda\in L\\ y=x\\ y\in Y(r)}} \Phi(\lambda, x, y) = \max_{\substack{\lambda\in L\\ y\in Y(r)}} \Phi(\lambda, y, y). \tag{11}$$

Indeed, $\forall x'$, $\forall y' \in Y(r)$, $\forall \lambda' \in L$ can be written

$$\Psi_m(r) = \max_{\substack{\lambda\in L\\ y\in Y(r)}} \Phi(\lambda, y, y) \geqslant \Phi(\lambda, y', y') \geqslant \Phi(\lambda, x', y').$$

Here the first inequality on the left is obvious, while the second follows from [1.2]. As C's efficiency criterion in evaluating the operating mechanism $\Sigma = \langle W, B, \pi \rangle$

$$K_\Sigma = \min_{r\in\Omega} \frac{\tilde{\Psi}(r)}{\Psi_m(r)} = \min_{r\in\Omega} \frac{\min_{\hat{s}(r)\in R_{(s)}} \Psi(\lambda(\hat{s}(r)), x(\hat{s}(r)), r)}{\Psi_m(r)} = \min_{r\in\Omega} \frac{\min_{(\hat{s}(r),\ \hat{y}(r))\in R} \Phi(\lambda(\hat{s}(r)), x(\hat{s}(r)), \hat{y}(r))}{\Psi_m(r)}. \tag{12}$$

3. Problems of Analysis of the AS Operating Mechanism. Assume that the AS model and its operating mechanism $\Sigma = \langle W, B, \pi \rangle$ with the two-way method of data generation are specified. We need to determine the following:

a) the degree of distortion Δ of the information communicated to the AE:

$$\Delta = \max_{r\in\Omega} \max_{\hat{s}(r)\in R_{(s)}} \|\hat{s}(r) - \hat{r}\|. \tag{13}$$

This is the guaranteed value of the norm of the difference $\|s(r)-r\|$ with respect to $r\in\Omega$ and $\hat{s}(r)\in R_{(s)}$. If $\Delta = 0$ (a unique solution of the AE game $\hat{s}(r) = r$ exists), then the information communicated to the AE will be regarded as reliable. Other methods of determining the degree of reliability of the information $\hat{s}(r)$ communicated in the decision of the AE game with the real value of the corresponding parameters r, then determine the maximum guaranteed value of the comparison result with respect to $r\in\Omega$ and $s(r)\in R_{(s)}$;

b) degree of divergence between the plan and realization chosen by the AE:

$$\delta = \max_{r\in\Omega} \max_{\hat{s}(r)\in R_{(s)}} \max_{\hat{y}(r)\in R_{(y)}(\hat{s}(r))} \|x(\hat{s}(r)) - \hat{y}(r)\|. \tag{14}$$

The same reasoning that applies to Δ also applies to δ;

c) the value of C's efficiency criterion (12).

4. Synthesis of AS Operating Mechanism with the Two-Way Method of Data Generation. Assume that we are given the model and the set G_Σ of possible AS operating mechanisms using the two-way method of data generation. We are to determine on G_Σ the operating mechanisms $\Sigma^* = \langle W^*, B^*, \pi^*\rangle$ such that C's efficiency criterion (12) is maximized for Σ^*:

$$K_{\langle W^*,B^*,\pi^*\rangle} = \max_{\langle W,B,\pi\rangle\in G_\Sigma} K_{\langle W,B,\pi\rangle}. \tag{15}$$

If in the statement of problem (14), C chooses only:

a) sets $B = \{B_i\}\in G_B$, then we obtain the control problem by introducing constraints;

b) target functions $W = \{W_i\}\in G_W$, then we obtain the criterial control problem of [6];

c) control law $\pi(s)\in G_\pi$, then we obtain the problem of choice of control law of [7].

Solution Σ^* of problem (15) will be called an optimum operating mechanism. Generally speaking, because of the existing constraints on control laws G_π, AE target functions G_W, and so forth, the value of K_{Σ^*} can be less than 1, i.e., $K_{\Sigma^*}\le 1$. Solution Σ^* of problem (15) will be called an absolutely optimum operating mechanism if $K_{\Sigma^*} = 1$.

5. Operating Mechanisms with Penalties for Distortion of Information [1]. The target function of the i-th AE with penalty function $\chi_i(s_i, \theta_i)$ for distortion of information has the form $W_i = f_i^{pd}(\lambda, x_i, y_i, \chi_i(s_i, \theta_i)) = f_i^{pd}(\lambda, x_i, y_i, \lambda_i(s_i, \sigma_i(s_i, y_i)))$. Here $\theta_i = \sigma_i(s_i, y_i)$ is the generation operator for estimates θ_i based on observation of the realization of the AE. Similarly to (11), we have the following condition for the target function of AE with penalties for distortion of information:

$$f_i^{pd}(\lambda, x_i, y_i, \chi_i(s_i, \theta_i)) \begin{cases} < f_i(\lambda, x_i, y_i), & \text{if} \quad s_i \ne \theta_i, \\ = f_i(\lambda, x_i, y_i), & \text{if} \quad s_i = \theta_i, \quad i\in I. \end{cases} \tag{16}$$

The efficiency criterion for the i-th AE, taking account of the selection rule for the realizations in an AS with penalties for distortion of information, can be written in the following form (which will subsequently prove convenient):

$$\max_{y_i\in B_i(x_i(s), r_i)} f_i^{pd}(\lambda, x_i, y_i, \chi_i(s_i, \sigma_i(s_i, y_i))) = f_i^{pd}(\lambda, x_i, \hat{y}_i, \chi_i(s_i, \sigma_i(s_i, \hat{y}_i))) = f_i^{pd}(\lambda, x_i, \hat{y}_i, \chi_i(s_i, \theta_i)) = \varphi_i^{pd}(\lambda, x_i, r_i, s_i, \chi_i(s_i, \theta_i)) \tag{17}$$

where, similarly to (16), we have

$$\varphi_i^{pd}(\lambda, x_i, r_i, s_i, \chi_i(s_i, \theta_i)) \begin{cases} < \varphi_i(\lambda, x_i, r_i), & \text{if} \quad s_i \ne \theta_i, \\ = \varphi_i(\lambda, x_i, r_i), & \text{if} \quad s_i = \theta_i. \end{cases} \tag{18}$$

The statement of the analysis and synthesis problems for the operating mechanisms in AS with penalties for information distortion can be made similar to those given in 3 and 4 of this section, with appropriate modification of the principles for selecting rational first- and second-move strategies (estimates s_i and realizations y_i).

Illustrations of general formulations of control problems in AS, using simple examples, can be found in [7, 8].

APPENDIX

In relation to the game considered in §2, let us consider a number of principles for selecting rational first-move strategies and the solutions to which their use by the players leads.

Example 1. Principle of maximum guaranteed result when there is no information about the strategies of the remaining players [3]. In relation to the indeterminacy associated with his ignorance of the others' strategies, player i proceeds from the guaranteed-result principle: $\varphi_i(u) \overset{\Pi_i}{\Rightarrow} \min_{u(i)\in U(i)} \varphi_i(u) \underset{u_i\in U_i}{\longrightarrow} \max$. Here $U(i)=\prod_{j\neq i} U_j$.

Information about preceding periods is not utilized. If all players adhere to the principle of maximum guaranteed result in choosing strategies, the game solutions are situations $\hat{u}$ such that

$$\forall i: \min_{u(i)\in U(i)} \varphi_i(u_1,\ldots,u_{i-1},\hat{u}_i,u_{i+1},\ldots,u_n) = \max_{u_i\in U_i} \min_{u(i)\in U(i)} \varphi_i(u).$$

Example 2. Coalition principle of achieving Nash equilibrium [3]. All players agree to implement some Nash equilibrium situation $\hat{u}$, i.e., a situation satisfying the conditions $\forall i: \varphi_i(\hat{u}) = \max_{u_i\in U_i} \varphi_i(\hat{u}_1,\ldots,\hat{u}_{i-1},u_i,\hat{u}_{i+1},\ldots,\hat{u}_n)$.

Thus in this case each player knows the strategies chosen by the others (player i knows strategies $\hat{u}_j$, $j\neq i$). In this case the principle of choosing rational strategies for the i-th player can be represented as follows:

$$\varphi_i(u) \overset{\Pi_i}{\Rightarrow} \varphi_i(\hat{u}_1,\ldots,\hat{u}_{i-1},u_i,\hat{u}_{i+1},\ldots,\hat{u}_n) \underset{u_i\in U_i}{\longrightarrow} \max.$$

The solution of the game is a Nash equilibrium situation chosen by the players in coalition. Another version of the coalition principle of achieving Nash equilibrium was given in [3].

Example 3. "Absolutely Optimal" Strategies [3]. Strategy $\hat{u}_i$ of player i is called "absolutely optimal" if $\forall u_j\in U_j, j\neq i: \varphi_i(u_1,\ldots,u_{i-1},\hat{u}_i,u_{i+1},\ldots,u_n) = \max_{u_i\in U_i}\varphi_i(u)$. If an "absolutely optimal" strategy $\hat{u}_i$ exists for player i, a principle for choosing a rational strategy for player i can be set up without resorting to the rule for elimination of indeterminacy: $\forall u_j\in U_j, j\neq i: \varphi_i(u) \underset{u_i\in U_i}{\longrightarrow} \max$. If "absolutely optimal" strategies exist for all players, the solutions of the game are situations such that

$$\forall i: \varphi_i(u_1,\ldots,u_{i-1},\hat{u}_i,u_{i+1},\ldots,u_n) = \max_{u_i\in U_i}\varphi_i(u).$$

It is not difficult to see that any such situation satisfies the conditions of Nash equilibrium.

Example 4. Indicator behavior in games with repeating sets [4, 5, 8] and so forth. The indicator rule of choosing strategies for the i-th player in a game with repeating sets is described by the following iteration procedure:

$$\forall k: \hat{u}_i^k = \hat{u}_i^{k-1} + \gamma_i^k[\omega_i(\hat{u}^{k-1}(i)) - \hat{u}_i^{k-1}] = \hat{u}_i^{k-1} - \gamma_i^k(\tilde{u}_i^{k-1} - \hat{u}_i^{k-1}).$$

Here $\tilde{u}_i^{k-1} = \omega_i(\hat{u}^{k-1}(i))$ is the maximum point (position of the target) with respect to the intrinsic variable u_i^{k-1} of the efficiency criterion for the i-th player in the (k−1)-th set of the game, given an array of strategies $\hat{u}^{k-1}(i)$ for the other players:

$$\varphi_i(\hat{u}^{k-1}) = \max_{u_i^{k-1}\in U_i} \varphi_i(\hat{u}_i^{k-1},\ldots,\hat{u}_{i-1}^{k-1},u_i^{k-1},\hat{u}_{i-1}^{k-1},\ldots,\hat{u}_n^{k-1}).$$

As a rule, it is assumed that a target position for the player exists and is unique. The specific γ_i^k value that determines the step size $\Delta u_i^k = \hat{u}_i^k - \hat{u}_i^{k-1}$ may depend on time, the current state, or certain other (e.g., random or external) parameters. The indicator principle of choice of strategy presupposes that a player in the k-th set, who knows the strategies of all players $\hat{u}^{k-1}$ in the (k−1)-th set, can determine the way in which his efficiency criterion depends on his own variable u_i^{k-1} in some neighborhood $U_i^k \subset U_i$ of point $\hat{u}^{k-1}$, and moves towards increasing his target function, assuming that the strategies of the others remain unchanged. If for $u_i^{k-1} = \hat{u}_i^{k-1}$ the target function of player i is maximized on U_i^k, the player maintains strategy $\hat{u}_i^k = \hat{u}_i^{k-1}$. The assumptions regarding set U_i^k which limits the step size of the i-th player $\Delta u_i^k = \hat{u}_i^k - \hat{u}_i^{k-1}$, where $\hat{u}_i^k, \hat{u}_i^{k-1} \in U_i^k$ are analogous to the assumptions regarding γ_i^k. Thus the indicator principle of choice of strategy can be schematized as follows:

$$\varphi_i(u^k) \overset{\Pi_i}{\Rightarrow} \varphi_i(\hat{u}_1^{k-1}, \ldots, \hat{u}_{i-1}^{k-1}, u_i^k, \hat{u}_i^{k-1}, \ldots, \hat{u}_n^{k-1}) \underset{u_i^k \in U_i^k}{\longrightarrow} \max.$$

If $\forall k$ set U_i^k coincides with $U_i : U_i^k = U_i$, we obtain the Cournot principle of choice of rational strategies [8]. An important case is that in which the observed quantities for the player are not the vector strategies of other players but only certain aggregates of these strategies $\varkappa_i(u) = \{\varkappa_{i1}(u), \varkappa_{i2}(u), \ldots, \varkappa_{im}(u)\}$ ([5], IV). Precise determination of the position of the target is frequently quite complicated in this case. A player in this situation can orient himself toward determining the approximate position of the target $\tilde{\omega}_i(\varkappa_i(u))$. This approach is more realistic when the number of players is large, since the number of observed aggregates sufficient for indicator choice of strategies may be much smaller than the number of variables that describe the strategies of all players in the game. The presence of a large number of players may cause the strategy of an individual player to have little effect on a number of aggregates, and this can also simplify the determination of the target's position.

Stable game solutions for the case of indicator behavior of all players and a single target position for player are Nash equilibrium situations. Sufficient conditions for global stability of Nash equilibrium for indicator behavior of the players have been considered in a number of studies, [4, 5, 8] and others.

LITERATURE CITED

1. V. N. Burkov and V. V. Kondrat'ev, "Two-level active systems, I: Basic concepts and definitions," Avtom. Telemekh., No. 6, 64-72 (1977).
2. Yu. B. Germeier, Games with Nonconflicting Interests [in Russian], MGU (1972).
3. Yu. B. Germeier, I. A. Vatel', F. I. Ereshko, and A. F. Kononenko, "Games with nonconflicting interests," Proc. All-Union Seminar Workshop on Control of Large Systems [in Russian], Metsniereba, Tbilisi (1973), pp. 88-136.
4. A. V. Malishevskii, "Models of combined operation of many purposeful elements, I. II," Avtom. Telemekh., No. 11, 92 (1972); No. 12, 108 (1972).
5. V. I. Opoitsev, "Dynamics of team behavior, I-IV," Avtom. Telemekh., No. 4, 157-168 (1974); No. 6, 133-144 (1974); No. 1, 124-138 (1975); No. 7, 108-119 (1975).
6. V. N. Burkov and V. I. Opoitsev, "Meta-game approach to control of hierarchical systems," Avtom. Telemekh., No. 1, 103-114 (1974).
7. S. V. Emel'yanov and V. N. Burkov, "Control of active systems," in: Active Systems [in Russian], IAT, Moscow (1973), pp. 3-39.
8. M. Intrilligator, Mathematical Optimization and Economic Theory, Prentice-Hall, New Jersey (1971).